Reading: PYTHIA manual, as needed.

Problem 1. Particle rapidity. The rapidity $y$ of a particle of mass $M$ and 4-momentum $P^\mu = (E, p_x, p_y, p_z) = (E, \vec{p}_T, p_z)$ is defined as

$$y \equiv \frac{1}{2} \ln \frac{E + p_z}{E - p_z}.$$

(a) Show that the 4-momentum can be written as

$$P^\mu = \left( e_T \cosh(y), p_T \cos(\varphi), p_T \sin(\varphi), e_T \sinh(y) \right),$$

where $e_T = \sqrt{M^2 + p_T^2}$ and $\varphi$ is the usual azimuth angle.

(b) Show that the phase space element can be expressed as

$$\frac{d^3 \vec{p}}{E} = p_T dp_T d\varphi dy = e_T d\sigma_T d\varphi dy.$$

Problem 2. Single $W$ production at the LHC. Consider only leptonic decays of the $W$ and plot the $p_T$ distribution of the lepton (either $e$ or $\mu$) for two cases:

(a) initial state radiation “OFF” (use subprocess 2).

(b) initial state radiation “ON” (use subprocesses 16 and 31).

(Make sure to use only leptons within the geometrical acceptance of the detector, i.e. impose the acceptance cut of $|\eta| < 2.5$.) Discuss the effect of the $W$ transverse motion on the Jacobian peak.

Problem 3. Higgs boson in an $s$-channel. Consider $pp \rightarrow \mu^+\mu^-$ scattering mediated by a virtual Higgs boson exchange only (i.e., $s$-channel gluon fusion to a Higgs boson, with the Higgs decaying to muon pairs). Use PYTHIA to plot the angular distribution of the muon (i.e. the differential cross-section $d\sigma/d\cos\theta$) in the CM frame. Compare to the analogous result from Section 5.1 in Peskin&Schroeder for $e^+e^- \rightarrow \mu^+\mu^-$ which was mediated by a virtual photon only. Can we use this angular distribution as an indicator of the spin of the intermediate particle?

Problem 4. $Z'$ production. Repeat the previous problem, only this time consider an intermediate $Z'$ vector boson resonance. (Use the default $Z'$ model in PYTHIA: read section 8.4.2.)