PHZ7427: Spring 2014
FINAL EXAM: Solutions
Tuesday, 04/29/2014
Instructor: D. L. Maslov
maslov@phys.ufl.edu 392-0513 Rm. 2114

In multiple-choice problems, you can choose more than one answer, if necessary.
If you forgot a particular formula, use dimensional analysis to derive it (up to a number).

Some fundamental constants:

- $e = 1.6 \times 10^{-19}$ C (SI)=4.8×10^{-10} statcoulombs (CGS)
- $m = 9.1 \times 10^{-31}$ kg
- $\hbar = 1.1 \times 10^{-34}$ J· s
- $k_B = 1.4 \times 10^{-23}$ J/K
- $c = 3.0 \times 10^{8}$ m/s
- $\epsilon_0 = 8.9 \times 10^{-12}$ F/m (if you choose to work in SI)

P1 25 points An infinite superconducting slab of thickness d is carrying current I per unit width (units of I: A/m). Find the current density inside the slab and the magnetic field both inside and outside the slab. Sketch the field and the current as a function of z. The slab is infinite in the x and y directions (cf. Fig. 2). Assume that the superconductor can be described by the London equation: $\nabla^2 \mathbf{B} = \mathbf{B}/\Lambda^2$, where Λ is the penetration depth.

Solution: Let $j(z)$ be the current density. By Amperes law, the field outside the superconductor is found from

$$2B_0 = \frac{4\pi}{c}\int_{-d/2}^{d/2} dz j(z) = \frac{4\pi}{c} I \to B_0 = \frac{2\pi}{c} I.$$

Obviously, the field is dependent of the distance from the slab. For $z > 0$, the field is directed opposite to the x-axis; for $z < 0$ at is along the x axis. The field inside the slab is given by the fundamental solution of the London equation:

$$B(z) = a\exp(z/\Lambda) + b\exp(-z/\Lambda).$$

The boundary conditions are $B(d/2) = -B_0$ and $B(-d/2) = B_0$. Solving for a and b, we obtain

$$B(z) = -B_0 \frac{\sinh(z/\Lambda)}{\sinh(d/2\Lambda)}.$$

The current density is found as

$$j(z) = -\frac{c}{4\pi}\partial_z B(z) = \frac{I}{2\Lambda}\frac{\cosh(z/\Lambda)}{\sinh(d/2\Lambda)}.$$

P2 10 points Which of the plots in Fig. 3 corresponds to (the electronic part of) the specific heat of a d-wave superconductor? The asymptotic behavior of $C(T)$ at $T \to 0$ is indicated in the plots.

Answer:

a) Because of the nodes, the specific heat goes to zero in a power-law rather than the exponential manner.
b)
c)

P3 20 points Estimate (within an order of magnitude) the penetration depth of a superconductor at $T = 0.99T_c$. View the superconductor as a "good" metal with the number density $n = 10^{22}$ cm^{-3} and the effective mass $m^* = m$.

Answer:

a) 10^{-2} cm

b) 10^{-6} cm

c) 10^{-4} cm $\Lambda(T) = \Lambda(0)\sqrt{\frac{T_c - T}{T_c}} = 10\Lambda(0) \approx 0.5 \times 10^{-4}$, where $\Lambda(0) = c/\sqrt{4\pi ne^2/m} \approx 0.5 \times 10^{-5}$ cm.

P4 25 points Consider a one-dimensional monoatomic chain with electron number density $n = 1/a$ (per unit length), where a is the lattice spacing. The Debye temperature of the chain is T_D. Which of the following is true?

a) The resistivity of the wire scales as T for $T \gg T_D$ and as $\exp(-cT_D/T)$ for $T \ll T_D$, where c is a number. In a 1D, the only scattering processes are the ones with momentum transfer near 0 and near $2k_F$ (forward and backscattering, correspondingly). Forward scattering obviously does not contribute to the resistivity. To initiate a backscattering process, the energy of the phonon must "just right", i.e., $\hbar\omega = \hbar sq = 2\hbar sk_F$. There is no averaging over angles in 1D, hence the Bose function will be exponentially small if the temperature is below $2\hbar sk_F/k_B$. Since the number density in the wire is high, this temperature (the Bloch-Grueneisen temperature) is the same at the Debye temperature.

b) The resistivity of the wire scales as T for $T \gg T_D$ and as T^3 for $T \ll T_D$.

c) The resistivity of the wire scales as T for $T \gg T_D$ and as T^5 for $T \ll T_D$.

P5 20 points Charge carriers in graphene are described by a linear dispersion relation near the K points of the Brillouin zone: $\varepsilon = \pm\hbar v_F k$. According to the Landau criterion for superfluidity, a system with linear dispersion must sustain a superflow with velocity less than v_F. Nevertheless, graphene is definitely not a superfluid. This is so because:

a) The critical temperature is very low and we have not reached it yet.

b) The electron current is compensated by the hole current.

c) Graphene is not a Galilean-invariant system. Indeed, it is not not. The Dirac-like electron and holes near the K points occur because a special lattice structure of graphene.

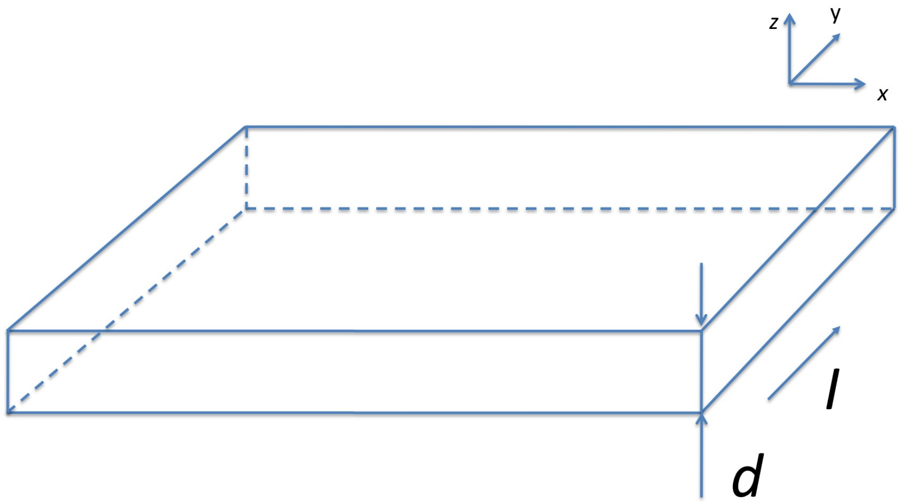

FIG. 1: Problem 1.

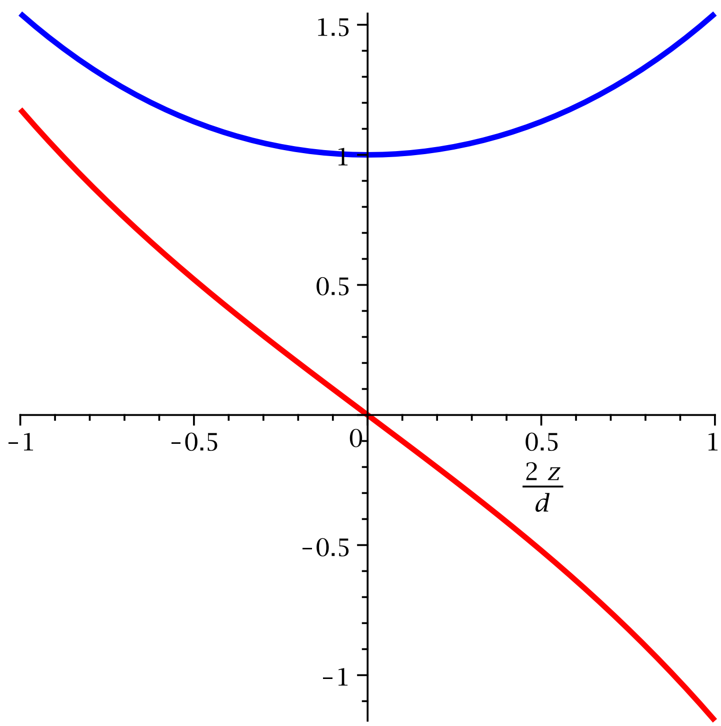

FIG. 2: The magnetic field (red) and current density (blue) distributions inside the slab.

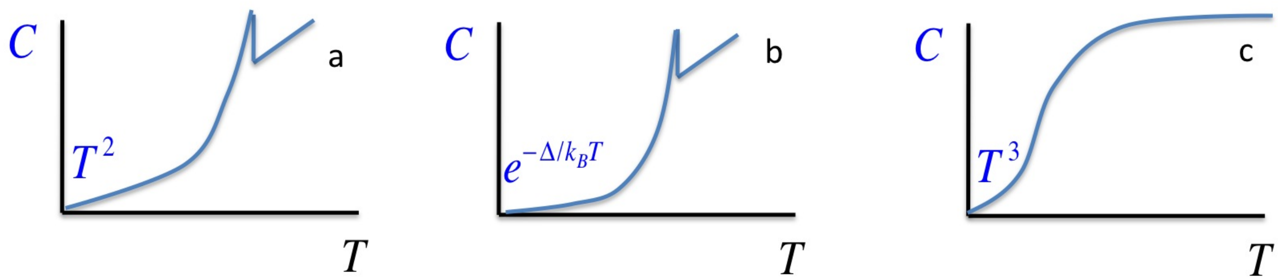

FIG. 3: Problem 2.