

PHY 6648: Quantum Field Theory I
Fall Term 2005
Homework Set 1

Due Friday, September 2 2005

Reading: “Notations and Conventions” section starting on page xix. Chapters 1 and 2 from the textbook, although they are not directly related to this homework, which is designed more as a diagnostic check.

Problem 1. Relativistic kinematics.

(a) A particle of mass M decays to a particle of mass m and a massless particle. Find the energies and momenta of the two final particles in the CM frame of M . Use $c = 1$ units. Check that the answers make sense in the limit $m \rightarrow 0$.

(b) A particle of mass M decays to two particles, one with mass m_1 , and the other with mass m_2 . Find the energies and momenta of the two final particles (in the CM frame of M , using $c = 1$ units). Check that your answers agree with those from part (a) in the limit $m_2 = 0$. Check that they also make sense in the limit $m_1 = m_2$.

(c) A particle of mass M decays to three massless particles. Define the scaling variables $x_i \equiv 2E_i/M$, which parametrize the energies of the final particles in the rest frame of M . Notice that $\sum_i x_i = 2$, so that only two of the x_i 's are independent free parameters. For example, let us consider the (x_1, x_2) plane. Find the region which is kinematically allowed in this decay.

(d) Repeat part (c), allowing *one* of the final particles, say #3, to have nonzero mass m . Show that again $\sum_i x_i = 2$. Find the kinematically allowed region for (x_1, x_2) . Show that its boundary is formed from a straight line and a hyperbola and find their parametric equations.

Problem 2. Complex variables and contour integration.

(a) Evaluate:

$$\int_0^\infty dx \frac{x^2}{(a^2 + x^2)(b^2 + x^2)}$$

(Answer: $\frac{\pi}{2(a+b)}$.)

(b) Evaluate:

$$\int_0^\infty dx \frac{x \sin x}{a^2 + x^2}$$

(Answer: $\frac{\pi}{2}e^{-a}$.)

(c) Evaluate:

$$\int_{-\infty}^\infty \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{\omega - (\Omega - i\Gamma)}$$

where Ω and Γ are real and positive. Consider the cases $t < 0$ and $t > 0$ separately. (Answer: 0, for $t < 0$; $-ie^{-i\Omega t - \Gamma t}$, for $t > 0$.)

Problem 3. Rotations and spin.

(a) Write the 3×3 rotation matrix for a rotation by θ about the y axis which carries the vector $(0, 0, 1)$ into $(\sin \theta, 0, \cos \theta)$.

(b) Show that this matrix is represented by $R(\theta) = \exp(-i\theta J^{(2)})$, where $J_{13}^{(2)} = -J_{31}^{(2)} = i$ and all other elements are zero. *Hint: Show that R satisfies the equation $dR(\theta)/d\theta = -iJ^{(2)}R(\theta)$.*

(c) The corresponding rotation matrix in the spin- $\frac{1}{2}$ representation is generated by $J^{(2)} = \sigma^{(2)}/2$. Write this matrix explicitly as a 2×2 matrix.

Problem 4. Particle physics units. This problem builds familiarity with the convention $\hbar = c = 1$, in which all physical quantities are expressed in units of energy to the appropriate power. In high-energy particle physics a typical unit of energy is $GeV = 1 \text{ Giga } eV = 10^9 eV$.

(a) What is Newton's constant G_N in units of the appropriate power of GeV ? What mass in GeV would an electron need to have in order for the gravitational attraction between two electrons to be of the same order as the Coulomb repulsion between them? What is the actual electron mass? (The large discrepancy between these two masses is a manifestation of the *gauge hierarchy problem*.)

(b) The energy scale of G_N derived in (a) also defines unique mass, time, and length scales, known as the Planck mass, Planck time, and Planck length. What are these in everyday units (say, grams, seconds, and meters)? (Simple dimensional analysis implies that all quantities in our universe should be of order these fundamental scales. The large discrepancy between the Planck time and the age of the Universe is a manifestation of the *cosmological constant problem*.)

Problem 5. Biosketch. The author of the article for inspirational reading (hep-th/9803075) handed out in class is Frank Wilczek. Write a couple of paragraphs explaining to an undergraduate premed student who Frank Wilczek is and what is his claim to fame.