Physics 4222 Mechanics 2
Spring Term 2024 - Exam 3

Name: ______________________________ UFID: ____________________

This is a 50 minute closed book exam consisting of **twenty** multiple choice questions. Each question is worth 1 point, so the maximum number of points on this test is 20. The exam is closed book, and no calculators or other electronic devices will be allowed. There is no penalty for wrong answers, so it is in your best interest to provide answers to all questions. If more than one answer appears plausible, choose the best one.

Useful formulas and equations (Chapters 11, 14, 15 and 16)

$$\mathbf{M}\ddot{\mathbf{q}} = -\mathbf{K}\mathbf{q} \qquad T = \frac{1}{2}\sum_{i,j} M_{ij}\dot{q}_i\dot{q}_j \qquad U = \frac{1}{2}\sum_{i,j} K_{ij}q_iq_j \qquad (\mathbf{K} - \omega^2\mathbf{M})\mathbf{a} = 0$$

$$N_{sc} = N_{inc}n_{tar}\sigma_{sc} \qquad N_{sc}(\text{into } d\Omega) = N_{inc}\, n_{tar}\,\frac{d\sigma}{d\Omega}(\theta,\phi)\, d\Omega$$

$$\theta = \pi - 2\int_{r_{min}}^{\infty} \frac{(b/r^2)dr}{\sqrt{1-(b/r)^2 - U(r)/E}}$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| \qquad \frac{d\sigma}{d\Omega} = \left(\frac{kqQ}{4E\sin^2(\theta/2)}\right)^2$$

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} \qquad c = \sqrt{\frac{T}{\mu}} \qquad u(x,t) = f(x-ct) + g(x+ct)$$

$$u(0,t) = u(L,t) = 0 \qquad u_n(x,t) = A_n\sin(k_nx)\cos(w_nt - \delta) \qquad k_n = n\frac{\pi}{L} \qquad \omega_n = n\frac{\pi c}{L}$$

$$\frac{\partial^2 p}{\partial t^2} = c^2\nabla^2 p \qquad \nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \qquad p = f(\mathbf{n}\cdot\mathbf{r} - ct) \qquad p = \cos\left[k(\mathbf{n}\cdot\mathbf{r} - ct)\right]$$

$$\frac{\partial^2 p}{\partial t^2} = c^2\frac{1}{r}\frac{\partial^2}{\partial r^2}(rp) \qquad p(r,t) = \frac{1}{r}\left[f(r-ct) + g(r+ct)\right]$$

$$x = (x_1, x_2, x_3, x_4) = (x, y, z, ct) \qquad x\cdot y = x_1y_1 + x_2y_2 + x_3y_3 - x_4y_4 \qquad q' = \Lambda q$$

$$\Lambda = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-v^2/c^2}}$$

Light cone at point Q : all points P such that $(x_P - x_Q)^2 = 0$

$$p = \left(\mathbf{p}, \frac{E}{c}\right) \qquad E^2 = (mc^2)^2 + (\mathbf{p}c)^2$$

Useful formulas and equations (Chapters 8, 9, 10, 12 and 13)

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \qquad M = m_1 + m_2 \qquad \mu = \frac{m_1 m_2}{m_1 + m_2} \qquad \mu r^2 \dot{\phi} = \text{const} = \ell \qquad F_{\text{cf}} = \mu r \dot{\phi}^2$$

$$U_{\text{eff}}(r) = U(r) + U_{\text{cf}}(r) = U(r) + \frac{\ell^2}{2\mu r^2} \qquad u = \frac{1}{r} \qquad u''(\phi) = -u(\phi) - \frac{\mu}{\ell^2 u(\phi)^2} F$$

$$F = \frac{G m_1 m_2}{r^2} = \frac{\gamma}{r^2} \qquad r(\phi) = \frac{c}{1 + \epsilon \cos\phi} \qquad c = \frac{\ell^2}{\gamma\mu} \qquad a = \frac{c}{1 - \epsilon^2} \qquad b = \frac{c}{\sqrt{1 - \epsilon^2}}$$

$$d = a\epsilon \qquad E = \frac{\gamma^2 \mu}{2\ell^2}\left(\epsilon^2 - 1\right) \qquad \tau^2 = 4\pi^2 \frac{a^3 \mu}{\gamma} \qquad v_2 = \lambda v_1 \qquad \epsilon_2 = \lambda^2 \epsilon_1 + \lambda^2 - 1$$

$$\ddot{\phi} + 2\beta\dot{\phi} + \omega_0^2 \sin\phi = \gamma\omega_0^2 \cos\omega t \qquad \gamma = \frac{F_0}{mg} \qquad x_{t+1} = r x_t (1 - x_t)$$

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \qquad \mathcal{H} = \sum_{i=1}^{n} p_i \dot{q}_i - \mathcal{L} \qquad \dot{q}_i = \frac{\partial \mathcal{H}}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial \mathcal{H}}{\partial q_i}$$

$$m\ddot{\mathbf{r}} = \mathbf{F} + \mathbf{F}_{\text{inertial}} \qquad \mathbf{F}_{\text{inertial}} = -m\mathbf{A} \qquad \boldsymbol{\omega} = \omega \mathbf{u} \qquad \boldsymbol{v} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\left(\frac{d\mathbf{Q}}{dt}\right)_{S_0} = \left(\frac{d\mathbf{Q}}{dt}\right)_{S} + \boldsymbol{\Omega} \times \mathbf{Q} \qquad m\ddot{\mathbf{r}} = \mathbf{F} + \mathbf{F}_{\text{cor}} + \mathbf{F}_{\text{cf}}$$

$$\mathbf{F}_{\text{cor}} = 2m\dot{\mathbf{r}} \times \boldsymbol{\Omega} \qquad \mathbf{F}_{\text{cf}} = m(\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega} \qquad \mathbf{g} = \mathbf{g}_0 + (\boldsymbol{\Omega} \times \mathbf{R}) \times \boldsymbol{\Omega}$$

$$\mathbf{L} = \mathbf{L}(\text{motion of CM}) + \mathbf{L}(\text{motion relative to CM})$$

$$T = T(\text{motion of CM}) + T(\text{motion relative to CM})$$

$$\mathbf{L} = \mathbf{I}\boldsymbol{\omega} \qquad I_{xx} = \sum_{\alpha} m_\alpha (y_\alpha^2 + z_\alpha^2), \text{ etc.} \qquad I_{xy} = -\sum_{\alpha} m_\alpha x_\alpha y_\alpha, \text{ etc.}$$

$$\mathbf{L} = \lambda\boldsymbol{\omega} \qquad \mathbf{I}' = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Useful formulas and equations (Chapters 1, 2, 3, 4, 5, 6, 7)

$\mathbf{r} \cdot \mathbf{s} = rs\cos\theta = r_x s_x + r_y s_y + r_z s_z$

$\mathbf{r} \times \mathbf{s} = (r_y s_z - r_z s_y, r_z s_x - r_x s_z, r_x s_y - r_y s_x) \qquad |\mathbf{r} \times \mathbf{s}| = rs\sin\theta$

Cartesian (x, y, z) : $\quad F_x = m\ddot{x} \qquad F_y = m\ddot{y} \qquad F_z = m\ddot{z}$

2D Polar (r, ϕ) : $\quad F_r = m(\ddot{r} - r\dot{\phi}^2) \qquad F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi})$

Cylindrical Polar (ρ, ϕ, z) : $\quad F_\rho = m(\ddot{\rho} - \rho\dot{\phi}^2) \qquad F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}) \qquad F_z = m\ddot{z}$

$\mathbf{f} = -f(v)\hat{\mathbf{v}} \qquad f(v) = f_{lin} + f_{quad} \qquad f_{lin} = bv = \beta D v \qquad f_{quad} = cv^2 = \gamma D^2 v^2$

$\beta = 1.6 \times 10^{-4}\ \mathrm{N \cdot s/m^2} \qquad \gamma = 0.25\ \mathrm{N \cdot s^2/m^4} \qquad m\dot{v} = -\dot{m}v_{ex} + F^{ext}$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad \mathbf{R} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \mathbf{r}_\alpha = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \ldots + m_N\mathbf{r}_N}{m_1 + m_2 + \ldots + m_N}$$

$$\vec{\ell} = \vec{r} \times \vec{p} \qquad \mathbf{L} = \sum_{\alpha=1}^{N} \vec{\ell}_\alpha = \sum_{\alpha=1}^{N} \vec{r}_\alpha \times \vec{p}_\alpha \qquad \dot{\mathbf{L}} = \mathbf{\Gamma}^{ext}$$

$$\Delta T \equiv T_2 - T_1 = \int_1^2 \mathbf{F} \cdot d\mathbf{r} \equiv W(1 \to 2) \qquad U(\mathbf{r}) = -W(\mathbf{r}_0 \to \mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{F}(\mathbf{r}') \cdot d\mathbf{r}'$$

$\mathbf{F} = -\nabla U \qquad E = T + U_1 + U_2 + \ldots + U_n \qquad \mathbf{F}(\mathbf{r}) = f(\mathbf{r})\hat{\mathbf{r}}$

$$U = U^{int} + U^{ext} = \sum_\alpha \sum_{\beta > \alpha} U_{\alpha\beta} + \sum_\alpha U_\alpha^{ext} \qquad (\text{net force on } \alpha) = -\nabla_\alpha U \qquad T + U = const$$

$$F = -kx \Longleftrightarrow U = \frac{1}{2}kx^2 \qquad \ddot{x} = -\omega^2 x \Longleftrightarrow x(t) = A\cos(\omega t - \delta)$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \leftrightarrow x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta); \beta = \frac{b}{2m}; \omega_0 = \sqrt{\frac{k}{m}}; \omega_1 = \sqrt{\omega_0^2 - \beta^2}$$

with $F(t) = mf_0\cos(\omega t) \Longleftrightarrow x(t) = A\cos(\omega t - \delta)$(long term solution)

$$A^2 = \frac{f_0^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2} \qquad \tan\delta = \frac{2\beta\omega}{\omega_0^2 - \omega^2} \qquad \omega_2 = \sqrt{\omega_0^2 - 2\beta^2}$$

$$\text{Extremal } S = \int_{u_1}^{u_2} f[x(u), y(u), x'(u), y'(u), u]du \quad \Longleftrightarrow \quad \frac{\partial f}{\partial x} = \frac{d}{du}\frac{\partial f}{\partial x'}, \quad \frac{\partial f}{\partial y} = \frac{d}{du}\frac{\partial f}{\partial y'}$$

$$\mathcal{L} = T - U \qquad \frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \qquad p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \qquad \mathcal{H} = \sum_{i=1}^{n} p_i\dot{q}_i - \mathcal{L}$$