SUSY and Cosmology

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The Dark Matter Problem

- The SM is incomplete and leaves open a lot of questions

My personal view is that [the breaking of the gauge symmetry] is the greatest of the Great Puzzles.

M. Peskin, last Thursday

- SM answer: new particle, Higgs boson. Will be discovered/studied at LHC/LC. But... gauge hierarchy problem.

- Many models BSM were proposed to address this issue, collider experiments will tell us which one is right in our lifetime. Generic prediction: more new particles.

- With all due respect to the gauge hierarchy problem, dark matter is our best experimental evidence for new physics BSM.
  - We know how much DM there is (23 ± 4%). (talk: R. Gaitskell)
  - We don’t know what it is: a new equation (DLS?) or a new particle. (talk: E. Baltz)

- Perhaps the two puzzles are related?

- Tip for experimentalists: always ask model builders

  Who is the dark matter in your theory/model
  and can you calculate its relic abundance?

  How can we test this theory at the Tevatron/LHC/LC?
Dark matter and new physics BSM

- There are many possible DM candidates (talk: E. Baltz)
- WIMPs: motivated by both particle and astrophysics.
  - Predicted in many particle physics scenarios BSM.
  - Give the right order of magnitude $\Omega_{DM}$.

$$\Omega_{DM} h^2 \sim 0.1 \left( \frac{\sigma_{EW}}{\sigma_{ann}} \right)$$

Is this simply a coincidence?

- Potentially observable signals in DM detection expts. Since they must have been able to annihilate in the Early Universe, they should also produce observable signals in direct and indirect dark matter detection experiments.

- Potentially observable signals in collider expts.
How to build a model with DM WIMPs

- **Recipe for BSM dark matter**
  - invent a model with new particles
  - invent a symmetry which guarantees a stable new particle
  - fudge parameters until the lightest new stable particle is neutral and has the correct relic density

- **Example: SUSY**
  - who is the DM particle?
  - why is it stable?
  - which parts of the MSSM parameter space are singled out by cosmology?
  - how do they map onto the parameter spaces of specific models?
  - SUSY DM discovery prospects
    - at colliders
    - in DM detection experiments

- **Is it really SUSY?**
Supersymmetry is an extra dimension theory with new anticommuting coordinates $\theta_\alpha$:

$$\Phi(x^\mu, \theta) = \phi(x^\mu) + \psi^\alpha(x^\mu)\theta_\alpha + F(x^\mu)\theta^\alpha\theta_\alpha$$

- SUSY relates SM particles and their superpartners ($\phi \leftrightarrow \psi$)
  - quarks, leptons $\Leftrightarrow$ squarks, sleptons
  - gauge bosons: $g, W^\pm, W^0_3, B^0 \Leftrightarrow$ gauginos: $\tilde{g}, \tilde{w}^\pm, \tilde{w}^0, \tilde{b}^0$
  - Higgs bosons: $h^0, H^0, A^0, H^\pm \Leftrightarrow$ higgsinos: $\tilde{h}^\pm, \tilde{h}^0_u, \tilde{h}^0_d$
  - graviton: $G \Leftrightarrow$ gravitino: $\tilde{G}$

- The superpartners have
  - spins differing by 1/2
  - identical couplings
  - unknown masses (model-dependent)

- Discovering new particles with those properties IS discovering supersymmetry

- Potential SUSY DM candidates (neutral superpartners):
  - gauginos ($\tilde{w}^0, \tilde{b}^0$)
  - higgsinos ($\tilde{h}^0_u, \tilde{h}^0_d$)
  - sneutrinos ($\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$)
  - gravitino ($\tilde{G}$)
New supersymmetric Yukawa interactions violating baryon number and lepton number lead to too rapid proton decay.

\[
p^+ \begin{cases} 
  d_R \\
  u_R \\
  u 
\end{cases} \rightarrow \bar{s}_R \rightarrow e^+_L \bar{u}_L \pi^0
\]

Introduce $R$-parity $R = (-1)^{3(B-L) + 2S}$

- SM particles: $R = +1$
- superpartners: $R = -1$

Impose $R$-parity conservation $\Pi R_i = 1$ at each vertex $\implies$ eliminate all dangerous proton decay diagrams

Side benefits from $R$-parity conservation

- No tree-level contributions to precision EW observables
- Stable LSP (lightest superpartner) $\implies$ dark matter?
Relic density calculation

- At early times, the DM particles $\chi$ are in thermal equilibrium with the SM stuff:
  \[ \chi\chi \leftrightarrow X\bar{X} \]

- The process of freeze-out of thermal relics is described by the Boltzmann equation
  \[ \frac{dn_\chi}{dt} = -3H n_\chi - \langle \sigma_A v \rangle \left( n_\chi^2 - n_{eq}^2 \right) \]
  - $-3H n_\chi$ accounts for dilution due to the Hubble expansion
  - $-\langle \sigma_A v \rangle n_\chi^2$ describes depletion due to $\chi\chi \rightarrow X\bar{X}$
  - $+\langle \sigma_A v \rangle n_{eq}^2$ describes resupply due to $X\bar{X} \rightarrow \chi\chi$
  - $\sigma_A$ is the total DM annihilation cross-section:
  \[ \sigma_A \equiv \sum_X \sigma(\chi\chi \rightarrow X\bar{X}) \equiv a + bv^2 + O(v^4) \]
  - An approximate analytical solution
  \[ \Omega h^2 = 0.08 \frac{1 \text{ pb}}{a + (3b - 0.75a)x_F} \]
  where $x_F = T_F/M_\chi \sim 0.04$, with $T_F$ the freeze-out temperature.
What does WMAP tell us?

- Two classes of DM candidates:
  - $a \sim b$: “s-annihilators”
  - $a < bx_F << b$: “p-annihilators”

- The present amount of dark matter $\Omega_{DM} h^2 = 0.112 \pm 0.009$ tells us exactly the WIMP annihilation cross-section:
  
  Birkedal, KM, Perelstein, hep-ph/0403004

- The dependence on $M_\chi$ is only logarithmic and is offset by the change in effective number of dof in the SM at different $T_F$.

- How does SUSY map onto this picture?
The relic density calculation is a standard and tiresome tree-level computation which should be done on a computer!

There are readily available codes:

- **Neutdriver** (Jungman)
- **DarkSUSY** (Gondolo, Edsjo, Ullio, Bergstrom, Baltz)
- Micromegas (Belanger, Boudjema, Pukhov, Semenov)
- IsaRED (Baer, Balazs, Belyaev)
- SSARD (Ellis, Falk, Olive)
- Drees/Nojiri code
- Roszkowski code
- Arnowitt/Nath code
- Lahanas/Nanopoulos code
- Bottino/Fornengo *et al.* code

Homework: Do the subsequent neutralino examples on a computer!
Sneutrinos as DM candidates

- Problem: annihilate too fast, would have been detected:

Falk, Olive, Srednicki (1994)
Neutralinos as DM candidates

- The neutral gauginos and higgsinos mix:
  \[
  \begin{pmatrix}
  M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\
  0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\
  -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\
  M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \\
  \end{pmatrix}
  \]

  where \(c_W = \cos \theta_W, \ s_W = \sin \theta_W, \ c_\beta = \cos \beta, \ s_\beta = \sin \beta\).

- The lightest neutralino \(\tilde{\chi}_1^0\) is a mixture of \(\tilde{b}^0, \tilde{w}^0, h_d^0, \tilde{h}_u^0\):
  \[
  \tilde{\chi}_1^0 = a_1 \tilde{b}^0 + a_2 \tilde{w}^0 + a_3 \tilde{h}_d^0 + a_4 \tilde{h}_u^0
  \]

- The masses of the four neutralino eigenstates are approximately \(\{M_1, M_2, \mu, \mu\}\).

- The calculation of the neutralino relic density for the general case is rather complicated - involves many diagrams.

- For simplicity let us consider first the three limiting cases
  - pure Bino: \(M_1 << M_2, \mu \Rightarrow \tilde{\chi}_1^0 \approx b^0\)
  - pure Wino: \(M_2 << M_1, \mu \Rightarrow \tilde{\chi}_1^0 \approx w^0\)
  - pure Higgsino: \(\mu << M_1, M_2 \Rightarrow \tilde{\chi}_1^0 \approx (h_u^0 \pm h_d^0)/\sqrt{2}\)
Pure Bino dark matter

- Possible Bino annihilation channels:

  \[ \tilde{b}^0 \rightarrow f \tilde{f} \]
  \[ \tilde{b}^0 \rightarrow h \tilde{h}^0 \]

- Bino annihilation suffers from several suppression factors
  - No s-channel diagrams in the pure Bino limit.
  - No gauge boson final states.
  - Sfermion mass suppression in (a)
  - Kinematic and Higgsino mass suppression in (b).
  - Either helicity or p-wave suppression in (a): \( \chi^0 \) are Majorana fermions \( \Rightarrow \) Pauli principle requires \( S = 0 \)
    - If S-wave, \( J = 0 \) and helicity flip required on \( f \) line
      (recall \( \pi^+ \) decay)
    - If P-wave, \( v^2 \) suppression and \( bx_F \gg a \).
  - Bino relic density tends to be too large, unless perhaps the sfermions are light. Upper bounds on SUSY masses?
Pure Wino dark matter

- Possible Wino annihilation channels:

\[ \tilde{w}^0 \rightarrow \tilde{f} \tilde{f} \quad (a) \]
\[ \tilde{w}^0 \rightarrow \tilde{h} \tilde{h} \quad (b) \]
\[ \tilde{w}^0 \rightarrow W^+ W^- \quad (c) \]

- Diagrams (a) and (b) suffer from the same suppression as before, but the new process (c) is unsuppressed and allows rapid annihilation.

- One can adjust parameters close to the \( \tilde{w}^0 \tilde{w}^0 \rightarrow W^+ W^- \) threshold by taking \( M_{\chi_1^0} \sim M_2 \) near \( M_W \), but then by SU(2) symmetry the associated lightest chargino mass \( M_{\tilde{\chi}_1^\pm} \sim M_2 \) is below the LEP bound.

- Bottomline: the Wino relic density is too small.
• Possible Higgsino annihilation channels:

(a) $\tilde{h}^0 \rightarrow f \tilde{f}$

(b) $\tilde{h}^0 \rightarrow h b^0, \tilde{w}^0$

(c) $\tilde{h}^0 \rightarrow W^+ h$

(d) $\tilde{h}^0 \rightarrow Z h$

• Diagrams (c) and (d) are unsuppressed and allow rapid annihilation.

• One can adjust parameters close to the $\tilde{h}^0 \tilde{h}^0 \rightarrow W^+ W^-$ threshold by taking $M_{\chi^0_1} \sim \mu$ near $M_W$, but then by SU(2) symmetry the associated lightest chargino mass $M_{\tilde{\chi}^\pm_1} \sim \mu$ is below the LEP bound.

• Bottomline: the Higgsino relic density is too small.
Good neutralino DM candidates

- We have seen that the pure interaction neutralino eigenstates are not good DM candidates (homework):
  - Bino: annihilates too slowly, gives too much DM or requires light scalar superpartners.
  - Wino: annihilates too fast, too little DM.
  - Higgsino: annihilates too fast, too little DM.
- What about mixed cases?
  - mixed Wino-Higgsino LSP ($M_2 \sim \mu << M_1$). Nothing new, still too little DM.
  - mixed Bino-Higgsino LSP ($M_1 \sim \mu << M_2$). Should work (Focus Point SUSY).
  - mixed Bino-Wino LSP ($M_1 \sim M_2 << \mu$). Should work (rSUGRA, non-universal gaugino masses).
- Other options: “accidental degeneracies”, leading to an enhancement of the Bino annihilation rates due to
  - a resonance in $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow Res \rightarrow f \bar{f}$
  - coannihilations with other superpartners
- Let us now illustrate all this with a specific SUSY model.
Focus point region

- MSUGRA can accommodate all but one of the cases above.
- Gaugino fraction $R_\chi$ of the LSP:
  \[ R_\chi \equiv |a_1|^2 + |a_2|^2 \approx |a_1|^2. \]
  \[ \tilde{\chi}^0_1 = a_1 \tilde{b}^0 + a_2 \tilde{w}^0 + a_3 \tilde{h}_d^0 + a_4 \tilde{h}_u^0. \]

Feng, KM, Wilczek (2000)

- Focus point region: large $m_0$, mixed Bino-Higgsino LSP.
- “Bulk” region: small $m_0$, Bino LSP, light scalars.
The exceptional cases

- If other superpartners are nearly degenerate with the (Bino) LSP, they can help it annihilate. In addition to Bino annihilation, we also have conversion and associated annihilation:

\[ \tilde{b}^0 \rightarrow f \tilde{f} \]
\[ f \rightarrow \gamma \]

- The required degeneracy is roughly \( \Delta M < T_F \sim M_\chi/25 \).
- \( f \) can be a lepton or quark.
- Finally, a resonant enhancement appears for \( 2M_\chi \sim M_{res} \)

\[ \tilde{\chi}^0 \rightarrow f \tilde{f} \]
\[ f \rightarrow \gamma \]

- The resonance can be \( h, A, Z \)
SUSY DM in MSUGRA

- Minimal Supergravity illustration
  - bulk
  - focus point
  - stau coannihilation
  - Higgs funnel

Baer, Balazs, Belyaev (2002)

\[
\begin{align*}
1 & < \Omega h^2 < 1.0 \\
1 & < \Omega h^2 < 0.1
\end{align*}
\]

- A stop coannihilation region also exists (at large $|A_0|$).
The case of a Bino-Wino mixed LSP does not arise in MSUGRA or models with gaugino unification, where

$$\frac{M_2}{M_1} = \frac{\alpha_2}{\alpha_1} \approx 2.$$ 

Relaxing the gaugino unification assumption leads to rSUGRA

$$r \equiv \frac{M_2(M_{GUT})}{M_1(M_{GUT})} \neq 1$$

and plausible relic densities for $r \sim 0.6$. (Birkedal, Nelson 2002)

Gravitino LSP (aka superWIMP)? (Feng, Rajaraman, Takayama 2003)

$\Omega_{NLSP}$ is determined by the NLSP annihilation rate. Later on, the NLSP decays, e.g. $\tilde{\chi}_1^0 \rightarrow \tilde{G}\gamma$ and the superWIMP automatically inherits the NLSP relic density scaled by $M_{\tilde{G}}/M_{NLSP}$.

Formerly ruled out “charged LSP” is now OK, e.g. $\tilde{\tau} \rightarrow \tilde{G}\tau$.

SuperWIMPs are disastrous for DM searches, but the products of the NLSP decays may have observable astrophysical signatures. (J. Feng, SSI 2003)
SUSY DM detection at colliders

- SUSY searches at colliders are done for particles other than the LSP and are very model-dependent (see talks this afternoon).
- Can cosmology teach us about true DM signals at colliders? It depends what the annihilation products are. For example:

\[
\begin{align*}
\tilde{\chi}^0 + \tilde{\chi}^0 &\rightarrow e^+ e^- \\
\tilde{\chi}^0 + \tilde{\chi}^0 &\rightarrow e^+ e^-
\end{align*}
\]

- The two rates are related by detailed balancing:

\[
\frac{\sigma(\chi + \chi \rightarrow X_i + \bar{X}_i)}{\sigma(X_i + \bar{X}_i \rightarrow \chi + \chi)} = 2 \frac{v_X^2 (2S_X + 1)^2}{v_X^2 (2S_\chi + 1)^2}
\]

and we predict the WIMP production rate at an \( X_i \bar{X}_i \) collider:

\[
\sigma(X_i \bar{X}_i \rightarrow 2\chi) = 2^{2(J_0-1)} \kappa_i \sigma_{\text{an}} \left( \frac{2S_X + 1}{2S_\chi + 1} \right)^2 \left( 1 - \frac{4M_\chi^2}{s} \right)^{1/2+J_0}
\]

where \( \kappa_i \) is the annihilation fraction for \( \tilde{\chi}^0 \tilde{\chi}^0 \rightarrow X_i \bar{X}_i \).
- Known: \( \{\sigma_{\text{an}}, S_X, s\} \), unknown: \( \{\kappa_i, M_\chi, S_\chi, J_0\} \).
- Applicable only close to 2\( \chi \) threshold (\( v_\chi \ll 1 \)).
- But... no trigerrable signature!
Photon tagging from ISR

What to do?

- Give up model-independence, consider production of heavier particles (superpartners, KK-modes etc.)
- Tag the known initial state with a photon (gluon)

\[
\begin{align*}
\frac{d\sigma(e^+e^- \rightarrow 2\chi + \gamma)}{dx d\cos \theta} &= \frac{\alpha 1 + (1 - x)^2}{\pi x} \frac{1}{\sin^2 \theta} \tilde{\sigma}(e^+e^- \rightarrow 2\chi)
\end{align*}
\]

where \( x \equiv 2E_\gamma/\sqrt{s} \), \( \tilde{\sigma} \) is evaluated at \( \hat{s} = (1 - x)s \).

- Analogously, WIMP production at hadron colliders can be predicted as a jet+MET signature. Very challenging experimentally...
• Discovery reach of LC500. Cuts:
  • $\sin \theta_{\gamma} > 0.1$, $p_{T,\gamma} > 7.5$ GeV (suppress Bhabha).
  • $\chi$’s must be non-relativistic and $E_{\gamma}$ below threshold:
    \[
    \frac{\sqrt{s}}{2} \left(1 - \frac{8M_{\chi}^2}{s}\right) \leq E_{\gamma} \leq \frac{\sqrt{s}}{2} \left(1 - \frac{4M_{\chi}^2}{s}\right).
    \]
  • No polarization. $L = 500$ fb$^{-1}$.

• In the “bulk” region $\kappa_e \sim 25\%$. Polarization would help.
- Combination of “all” pre-LHC experiments
  - Direct SUSY searches: Tevatron
  - Indirect SUSY searches: E827, B-factories
  - Direct WIMP searches: CDMS, CRESST, GENIUS
  - Indirect WIMP searches: Amanda, AMS, GLAST

- Many possible DM signals before 2007-08.
- Particle physics and astrophysics probes are highly complementary.
## SUSY and the competition

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