

Thermoelectric Pairs



The gradient of T leads not only to the heat current but also to the electric current.

If the circuit is open, the electric field arising due to the temperature gradient prevents the flow of electrons.

The proportionality coefficient between the built-up electric field and ∇T is called "thermopower":

definition

$$\vec{E} = \alpha \nabla T$$

Boltzmann equation:

$$\vec{\nabla} \cdot \vec{J} = I$$

$$f = f_0(T(r))$$

$$\vec{\nabla} f = \vec{\nabla} T \frac{\partial f_0}{\partial T} = \vec{\nabla} T \frac{\partial}{\partial T} \frac{1}{e^{\frac{\epsilon - \epsilon_F}{k_B T}} + 1}$$

$$= \vec{\nabla} T \frac{\epsilon - \epsilon_F}{k_B T^2} \frac{e^{-\frac{\epsilon - \epsilon_F}{k_B T}}}{\left(e^{\frac{\epsilon - \epsilon_F}{k_B T}} + 1 \right)^2} \quad \left. \vphantom{\frac{\epsilon - \epsilon_F}{k_B T^2}} \right\} \text{in all places replace } T \text{ by its average value}$$

$$\frac{\partial f_0}{\partial \epsilon} = -\frac{1}{k_B T} \frac{e^{-x}}{(e^x + 1)^2}; \quad x \equiv \frac{\epsilon - \epsilon_F}{k_B T}$$

$$\vec{\nabla} f = -(\epsilon - \epsilon_F) \frac{\vec{\nabla} T}{T} \frac{\partial f_0}{\partial \epsilon}$$

$$\vec{\nabla} \cdot \vec{\nabla} f = \cancel{e^{-x}/k} - (\epsilon - \epsilon_F) \frac{\vec{\nabla} \cdot \vec{\nabla} T}{T} \frac{\partial f_0}{\partial \epsilon} \equiv -e v \cdot \vec{F} \frac{\partial f_0}{\partial \epsilon}$$

$$\vec{F} \equiv \text{pseudo-electric field.} \quad \vec{F} \equiv \frac{1}{e} (\epsilon - \epsilon_F) \frac{\vec{\nabla} T}{T}$$

$$\int d\varepsilon \frac{(\varepsilon - \varepsilon_F)^2}{k_B T^2} \frac{e^{\frac{\varepsilon - \varepsilon_F}{k_B T}}}{\left(e^{\frac{\varepsilon - \varepsilon_F}{k_B T}} + 1\right)^2} = k_B^2 T \int_{-\infty}^{\infty} dx x^2 \frac{e^x}{(e^x + 1)^2} = k_B^2 T \frac{\pi^2}{3} \quad (3)$$

$$Q = \frac{k_B^2 \pi^2}{e} T \frac{\sigma'}{\sigma} = \frac{k_B^2 \pi^2}{e} T \frac{d}{d\varepsilon_F} k_B \sigma \quad (\text{Mott formula})$$

If T is energy independent

$$\sigma(\varepsilon) \propto v(\varepsilon_f) v(\varepsilon_f) \propto \varepsilon_f^{3/2}$$

$$v(\varepsilon_f) \propto \frac{v}{\varepsilon_f} \propto \frac{k_F^3}{\varepsilon_f} \propto \frac{\varepsilon_f^{3/2}}{\varepsilon_f} \propto \varepsilon_f^{1/2}$$

$$v^2(\varepsilon_f) \propto \varepsilon_f$$

$$\sigma(\varepsilon) = A \varepsilon_f^{3/2}$$

$$k_B \sigma = k_B A + \frac{3}{2} k_B \varepsilon_f$$

$$\frac{d}{d\varepsilon_f} k_B \sigma = \frac{3}{2} \frac{1}{\varepsilon_f}$$

$$Q = \frac{k_B^2}{e} \left(\frac{A}{\varepsilon_f} \right) \frac{\pi^2}{2}$$

↓ small effect in metals ($k_B T \ll \varepsilon_f$)

units

$$E = Q \Delta T$$

or

$$-V = Q \Delta T$$

$$[Q] = \frac{[V]}{[T]} = \frac{V}{K}$$

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$$Q = \frac{k_B \pi^2}{e^2} \left(\frac{k_B T}{E_F} \right)$$

$$\frac{k_B \pi^2}{e^2} \approx \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} \text{ S} \sim 4.5 \times 10^{-4} \frac{\text{V}}{\text{K}} = 0.45 \times \frac{\text{mV}}{\text{K}}$$

$$\frac{k_B T}{E_F} \sim 10^{-2} \quad T = 300 \text{ K}$$

$$[Q \sim \mu\text{V/K}]$$

Semiconductors, graphite \rightarrow small $E_F \rightarrow$ larger Q .