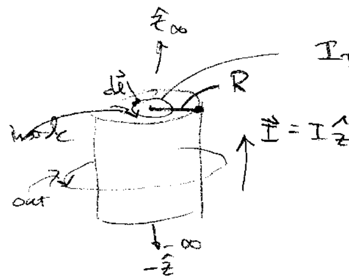
①

5.26 4th
5.25 3rd
5.23 2nd

$\vec{A} = ?$



I_{Total} evenly ~~across~~ through surface

(a) outside

(b) inside

Note: this is one approach to the problem and I prefer to start "inside".

(b) Note: $\vec{A} = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{\vec{J}}{\mathscr{r}}\, d\tau \Rightarrow \underline{\vec{A} = A(s)\,\hat{z}}$. w/ ϕ symmetry, z symmetry

as $\vec{J} \propto \vec{I}$.

know: $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc} \qquad \text{and} \qquad \hat{B} \propto \hat{\phi} \text{ only (not } \hat{z}, \text{ or } \hat{s})$$

$$B\, 2\pi s = \mu_0 \frac{I}{\pi R^2}\pi s^2$$

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{R^2} s\,\hat{\phi}$$

But $\vec{\nabla}\times\vec{A} = \vec{B} = \dfrac{\mu_0}{2\pi}\dfrac{I}{R^2} s\,\hat{\phi} = \left[\dfrac{1}{s}\dfrac{\partial A_z}{\partial \phi} - \dfrac{\partial A_\phi}{\partial z}\right]\hat{s} + \left[\dfrac{\partial A_s}{\partial z} - \dfrac{\partial A_z}{\partial s}\right]\hat{\phi}$

$$+ \frac{1}{s}\left[\frac{\partial}{\partial s}(s A_\phi) - \frac{\partial A_s}{\partial \phi}\right]\hat{z}$$

but already argued that $A_\phi = A_s = 0$ and $\underline{A_z \text{ has only } s \text{ depend}}$

$$= [0 - 0]\hat{s} + \left[0 - \frac{\partial A_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}[0 - 0]\hat{z}$$

$$\frac{\mu_0}{2\pi}\frac{I}{R^2} s = -\frac{\partial A_z}{\partial s} \Rightarrow \partial A_z = -\frac{\mu_0}{2\pi}\frac{I}{R^2} s\, \partial s$$

$$\Rightarrow \boxed{A_z^{(s)}{}_{in} = -\frac{\mu_0}{2\pi}\frac{I}{R^2}\frac{1}{2}s^2 + C'_{in}}$$

(c) outside $\vec{B} = \frac{\mu_0 I}{2\pi}\frac{1}{s}\hat{\phi}$ so $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl.}$

^ all of I

Same as above $\vec{\nabla}\times\vec{A} = \vec{B}_{out} \Rightarrow$

$$\frac{\mu_0 I}{2\pi}\frac{1}{s} = -\frac{\partial A_z}{\partial s} \Rightarrow \boxed{A_{z,out}(s) = -\frac{\mu_0 I}{2\pi}\ln(s) + C_{1,out}}$$

But A must be continuous at $s=R$.

$$A_{z,in}(s=R) = -\frac{\mu_0}{4\pi}\frac{I}{R^2}R^2 + C_{1,in} = -\frac{\mu_0 I}{2\pi}\ln(R) + C_{out} = A_{z,out}(s=R)$$

1 equation but 2 unknowns? ☺

Go back and recall better "form" for constants:

$$A_z(s)_{/out} = -\frac{\mu_0 I}{2\pi}\ln\left(\frac{s}{C_{in}}\right)$$

$$A_{z,in}(s) = -\frac{\mu_0}{4\pi}\frac{I}{\pi R^2}(s^2 - C_{out}^2)$$

$C_{in} = C_{out} = R$ works! as $A_{z,in}(s=R) = A_{z,out}(s=R) = 0$!

check

$\vec{\nabla}\cdot\vec{A} = 0$?

$\vec{\nabla}\times\vec{A} = B$?

ok.

so

$$\boxed{\begin{aligned}\vec{A}(s) &= -\frac{\mu_0 I}{2\pi}\ln\left(\frac{s}{R}\right)\hat{z};\quad s\geq R\\ &= -\frac{\mu_0}{4\pi}\frac{I}{\pi R^2}(s^2 - R^2)\hat{z};\quad s\leq R\end{aligned}}$$

I think the solution for "inside" has an extra factor of pi downstairs as a typo, so double-check....