PHY 6648: Quantum Field Theory I
Fall Term 2015
Homework Set 4

Due Friday, October 30 2015

Reading: Class notes, Chapters 2 and 3 from Peskin.

Problem 1. Symmetries of the Dirac Lagrangian. Consider the Dirac Lagrangian (3.34):

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \ .$$

(a) Show that the transformation

$$\psi(x) \to e^{i\alpha}\psi(x)$$

is a symmetry of the Lagrangian. Find the corresponding Noether current and check that it is indeed conserved.

(b) Now consider the transformation

$$\psi(x) \to e^{i\alpha\gamma^5}\psi(x)$$

and again find the corresponding Noether current. Is it conserved? Consider two cases: $m = 0$ and $m \neq 0$.

Problem 2. Parity. In this problem we will consider parity transformations, i.e. mirror reflections in spacetime:

$$x^\mu \ \to \ x'^\mu = (t, -\vec{x}). \tag{1}$$

(a) Let $\psi(x)$ satisfy the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0. \tag{2}$$

Show that

$$\psi'(x') \equiv \gamma^0 \psi(x) \tag{3}$$

satisfies the Dirac equation in the parity-reflected world:

$$(i\gamma^\mu \partial'_\mu - m)\psi'(x') = 0, \tag{4}$$

where

$$\partial'_\mu = \frac{\partial}{\partial x'^\mu} = (\frac{\partial}{\partial t}, -\frac{\partial}{\partial \vec{x}}).$$

(b) Now we can derive the P-parity assignments of the bilinears in the table on page 71, simply by comparing each one to its cousin in the parity-reflected world. Show that

$$
\begin{aligned}
\bar{\psi}'(x')\psi'(x') &= \bar{\psi}(x)\psi(x), & (5)\\
i\bar{\psi}'(x')\gamma^5\psi'(x') &= -i\bar{\psi}(x)\gamma^5\psi(x), & (6)\\
\bar{\psi}'(x')\gamma^\mu\psi'(x') &= g^{\mu\mu}\bar{\psi}(x)\gamma^\mu\psi(x), & (7)\\
\bar{\psi}'(x')\gamma^\mu\gamma^5\psi'(x') &= -g^{\mu\mu}\bar{\psi}(x)\gamma^\mu\gamma^5\psi(x), & (8)\\
\bar{\psi}'(x')\sigma^{\mu\nu}\psi'(x') &= g^{\mu\mu}g^{\nu\nu}\bar{\psi}(x)\sigma^{\mu\nu}\psi(x), & (9)
\end{aligned}
$$

where in the last three equations I have used $g^{\mu\mu}$ (no sum over μ) instead of the confusing $(-1)^\mu$ defined in the text.

Problem 3. Allowed interactions. If ϕ is a real scalar field and ψ is a Dirac field, which of the following terms can appear in the Lagrangian density of a sensible field theory in $3+1$ dimensions? Explain why or why not. Which ones can appear in principle, but become irrelevant at sufficiently low energies? In each case, the (real) coefficients c_i are chosen to have dimensions such that the term in the Lagrangian density has mass dimension 4.

(a) $c_1\phi^7(x)$;

(b) $c_2\partial_\mu\phi(x)\bar{\psi}(x)\gamma^\mu\psi(x)$;

(c) $c_3\phi(x)\psi^\dagger(x)\psi(x)$;

(d) $c_4\phi(x)i\bar{\psi}(x)\gamma^5\psi(x)$;

(e) $c_5(\bar{\psi}(x)\psi(x))^2$;

(f) $c_6\phi^2(x)\partial_\mu\phi(x)$;

(g) $c_7\phi(x)\bar{\psi}(x+\delta)\psi(x+\delta)$;

(h) $c_8\phi(x)\gamma^\mu\partial_\mu\psi(x)$.

Problem 4. Symmetry factors in φ^4 theory. Consider the second-order term in the expansion (4.43)

$$
\langle 0|T\left\{\phi(x)\phi(y)\frac{(-i)^2}{2!}\frac{\lambda}{4!}\frac{\lambda}{4!}\int d^4z\,\phi(z)\phi(z)\phi(z)\phi(z)\int d^4u\,\phi(u)\phi(u)\phi(u)\phi(u)\right\}|0\rangle\,.
$$

(a) Classify all topologically distinct classes of diagrams.

(b) By enumerating all possible contractions, calculate the symmetry factor for each class. *Hint: Check that the sum of the symmetry factors is equal to* $9!!/(2!4!4!) = 945/1152$.

(c) Check your answer from (b) by considering the symmetries of the diagrams and thus give an independent justification of your previous answers.

(d) Extra credit: Can you now handle the third-order term in (4.43)?