

PHY 6648: Quantum Field Theory I
Fall Term 2005
Homework Set 11

Due Monday, November 28 2005

Reading: Chapter 5 from the textbook and class notes. I will also accept homework solutions before Thanksgiving.

Problem 1. Feynman diagrams on a computer. Research the possible options for doing Feynman diagrams on a computer, i.e. automating the calculations from Chapter 5. Discuss the pros and cons of the different alternatives.

Problem 2. Playing with CompHEP. Use CompHEP to verify some of our results from Chapter 5. Use the various features of the program to make as many cross-checks of the final answers as you can think of. Consider

(a) $e^+e^- \rightarrow \mu^+\mu^-$.

(b) $e^-\mu^- \rightarrow e^-\mu^-$.

(c) Compton scattering.

(d) Pair annihilation.

(e) Bhabha scattering. *Hint: Be careful to regulate the cross-section by using a cut on the forward angle θ .*

(f) $e^+e^- \rightarrow \gamma + B$ from problem 5.5. Also try to verify the result for the decay width of B .

Problem 3. Monte Carlo Integration. Calculate π by means of Monte Carlo integration.

(a) To this end, consider a definite integral whose numerical value is proportional to π , then calculate the integral by Monte Carlo methods. For example, we know that the area of the unit circle is equal to π , hence the area of half a circle is $\pi/2$:

$$\frac{\pi}{2} = \int_{-1}^1 dx \sqrt{1-x^2} = 2 \int_0^1 dx \sqrt{1-x^2}, \quad (1)$$

and a convenient integral representation of π is

$$\pi = 4 \int_0^1 dx \sqrt{1-x^2}. \quad (2)$$

Now write a simple program (in your language of choice) which integrates the right-hand side. *Hint: For example, in fortran the function `rand()` generates a random number between 0 and 1.* Notice that the largest value of the integrand is $y = 1$, so it is convenient to take “the box” encompassing the integrand to be simply the unit square

$$0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

in which case we do not need to rescale the values returned by `rand()`. Investigate the numerical precision of the Monte Carlo integration as a function of the number of tries.

(b) For fun, you may try a different integral representation of π and see if you get the same answer. Now that you have a functioning code, that should be pretty easy.

(c) What are the advantages of Monte Carlo integration over the traditional integration methods (e.g. Simpson's rule, Gauss' rule etc.)?