Electrodynamics & Relativity

We know from electrodynamics:

* there are 2 vectors $\vec{E}, \vec{B}$
* the Lorentz force is

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

* the fields can be expressed in terms of potentials

$$\vec{E} = -\operatorname{grad}\varphi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

How can we guess the relativistic versions of those equations?

* try the same trick as before:

$$(\underbrace{\vec{E}}_{\text{3-vector}}, \underbrace{?}_{\text{scalar}})$$

scalar =? $c|\vec{B}|$, cB_x, ... ?

$$(\underbrace{\vec{B}}_{\text{3-vector}}, \underbrace{?}_{\text{scalar}})$$

scalar =? $\frac{|\vec{E}|}{c}$, $\frac{E_x}{c}$, ... ?

Must differ by units of velocity: $E \sim cB$

This approach does not work.

Perhaps we should have been looking at the potentials?

$$E \sim \frac{\varphi}{x} \sim \frac{A}{t}$$

φ and A differ by units of velocity!

We can form the 4-vector

$$\boxed{A_\mu = \left(\vec{A}\, , \frac{\varphi}{c}\right)}$$

The two 3-vectors $\vec{E}$ and $\vec{B}$ form an antisymmetric tensor

$$F = \begin{bmatrix} 0 & B_3 & -B_2 & -E_1/c \\ -B_3 & 0 & B_1 & -E_2/c \\ B_2 & -B_1 & 0 & -E_3/c \\ \frac{E_1}{c} & \frac{E_2}{c} & \frac{E_3}{c} & 0 \end{bmatrix}$$

Roughly speaking,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

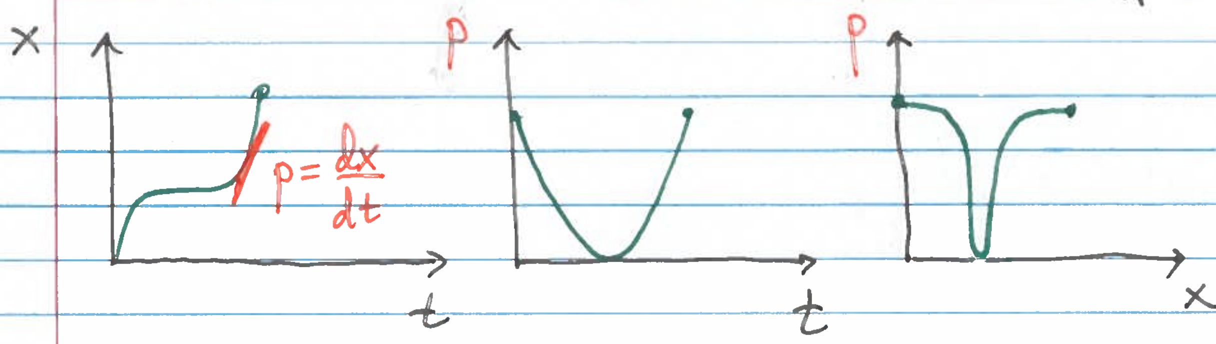
$\mu = 1,2,3,4$
$i,j,k = 1,2,3$

$$F_{ij} \sim \partial_i A_j - \partial_j A_i \sim (\nabla A)_k$$

$$F_{i4} \sim \partial_4 A_i - \partial_i A_4 .$$

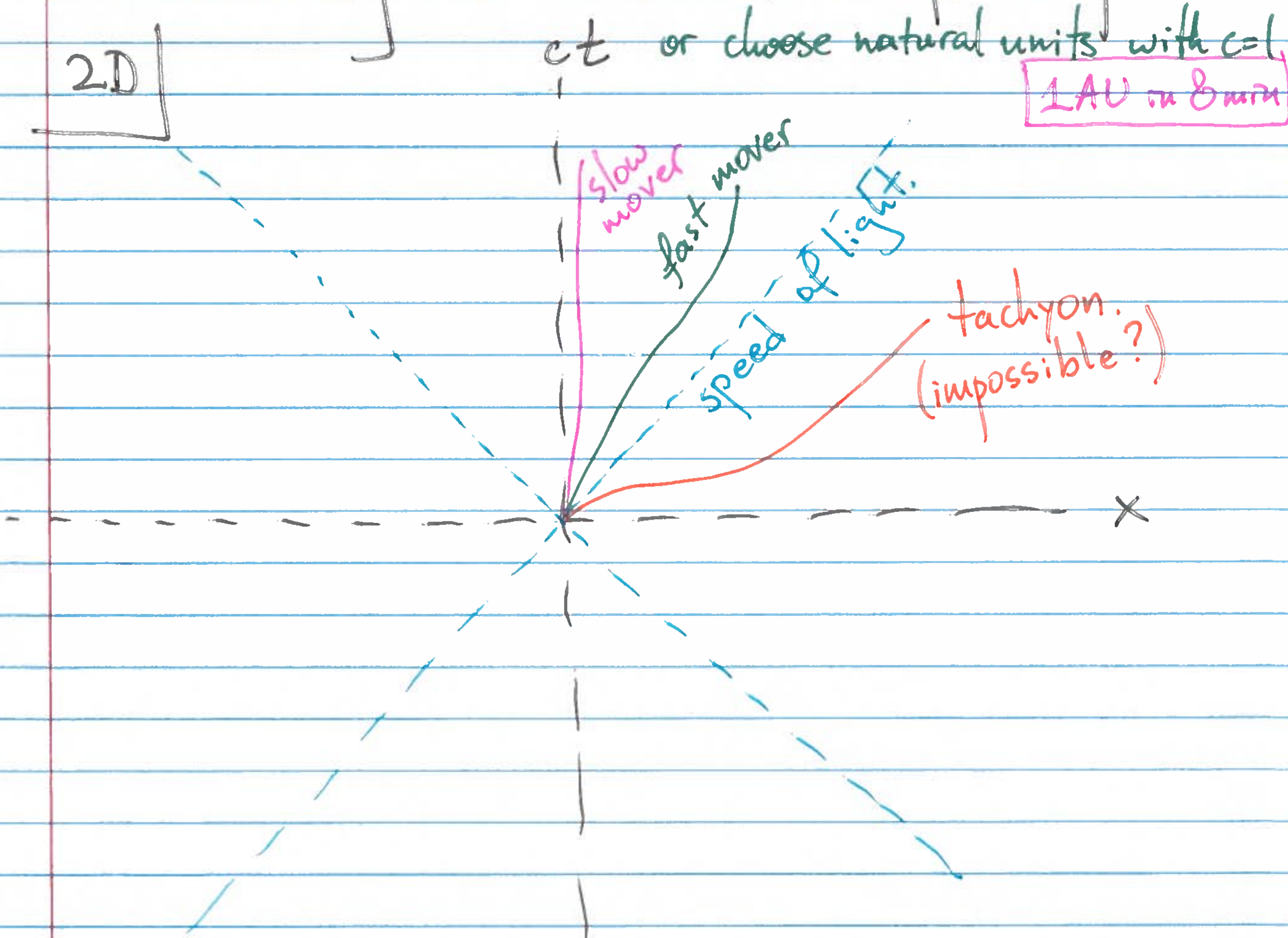
The Light Cone.

In Mechanics we visualize motion in different ways



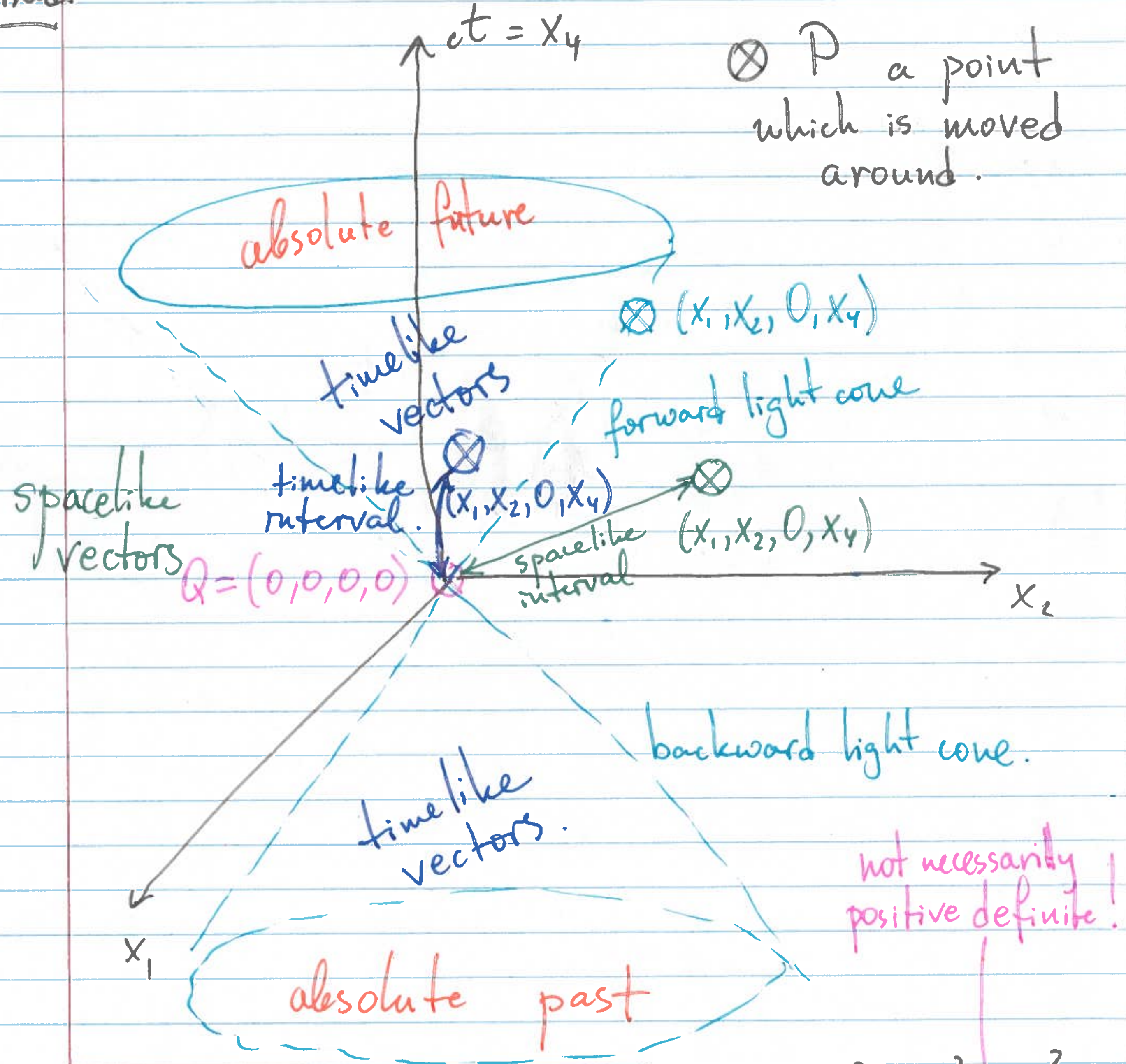
space (configuration space) | momentum space | phase space

In Relativity we use the first option: spacetime



$x_3 = \text{fixed}$.

3D Light Cone



Compute $(x_P - x_Q)^2 = x_P^2 = x_1^2 + x_2^2 + x_3^2 - x_4^2$.

On the light cone:

$$x_1^2 + x_2^2 + 0 - x_4^2 = r^2 - (ct)^2 = 0$$

because $r = ct$ is the distance travelled by light.

Time is relative.

Inside the light cone, speed is less than c, so

$$r^2 - (ct)^2 = (\bar{v}t)^2 - (ct)^2 < 0.$$

Outside the light cone, it is the opposite:

$$r^2 - (ct)^2 = (\bar{v}t)^2 - (ct)^2 > 0.$$

Therefore the "distance" in space-time between two events P and Q can be "+"ve, "−"ve or $\emptyset$.

$(X_P - X_Q)^2 = 0$ lightlike only light can connect them.

$(X_P - X_Q)^2 < 0$ time-like causally connected.

$t_Q = 0$ $t_P \equiv t > 0$: Q before P.

$X_P \cdot X_P < 0$.

A different observer S'.

$X'_P \cdot X'_P = X_P \cdot X_P < 0 \Rightarrow$ timelike

$X'_{4P} = \gamma(X_4 - \beta X_1) > 0$ because $\beta < 1$, $X_1 < X_4$. $\longrightarrow$ P after Q in S'

* Q can influence P
* information can be propagated.
* All inertial observers agree on the exact time sequence of events

$(X_P - X_Q)^2 > 0$ space-like causally disconnected.

* Q cannot influence P
* information cannot be sent from Q to P
* Observers can disagree on the time sequence Q, P