PHY 3323 "Electromagnetism 1" Fall 2018
Quiz 6 (30 points) 28 November 2018
Professor Mark W. Meisel

PLEASE PRINT and box final results. Use only pencil or blue or black ink. Show all work for full credit. Work must be clear and unambiguous for credit. Please place your name on the upper right-hand corner of every worksheet. Labels for your work must be unambiguous. Please USE ONLY ONE-SIDE of a sheet of paper. Calculators may NOT be used. This exam must be your own independent work. Unless otherwise stated, the notation is the same as used in lecture and the textbook.
If you agree with this statement: *"On my honor, I have neither given nor received unauthorized aid in doing this assignment."*; then please **PRINT** and **SIGN** your name.

NAME (Print and Signature): ______________________________

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Some integrals and expressions, used while working some problems, that you might wish to recall.

$$\int (\sin^2 ax)\, dx = \frac{1}{2}x - \frac{1}{4a}\sin 2ax \qquad \int (\cos^2 ax)\, dx = \frac{1}{2}x + \frac{1}{4a}\sin 2ax$$

$$\int (\sin ax)(\cos ax)\, dx = \frac{1}{2a}\sin^2 ax \qquad \int \frac{x\,dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{-1}{\sqrt{(x^2 \pm a^2)}}$$

$$\int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{(x^2 \pm a^2)}} \qquad \int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2}\,) \qquad \int \frac{x^3\,dx}{\sqrt{x^2 \pm a^2}} = \frac{1}{3}\sqrt{(x^2 \pm a^2)^3} \mp a^2\sqrt{x^2 \pm a^2}$$

$$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 + a^2}\,)\right]$$

$$\int e^{ax}\, dx = \frac{e^{ax}}{a} \qquad \int x\, e^{ax}\, dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^m e^{ax}\, dx = \frac{x^m e^{ax}}{a} - \frac{m}{a}\int x^{m-1} e^{ax}\, dx \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{\sin\theta\, d\theta}{(z^2 + R^2 - 2zR\cos\theta)^{1/2}} = \frac{1}{zR}(z^2 + R^2 - 2zR\cos\theta)^{1/2} \qquad \sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\int \frac{\sin\theta\, d\theta}{(a^2 + R^2 - 2aR\cos\theta)^{3/2}} = -\frac{1}{aR}(a^2 + R^2 - 2aR\cos\theta)^{-1/2} \qquad \cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\int_0^a \sin(n\pi y/a)\,\sin(n'\pi y/a)\, dy = \begin{cases} 0, & \text{if } n' \neq n, \\ \frac{a}{2}, & \text{if } n' = n. \end{cases} \qquad \int_0^a \sin(n\pi y/a)\, dy = \begin{cases} 0, & \text{for } n \text{ even}, \\ \frac{2a}{n\pi}, & \text{for } n \text{ odd}. \end{cases}$$

$$P_l(x) = \frac{1}{2^l\, l!}\left(\frac{d}{dx}\right)^l (x^2 - 1)^l, \quad P_0(x) = 1, \qquad P_1(x) = x, \quad P_2(x) = (3x^2 - 1)/2$$

$$\int_{-1}^{1} P_l(x)\, P_{l'}(x)\, dx = \int_0^\pi P_l(\cos\theta)\, P_{l'}(\cos\theta)\,\sin\theta\, d\theta = \begin{cases} 0, & \text{if } l' \neq l, \\ \frac{2}{2l+1}, & \text{if } l' = l. \end{cases}$$

$$V(r,\theta) = \sum_{k=0}^{\infty}\left(A_k r^k + \frac{B_k}{r^{k+1}}\right) P_k(\cos\theta)$$

$$V(s,\phi) = a_0 + b_0 \ln s + \sum_{k=1}^{\infty}\left(a_k s^k + \frac{b_k}{s^k}\right)(c_k \cos k\phi + d_k \sin k\phi)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_o}\sum_{n=0}^{\infty}\frac{1}{r^{(n+1)}}\int (r')^n P_n(\cos\theta')\,\rho(\vec{r}\,')\, d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint (r')^n P_n(\cos\theta')\, d\vec{l}\,'$$

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DO NOT TURN OVER until instructed.

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You may wish to recall

$$\int \sin^3 ax\,dx \;=\; -\frac{1}{3a}(\cos ax)(\sin^2 ax\,+\,2)\ ,\quad \int \cos^3 ax\,dx \;=\; \frac{1}{3a}(\sin ax)(\cos^2 ax\,+\,2)\ ,$$

$$\int_0^{\pi} \sin\theta\,d\theta \;=\; 2\ ,\quad \int_0^{\pi} \sin^3\theta\,d\theta \;=\; \frac{4}{3}\ ,\quad \text{and/or}\quad \int \frac{x^3\,dx}{\sqrt{(x^2\pm a^2)^3}} \;=\; \sqrt{x^2\pm a^2}\;\pm\;\frac{a^2}{\sqrt{x^2\pm a^2}}\ .$$

In case you may find it useful, especially when working Problem 6.25 (6.23), here is a discussion about one of the boundary conditions (abbreviated BC) for the magnetic scalar potential, $\Phi_{\rm m}$ where $\vec{H} = -\vec{\nabla}\Phi_{\rm m}$ (note: the textbook uses the notation W for $\Phi_{\rm m}$, which is my preference). Specifically, one BC (often referred to as "BC-IV" in lecture) is given by Eq. 6.24 (both editions) as

$$H^{\perp}_{\rm out}\;-\;H^{\perp}_{\rm in}\;=\;-\left(M^{\perp}_{\rm out}\;-\;M^{\perp}_{\rm in}\right)\quad . \tag{1}$$

When a material possesses a frozen magnetization, $\vec{M}$, this "BC-IV" can be written as

$$\left.\frac{\partial\Phi_{\rm m,out}}{\partial r}\right|_{\rm surface}\;-\;\left.\frac{\partial\Phi_{\rm m,in}}{\partial r}\right|_{\rm surface}\;=\;-\,\sigma_{\rm m}\quad . \tag{2}$$

In Eq. 2, $\sigma_{\rm m} = \vec{M}\cdot\hat{n}$, and $\sigma_{\rm m}$ is an effective magnetic surface charge that accounts for the discontinuity generated by the existing magnetization of the material. This reasoning follows from the analogy to the case of electric polarization.

On the other hand, when an unmagnetized material with a linear $\chi_{\rm m}$ is placed in an external magnetic field, "BC-IV", Eq. 1, can be easily manipulated to group the *out* terms to the LHS (left-hand-side) of the equation and the *in* terms to the RHS (right-hand-side) of the equation. The result is $B^{\perp}_{\rm out} \;=\; B^{\perp}_{\rm in}$. Consequently, in terms of $\Phi_{\rm m}$ for this specific case, "BC-IV" is

$$\mu_o\,\left.\frac{\partial\Phi_{\rm m,out}}{\partial r}\right|_{\rm surface}\;=\;\mu\,\left.\frac{\partial\Phi_{\rm m,in}}{\partial r}\right|_{\rm surface}\quad . \tag{3}$$

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1. (3 points) Have you placed your name and page number in the upper right-hand corner of each sheet of your work except for the cover sheet and the top of this flip-side of the cover sheet? Have you avoided placing any information or work in the upper left-hand corner of any sheet as this space is "reserved" for the staple (if needed)? Have you clearly identified the work associated with each problem? Have you used only one side of each sheet of paper excluding the cover sheet? Have you read the *Honor Code* statement and subsequently printed and signed your on the cover sheet? (Circle your response on this sheet.)

YES **NO**