PHY 6648: Quantum Field Theory I
Fall Term 2005
Homework Set 2

Due Friday, September 9 2005

Reading: Section 2.2 from the textbook and class notes.

Problem 1. Total divergences.

(a) Consider adding a total divergence to the Lagrangian density of a scalar field φ

$$\mathcal{L}(\varphi, \partial\varphi) = \mathcal{L}(\varphi, \partial\varphi) + \partial_\mu \Lambda^\mu(\varphi, \partial\varphi).$$

Show that the extra term satisfies identically the classical equations of motion and thus does not contain any physics. *Hint: If you are having trouble proving this for the general case, consider an explicit example, e.g.* $\Lambda^\mu = \varphi^2 \partial^\mu \varphi$. *If you are still having a problem, perhaps part (b) would help.*

(b) Generalize the Euler-Lagrange equation of motion derived in class to the case when the Lagrangian density depends on higher order derivatives of the field, e.g. up to n-th order:

$$\sum_{k=0}^{n} (-1)^k \partial_{\mu_1} \partial_{\mu_2} \ldots \partial_{\mu_k} \left\{ \frac{\partial \mathcal{L}(\varphi, \partial\varphi, \ldots, \partial^n \varphi)}{\partial(\partial_{\mu_1} \partial_{\mu_2} \ldots \partial_{\mu_k} \varphi)} \right\} \;=\; 0.$$

Problem 2. Scale invariance. Let φ be the Klein-Gordon field in $3+1$ dimensions, and consider the finite transformation corresponding to a change of length scale

$$x^\mu \;\rightarrow\; x'^\mu = \lambda x^\mu \,, \tag{1}$$

$$\varphi(x) \;\rightarrow\; \varphi'(x') = \lambda^{-d} \varphi(x) \,, \tag{2}$$

where d is the dimension of the field and λ is the parameter of the transformation.

(a) Use Noether's theorem to construct the current J^μ associated with this transformation.

(b) Consider the following Lagrangian density for the scalar field $\varphi(x)$

$$\mathcal{L} = \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - \frac{1}{2} m^2 \varphi^2 - \frac{1}{4!} g \varphi^4 \,.$$

What is the value of d? *Hint: Use dimensional analysis.*

(c) For what values of m and g will J^μ be conserved? Check that in that case the transformation (1-2) is indeed a symmetry of the Lagrangian.

Problem 3. Classical electromagnetism. Problem 2.1 in Peskin&Schroeder, supplemented with the following (see next page as well):

(c) Show that the extra term $\partial_\lambda K^{\lambda\mu\nu}$ which was added to the energy-momentum tensor in part (b), does not affect the conserved quantities (total energy and momentum), as long as $K^{\lambda\mu\nu}$ is antisymmetric in it first two indices ($\lambda\mu$).

(d) In part (a) you will only find two of Maxwell's equations. Show that

$$\varepsilon_{\mu\nu\rho\sigma}\partial^{\nu}F^{\rho\sigma} = 0$$

is an identity which provides the two missing equations.