The discovery of CP violation (2)

If $CP$ is conserved, the selection rules (11.23) and (11.24) immediately suggest the identification

$$K_S^0 = K_1^0 \quad \text{and} \quad K_L^0 = K_2^0.$$  \hspace{1cm} (11.31)

However, in 1964 Christenson, Cronin, Fitch and Turlay discovered that $K_L^0$ also decayed to two pions

$$K_L^0 \rightarrow \pi^+ + \pi^-,$$ \hspace{1cm} (11.32)

with a branching ratio of order $10^{-3}$. This result is clear evidence of $CP$ violation, since two-pion final states have $CP = 1$, whereas the dominant three-pion final states (11.29) have $CP = -1$, as we showed above.

The experiment of Christenson, Cronin, Fitch and Turlay where the CP violation has been discovered is discussed below.
The discovery of CP violation (3). Beam

The first task is to prepare a $K^0_L$ beam.

This was done by Christenson, Cronin, Fitch and Turlay in their 1964 experiment at Brookhaven proton accelerator, by colliding a 30 GeV proton beam with a metal target and forming a secondary beam from the produced particles from the produced particles using a lead collimator.

The charged particles were swept out of the beam by a bending magnet, while most of the photons were removed by passing it through a 4cm thick block of lead, which is not sufficient for kaons to interact hadronically (the radiation length in lead is significantly higher than hadronic interactions length, because of the difference in cross-sections of electromagnetic and hadronic interactions in lead). The bending magnet and 4cm block of lead are not shown in the picture of experimental apparatus on the next slide.

The beam, initially containing both $K^0_S$ and $K^0_L$ mesons (and neutral background particles), through a collimator C reached the detection apparatus 18 m away (shown on the next slide). By that time the short-lived $K^0_S$ component had ~ completely decayed (several exponents!), leaving a beam that consisted mainly of $K^0_L$ particles and background neutrons, together with a few background photons that had penetrated the lead block.
The discovery of CP violation (4) Apparatus

Figure 11.10 Schematic diagram of the apparatus used in the discovery of CP violation. The $K^0_L$ beam entered a helium-filled bag B (suppressing background from beam interactions in a bag gas) through a lead collimator C. The CP-violating decays in the shaded region in B were detected by the symmetrically spaced spectrometers, containing (each) a pair of spark chambers A separated by a magnet M. The spark chambers, recording tracks, have been triggered by coincidence of scintillation counters S and a water Cerenkov counters W, recording charged particles with velocities greater than 0.75c (including fast moving pions from the decay of kaons, check the kinematics). This trigger eliminated background events involving slow-moving particles produced, for example, due to the collisions of neutrons.
The discovery of CP violation (5) Results

The decay events occur only at zero angles to the beam and at an invariant mass of 498 MeV/c^2, and background events have a smooth dependence on this angle and on the invariant mass. There is a peak in the beam direction $\theta = 0^0$ for the invariant mass range $494 < m < 504$ GeV/c^2 (on the left part of Fig 11.11 below), which is from the CP-violating decay $K^0_L \rightarrow \pi^+ + \pi^-$.

Figure 11.11 Angular distribution of the $\pi^+ \pi^-$ pairs detected using the apparatus of Figure 11.10, where $\theta$ is the angle between the line-of-flight of the centre-of-mass of the pair and the initial beam direction. Results are shown for an invariant mass range including the $K^0_L$ mass (498 MeV/c^2), and for two neighbouring mass ranges. (J.H. Christenson et al. 1964. Reproduced with permission from the American Physical Society.)
CP-violating $K^0_L$ decays

Because CP is not conserved, the physical states with different lifetimes $K^0_S$ and $K^0_L$ need not correspond to the CP eigenstates $K^0_1$ and $K^0_2$ if CP is conserved but K-mixing happens (as assumed in 11.31). Now, they can contain small components of states with the opposite CP:

$$\left| K^0_S, p = 0 \right> = \frac{1}{(1 + |\varepsilon|^2)^{1/2}} \left[ |K^0_1, p = 0 \rangle + \varepsilon |K^0_2, p = 0 \rangle \right]$$  \hfill (11.33a)

$$\left| K^0_L, p = 0 \right> = \frac{1}{(1 + |\varepsilon|^2)^{1/2}} \left[ \varepsilon |K^0_1, p = 0 \rangle + |K^0_2, p = 0 \rangle \right]$$  \hfill (11.33b)

This can be verified also by measurements of the semileptonic decays:

$$K^0_L \rightarrow \pi^- + e^+ + \nu_e \quad \text{and} \quad K^0_L \rightarrow \pi^+ + e^- + \bar{\nu}_e.$$  \hfill (11.34)

Note, that the $K^0$ and anti-$K^0$ can decay by the semileptonic reactions:

$$K^0 \rightarrow \pi^- + e^+ + \nu_e \quad \text{and} \quad \bar{K}^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e,$$  \hfill (11.35)

whereas the corresponding reactions

$$K^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \quad \text{and} \quad \bar{K}^0 \rightarrow \pi^- + e^+ + \nu_e$$  \hfill (11.36)

are forbidden by the $\Delta S = \Delta Q$ selection rule (9.30), (These decays are possible only in higher orders of weak interactions, which are suppressed and thus unobservable, try quark diagrams)
Thus the relative yields $N^+$ and $N^-$ of positrons and electrons in the decays

yield a direct measure of the relative probabilities of finding a $K^0$ or anti-$K^0$

\begin{align*}
K^0_L &\rightarrow \pi^- + e^+ + \nu_e \quad \text{and} \quad K^0_L \rightarrow \pi^+ + e^- + \bar{\nu}_e. 
\end{align*}

(11.34)

d (11.21), see M&S, these are given by

$$N^\pm \propto |1 \pm \varepsilon|^2 (1 + |\varepsilon|^2)^{-1},$$

and if we neglect terms of order $|\varepsilon|^2$, the asymmetry for a $K^0_L$ beam is:

$$A \equiv (N^+ - N^-)/(N^+ + N^-) = 2 \text{Re} \varepsilon$$

(11.37)

Figure 11.12 shows data on the asymmetry (11.37) as a function of time in the rest frame of the decaying particles. The beam that is initially predominantly a $K^0$ state, which can be formed in strong interactions, conserving strangeness. After the initial oscillations, which occur because the $K^0_S$ decays faster than the $K^0_L$, there is an asymmetry whose value is $2 \text{Re} \varepsilon \approx 3.2 \times 10^{-3}$. 

![Figure 11.12](image-url)
The magnitude of $\epsilon$ can be deduced from the measured rates of the two CP-violating decays, which both have been measure after the initial discovery:

$$K^0_L \rightarrow \pi^+ + \pi^- \ (B = 2.0 \times 10^{-3}),$$
$$K^0_L \rightarrow \pi^0 + \pi^0 \ (B = 8.6 \times 10^{-4}) ,$$

These CP-violation decays can occur in two different ways: either (i) CP violation by mixing, in which the CP-forbidden $K^0_1$ component in the $K^0_L$ decays via the CP-allowed process (11.23), giving a contribution proportional to the probability $|\epsilon|^2/(1 + |\epsilon|^2) \approx |\epsilon|^2$ of finding a $K^0_1$ component in the $K^0_L$; or (ii) Direct CP violation, in which the $K^0_2$ component in the $K^0L$ decays directly to pion pairs via the CP-violating reactions. In practice, CP violation by mixing dominates (see M&S for a more detailed discussion).
Flavor oscillations and the CPT theorem

Although CP is not conserved, there is good reason to believe that all interactions are invariant under the combined operation of charge conjugation C, parity P and time reversal T. This result is called the CPT theorem and can be shown to hold in any relativistic quantum theory in which signals cannot propagate faster than the speed of light.

It has been shown that like CP, the combined operation of CPT converts particles to antiparticles, and invariance under this operation requires particles and antiparticles to have the same masses and lifetimes. This is in accord with experiment and is tested e.g. by observations on so-called ‘flavor oscillations’ in neutral meson systems. We will concentrate on the neutral kaons, where the CPT invariance has been tested with a very good precision.
When neutral kaons are produced in strong interaction processes, associated production means they are almost invariably produced with definite strangeness. For example, if the neutral kaon produced in the strong reaction

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

must be in a $K^0$ state with $S = 1$, in order to conserve strangeness. However, if the produced particle is allowed to travel through free space and its strangeness is measured, one finds that it no longer has a definite strangeness $S = 1$, but has components with both $S = 1$ and $S = -1$ whose intensities oscillate with time. It is called strangeness oscillations, it happens (similar to neutrino oscillations) when there is a mass difference between $K^0_\text{S}$ and $K^0_\text{L}$ particles. As a result, when measuring oscillations, a very mass difference can be be observed and measured with extraordinary precision. This is since oscillations is an amplitude (and not an amplitude squared) effect as we will now see.
The initial state produced in the reaction $\pi^- + p \rightarrow K^0 + \Lambda^0$ is (for simplicity we ignore small corrections from CP violation):

$$|K^0, p\rangle = \frac{1}{\sqrt{2}} \left[ |K^0_S, p\rangle + |K^0_L, p\rangle \right]$$

At later times, this will become:

$$\frac{1}{\sqrt{2}} \left[ a_S(t) |K^0_S, p\rangle + a_L(t) |K^0_L, p\rangle \right], \quad (11.45)$$

$$a_\alpha(t) = e^{-i m_\alpha t} e^{-\Gamma_\alpha t/2}, \quad (\alpha = S, L) \quad (11.46)$$

Here $m_\alpha$ and $\Gamma_\alpha$ are the mass and decay rate of the particle concerned. Note that the probability

$$\left| \frac{1}{\sqrt{2}} a_\alpha(t) \right|^2 = \frac{1}{2} e^{-\Gamma_\alpha t}, \quad (\alpha = S, L)$$

of finding a $K^0_S$ or $K^0_L$ decreases exponentially with a lifetime $\tau_\alpha = \Gamma_\alpha^{-1}, \quad (\alpha = S, L)$

Note, that because $\tau_S << \tau_L$, only the $K^0_L$ component survives, so intensities for the $K^0$ and anti-$K^0$ components become equal with time.
Flavor oscillations and the CPT theorem (3)

we are interested in the intensities of the $K^0$ and anti-$K^0$ components at shorter times, and we use

$$|K_S^0, p\rangle = |K_1^0, p\rangle \equiv \frac{1}{\sqrt{2}}[|K^0, p\rangle + |\bar{K}^0, p\rangle] \quad (11.47a)$$

$$|K_L^0, p\rangle = |K_2^0, p\rangle \equiv \frac{1}{\sqrt{2}}[|K^0, p\rangle - |\bar{K}^0, p\rangle], \quad (11.47b)$$

The expression $\frac{1}{\sqrt{2}} \left[ a_s(t)|K_S^0, p\rangle + a_L(t)|K_L^0, p\rangle \right]$ (11.45) above can be written as:

$$A_0(t)|K^0, p\rangle + \bar{A}_0(t)|\bar{K}^0, p\rangle, \quad (11.48)$$

Where $A_0(t) = \frac{1}{2}[a_s(t) + a_L(t)]$ \quad (11.49a) and $\bar{A}_0(t) = \frac{1}{2}[a_s(t) - a_L(t)].$ \quad (11.49b)

The intensities of the two components are then given by:

$$I(K^0 \rightarrow K^0) \equiv |A_0(t)|^2 = \frac{1}{4} \left[ e^{-\Gamma_s t} + e^{-\Gamma_L t} + 2 e^{-(\Gamma_s + \Gamma_L)t/2} \cos(\Delta mt) \right] \quad (11.50a)$$

and

$$I(K^0 \rightarrow \bar{K}^0) \equiv |\bar{A}_0(t)|^2 = \frac{1}{4} \left[ e^{-\Gamma_s t} + e^{-\Gamma_L t} - 2 e^{-(\Gamma_s + \Gamma_L)t/2} \cos(\Delta mt) \right], \quad (11.50b)$$
Flavor oscillations and the CPT theorem (4)

Oscillations allow to measure experimentally \( \Delta m = |m_S - m_L| \) \(^{(11.51)}\)

The strangeness oscillations \((11.50)\) have been observed and the mass difference \((11.51)\) measured in several experiments. At the CPLEAR experiment at CERN, protons and antiprotons were observed to annihilate at rest at the centre of a large 4\(\pi\) detector and events corresponding to the reactions

\[ p + \bar{p} \rightarrow K^- + \pi^+ + K^0, \quad K^+ + \pi^- + K^0 \]

selected, where the identity of the neutral kaon produced is deduced using strangeness conservation. In addition, the particles are produced with sufficiently low momenta that the charged particles can be observed in the same detector and the time between production and decay determined.

As a result of all experiments the mass difference \(11.51\) has been determined experimentally

\[ \Delta m = (3.483 \pm 0.006) \times 10^{-12} \text{ MeV}/c^2, \quad (11.54) \]
The result (11.54) leads to a very precise confirmation that the masses of particles and antiparticles are equal. The $K^0_S$ and $K^0_L$ are not antiparticles, but the $K^0$ and $\bar{K}^0$ are, as can be seen from (11.18), and the CPT theorem requires

$$m_{K^0} = m_{\bar{K}^0},$$  \hspace{1cm} (11.55)

(11.54) can be shown to arise solely from the possibility of transitions $K^0 \leftrightarrow \bar{K}^0$, whose magnitude can be calculated from diagrams like those shown in Figure 11.8. We shall not discuss this further, but merely note that the resulting agreement between the predicted and measured values confirms the identity (11.55) to better than one part in $10^{18}$. In contrast, the particle–antiparticle mass relation that has been most precisely tested by direct measurement is $m_{e^+} = m_{e^-}$, which is only verified to within an experimental error of order of one part in $10^8$.

We conclude by noting that similar flavour oscillations have also been observed in the $D^0 - \bar{D}^0$, $D^0_s - \bar{D}^0_s$, $B^0 - \bar{B}^0$ and $B^0_s - \bar{B}^0_s$ meson systems, and will be discussed for the $B^0 - \bar{B}^0$ case in Section 11.2.6.