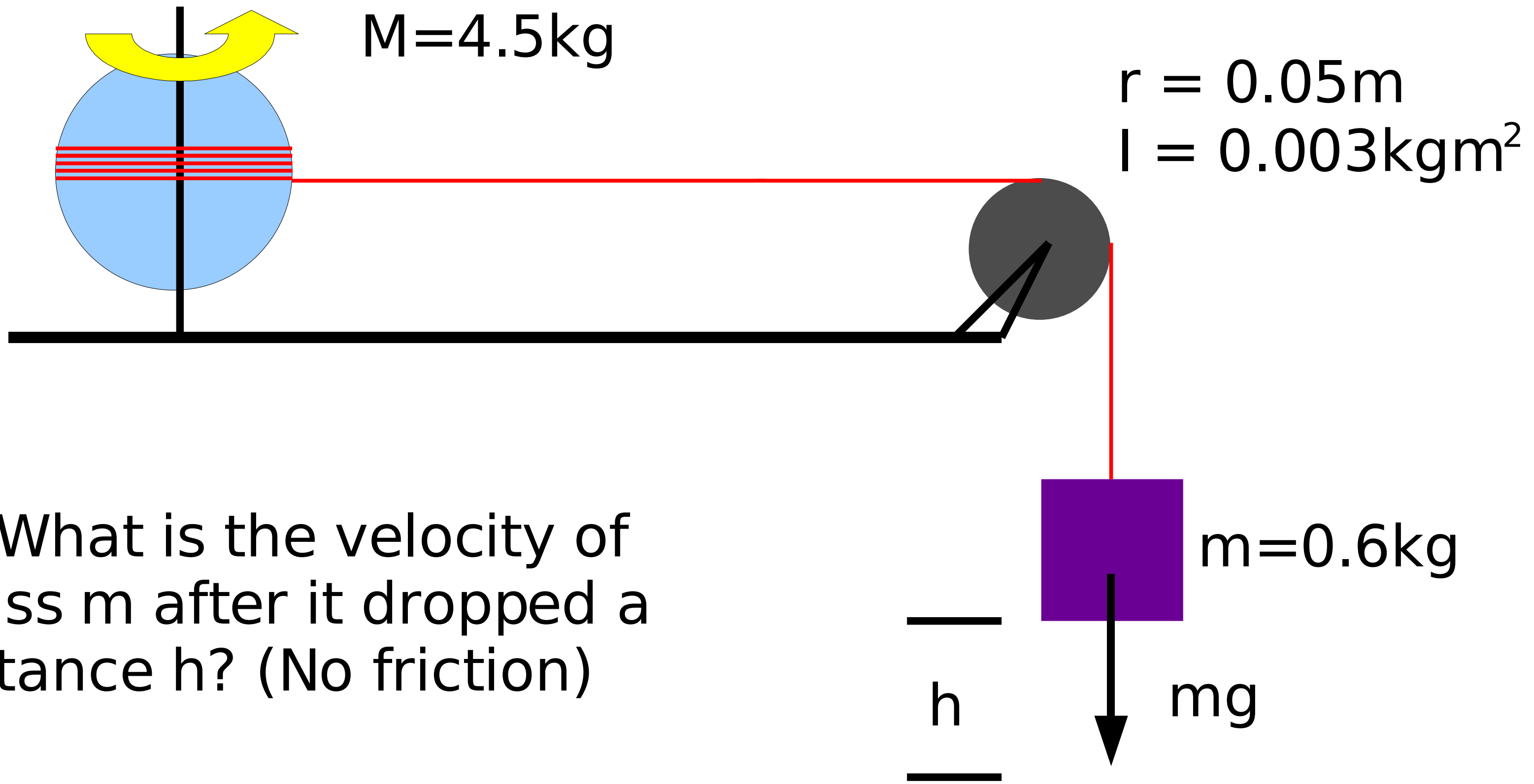
Work and kinetic Energy

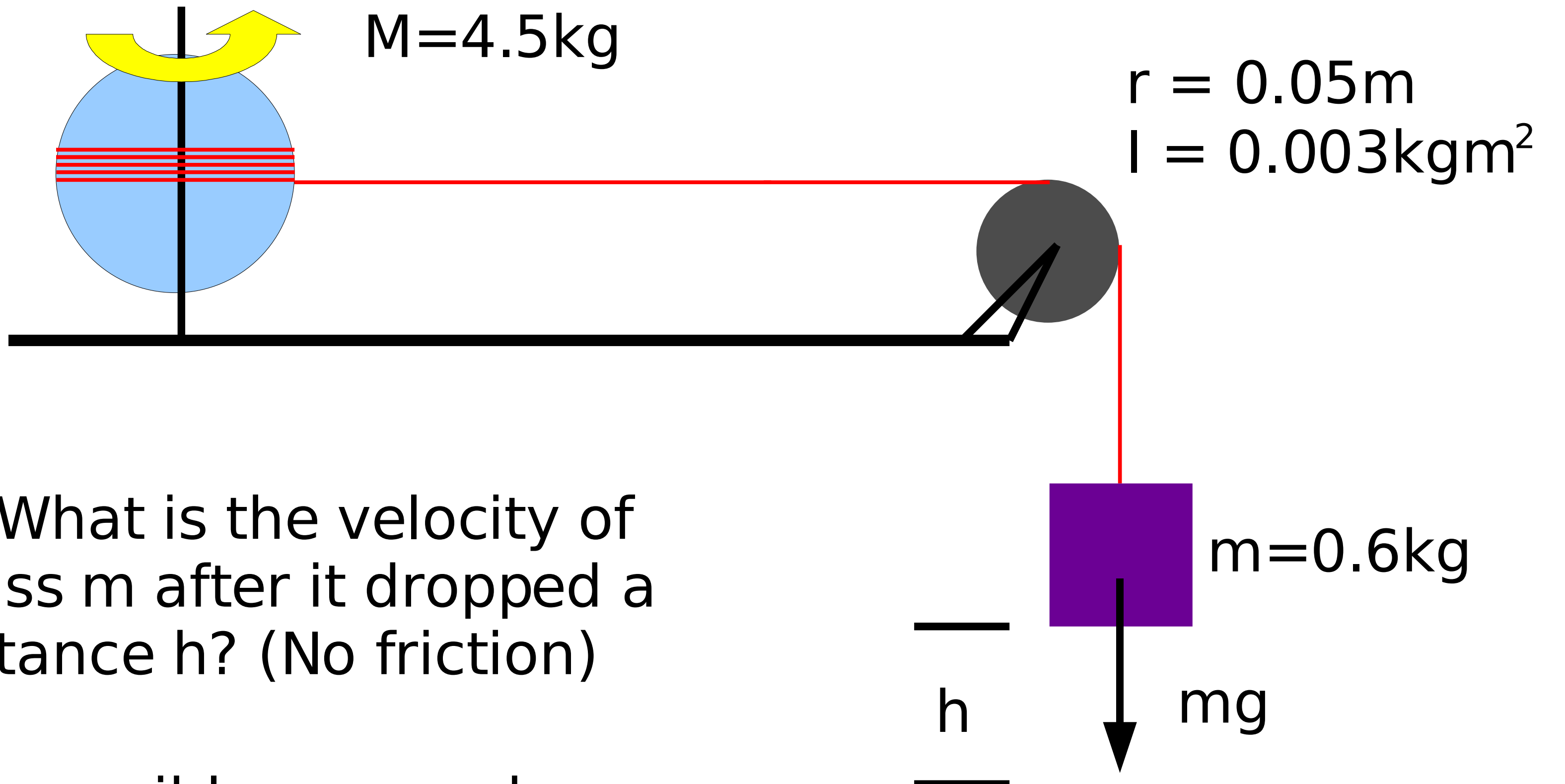
Problem 66.



Q: What is the velocity of mass m after it dropped a distance h? (No friction)

Work and kinetic Energy

Problem 66.

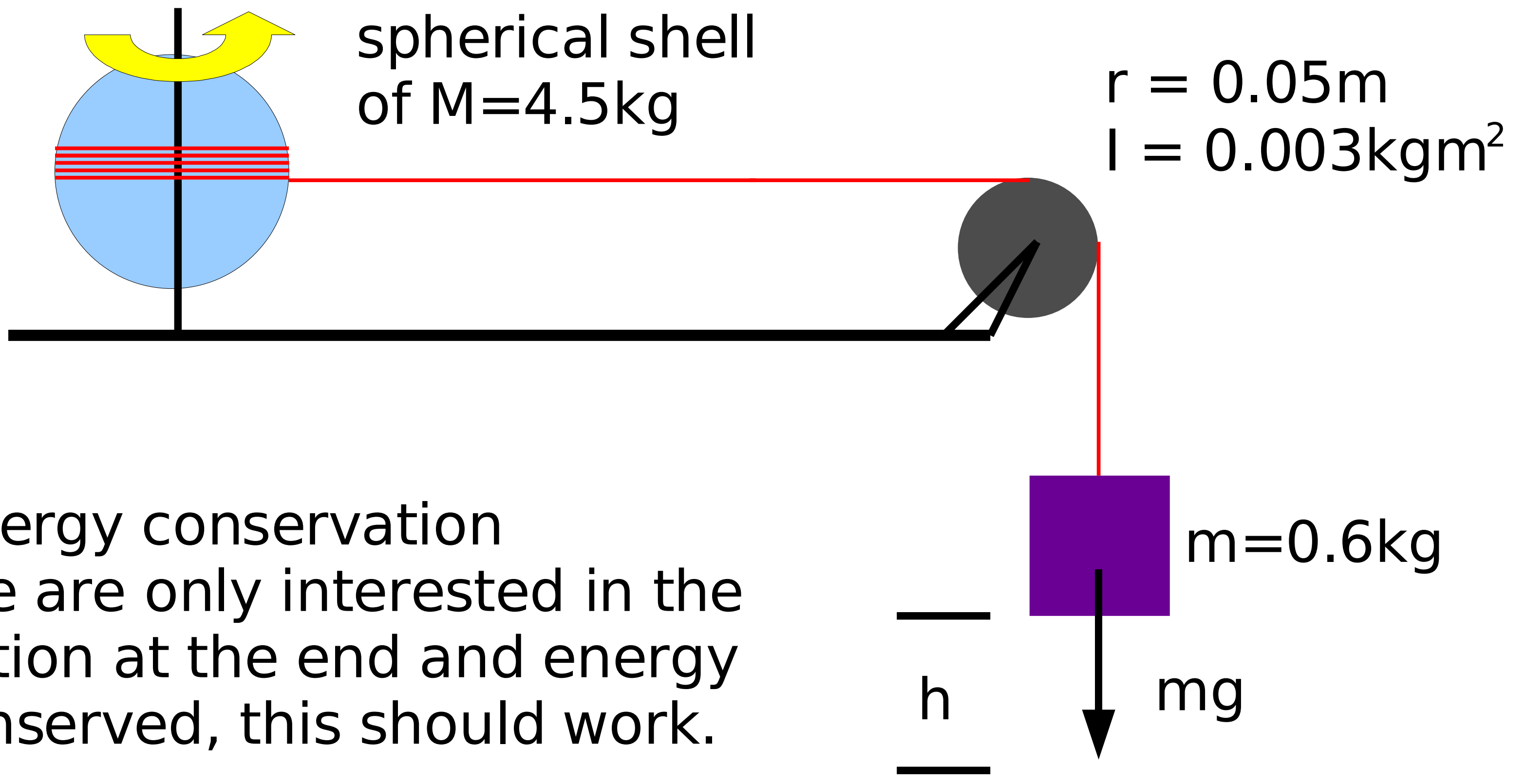


Q: What is the velocity of mass m after it dropped a distance h? (No friction)

Two possible approaches:
a) Use forces (gravity and tension) and torques
b) Energy conservation

Work and kinetic Energy

Problem 66.

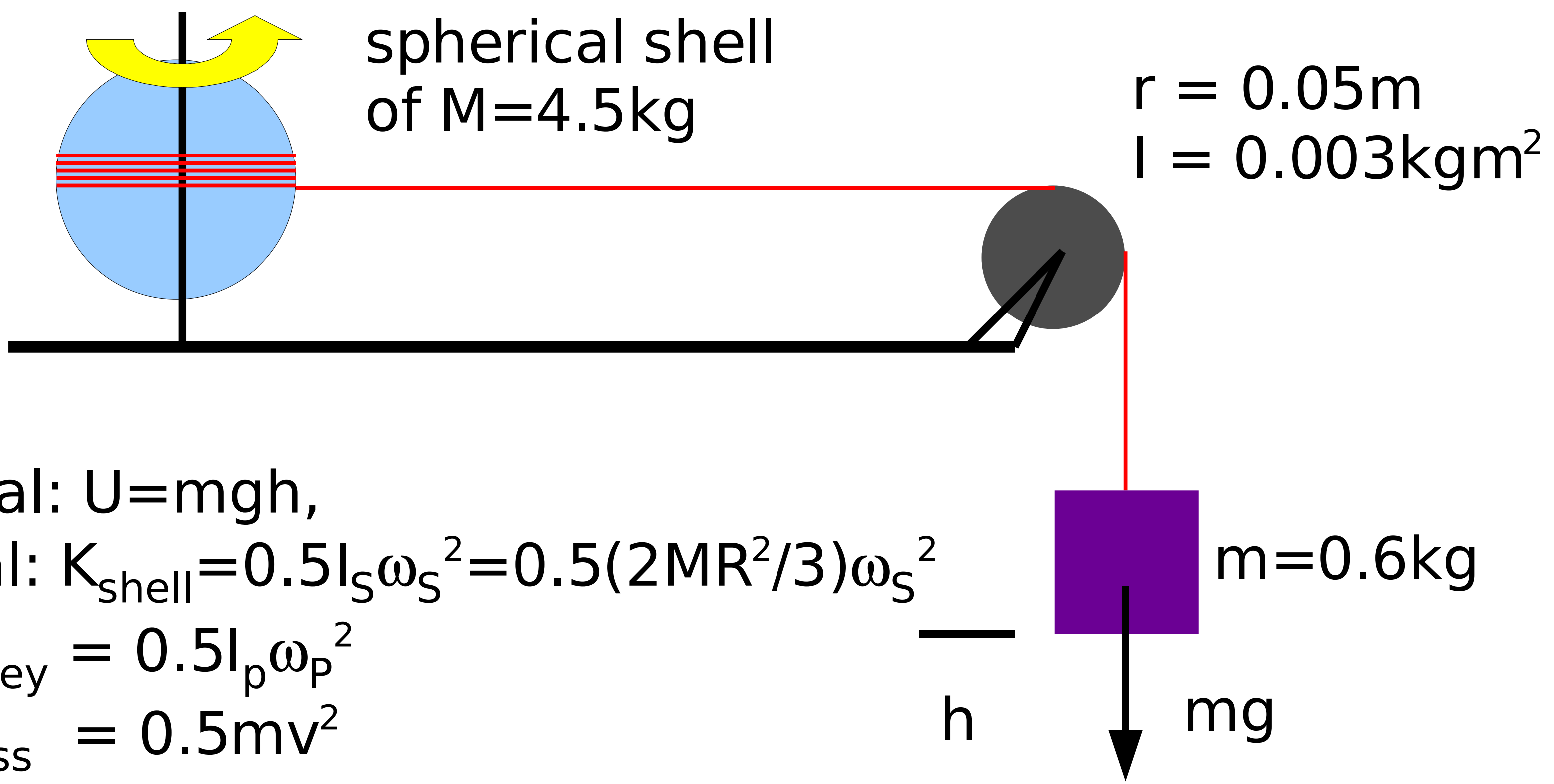


b) Energy conservation
As we are only interested in the situation at the end and energy is conserved, this should work.

Initial: U=mgh, Final: $K_{shell} + K_{pulley} + K_{mass}$

Work and kinetic Energy

Problem 66.



Initial: U=mgh,

Final: $K_{shell}=0.5I_S\omega_S{}^2=0.5(2MR^2/3)\omega_S{}^2$

$K_{pulley} = 0.5I_p\omega_P{}^2$

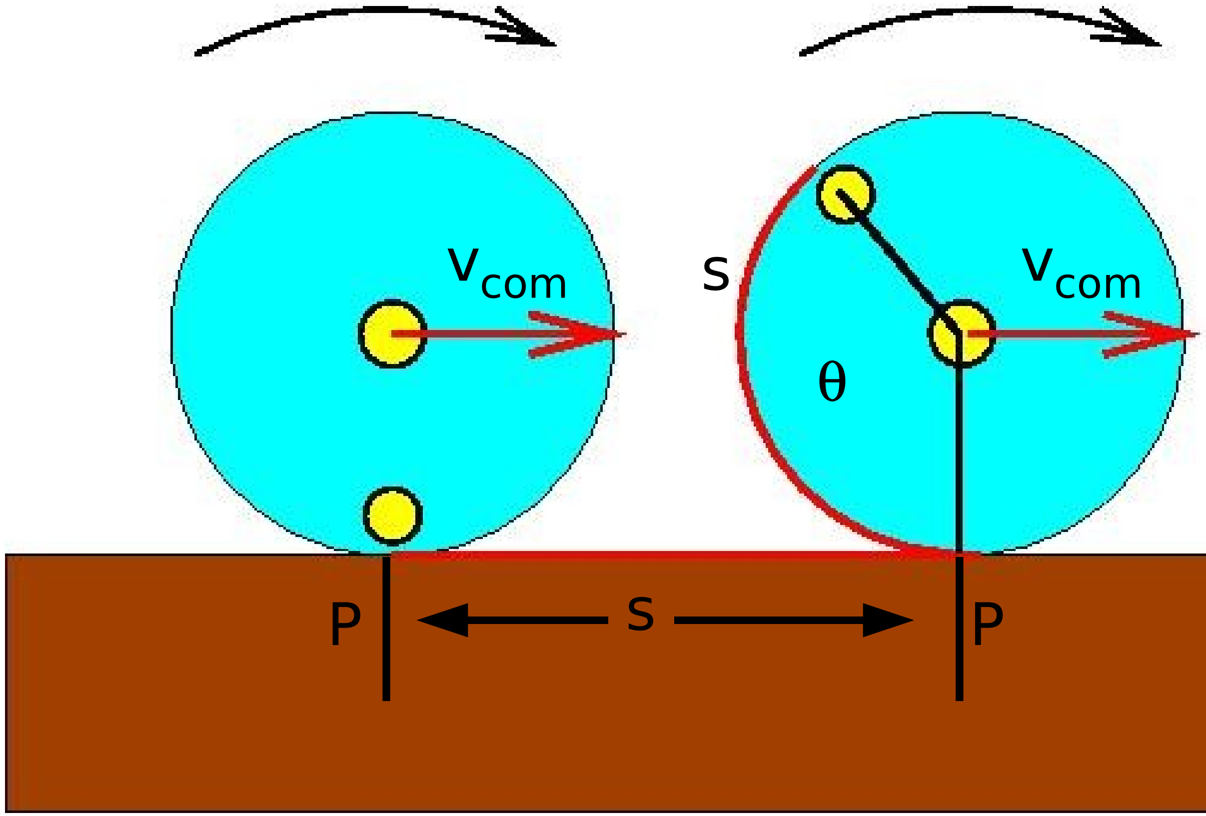
$K_{mass} = 0.5mv^2$

Also know that the 'length of the string is conserved':

$v=\omega_P r$, $v=\omega_S R$ Use this to replace the ω's in the kinetic energies

$mgh = Mv^2/3 + 0.5I_pv^2/r^2 + 0.5mv^2$ ➡ Solve for v = 1.4m/s

Rolling



$s = \theta R$

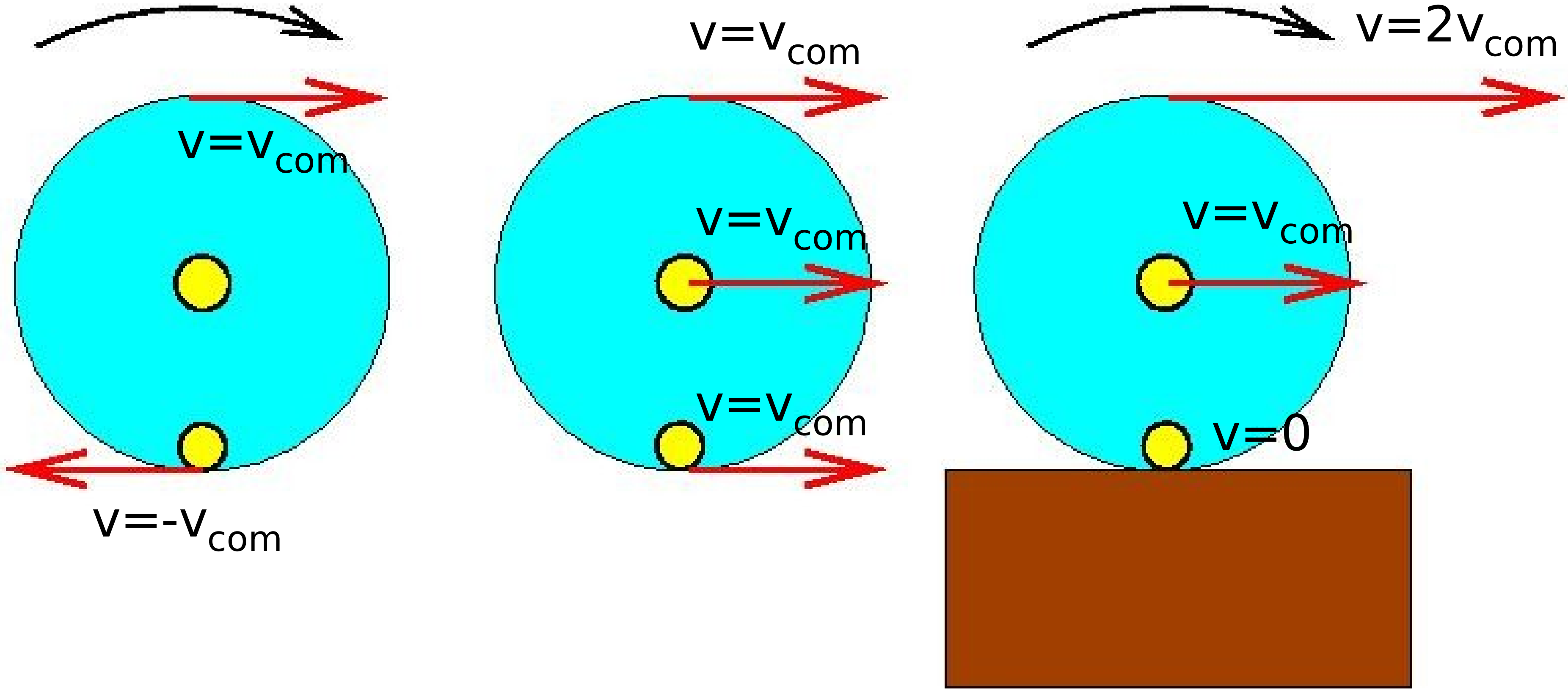
$v_{com} = ds/dt = \omega R$

smooth rolling,
no slipping or slidding

Starting point for understanding rolling: The length of the path traveled in one direction matches the length of the segment of the circumference which made contact with ground during that travel.

Different interpretations of Rolling

Pure Rotation + Pure Translation = Rolling



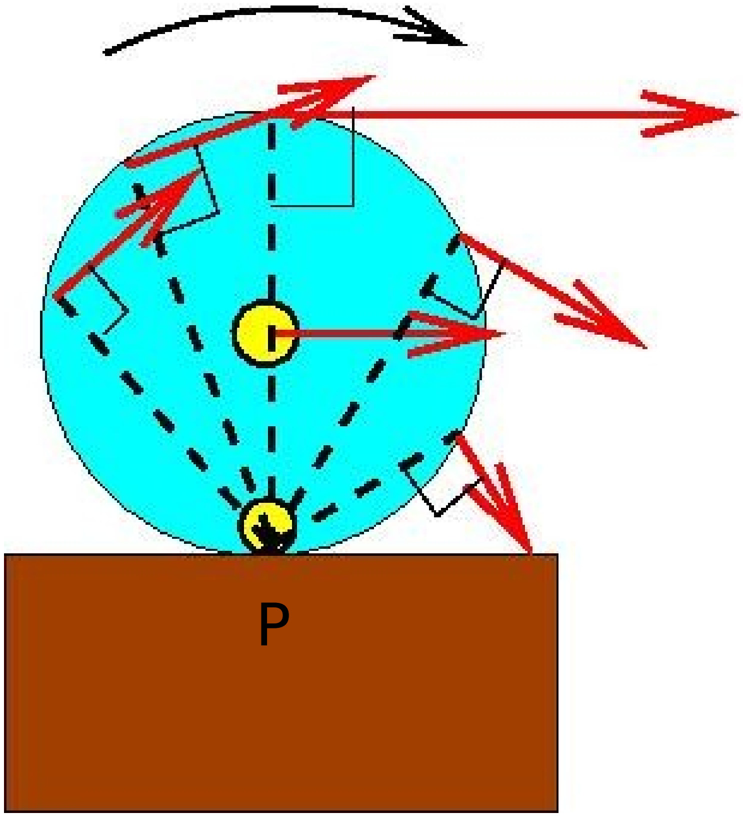
Rolling motion: The tangential velocity at the top of the wheel is twice as large as the COM velocity and zero at the bottom.

Rolling wheel:

Fastest at the top

Stands still at bottom

Different interpretations of Rolling



Or seen as a pure rotation around P.

Note:

- All points have different velocities
- The rotation axis
  - is not fixed within the rotating body
  - is now moving with v_{com}

Two different ways to look at rolling:

1. Translation + Rotation around the central axis
2. Pure rotation around a non-fixed axis which moves with v_{com}

Rolling

Kinetic Energy:

Pure rotational motion around axis on bottom:

$$K = 0.5\ I_P \omega^2$$

Moment of inertia for this case:

$$I_P = I_{COM} + MR^2$$

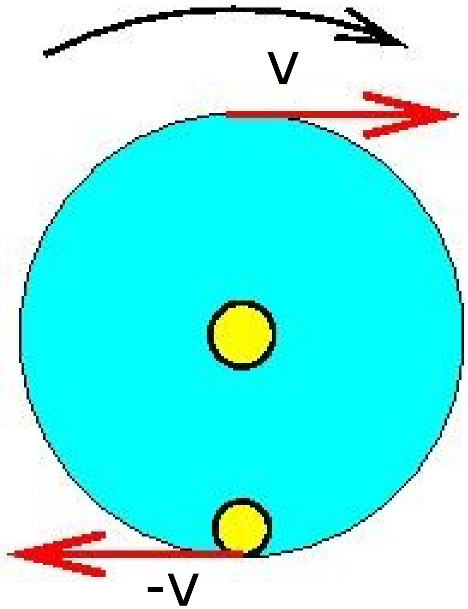
gives: $K = 0.5\ I_{COM}\omega^2 + 0.5MR^2\omega^2$

use $v_{com} = R\omega$ and $K = 0.5\ I_{com}\omega^2 + 0.5Mv_{com}{}^2$

Rotational energy + Translational energy

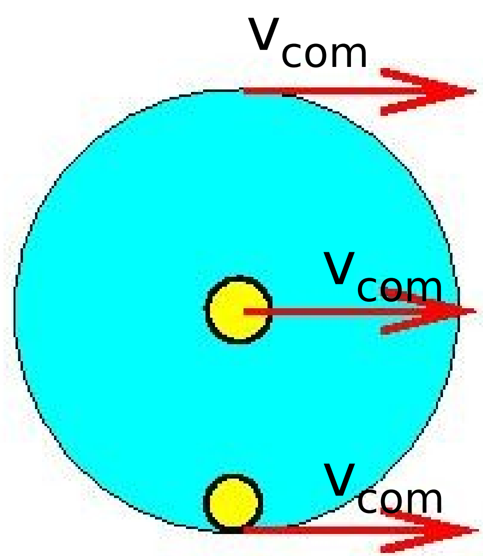
Both interpretations give same energy

Rolling and friction I: without external forces or torques



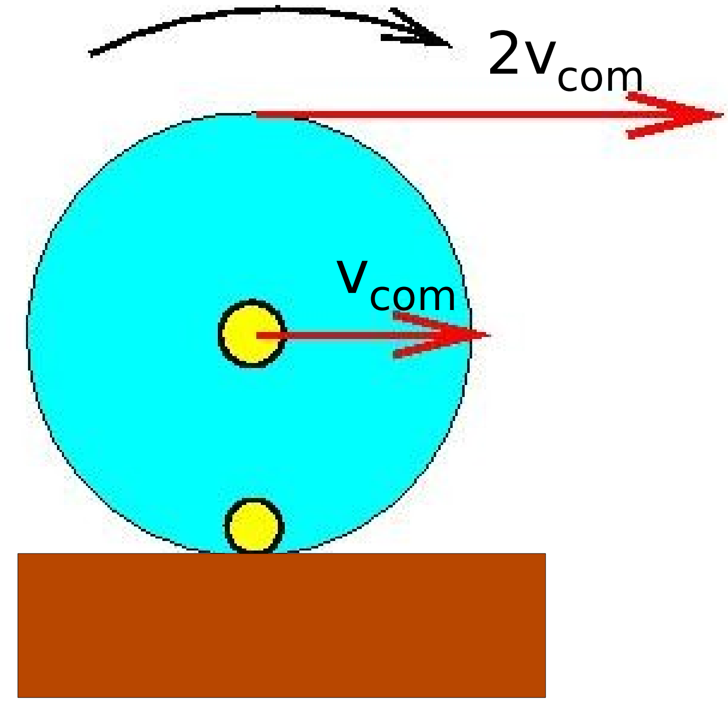
- Rotating wheel
- No friction

Wheel stays in place,
keeps rotating
Example: spinning ball



- Sliding wheel
- No friction

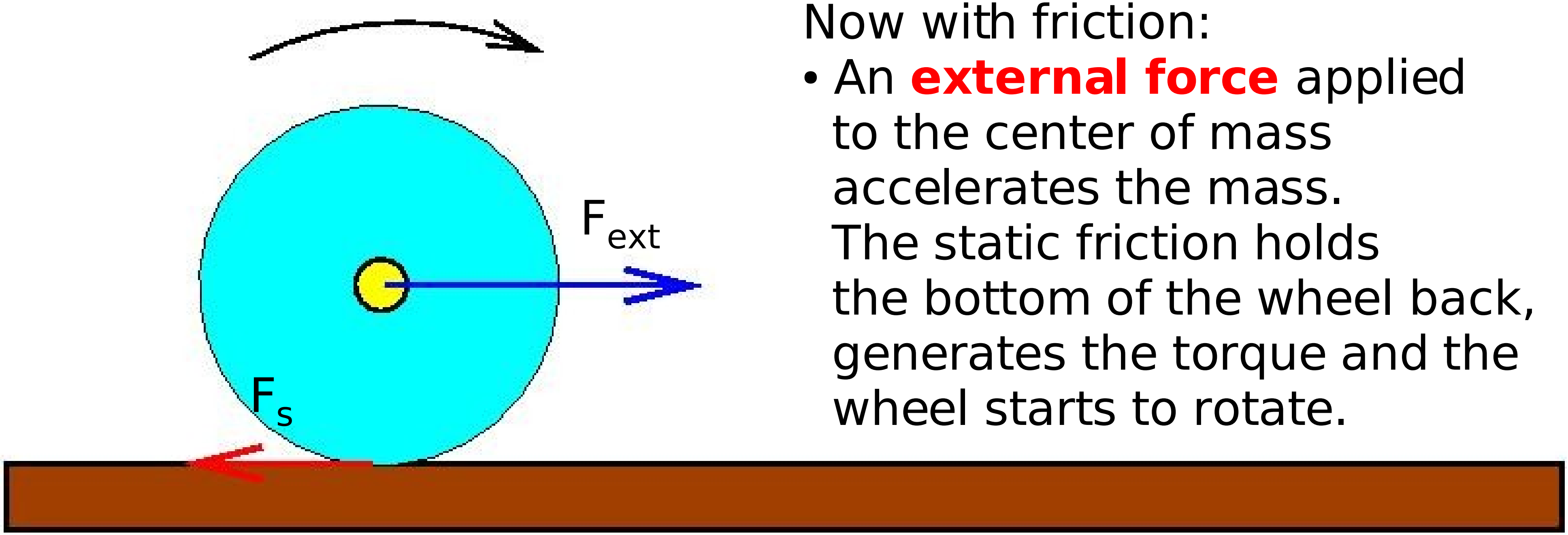
Wheel keeps on sliding
no rotation
Example: thrown ball w/o spin



- Rolling wheel
- No forces/no friction

Wheel continues to roll
Example: Cylinder on horizontal surface

Rolling and friction II: with external forces or torques



Now with friction:

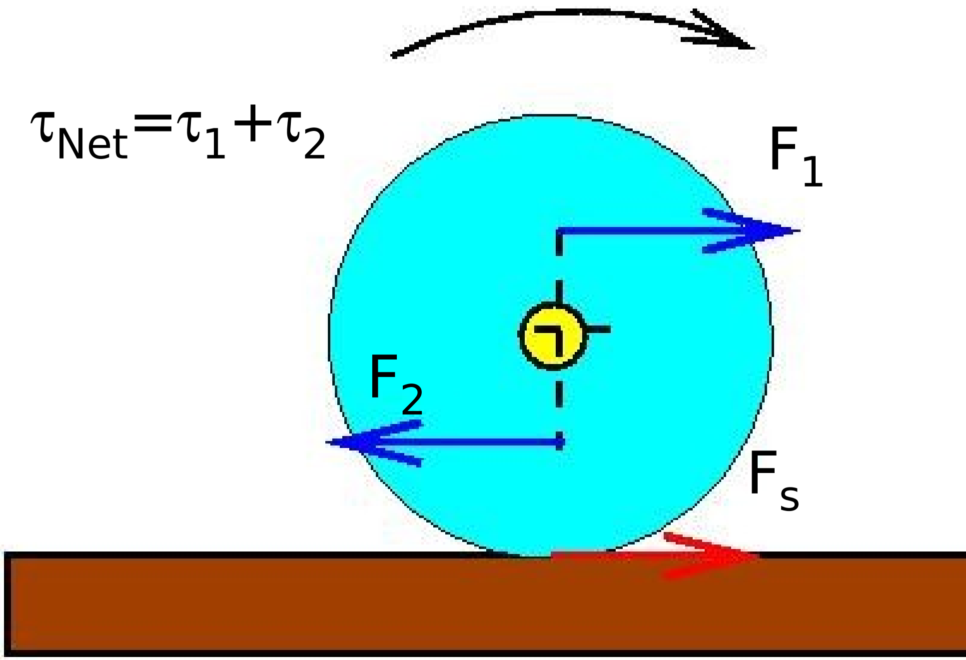
- An **external force** applied to the center of mass accelerates the mass. The static friction holds the bottom of the wheel back, generates the torque and the wheel starts to rotate.

If the wheel rotates w/o slipping: static friction (most of our cases)
If the wheel rotates with slipping: kinetic friction

Example: Pushing a car or a bike.

Direction of the friction points **backwards**!

Rolling and friction II: with external forces or torques



Now with friction:

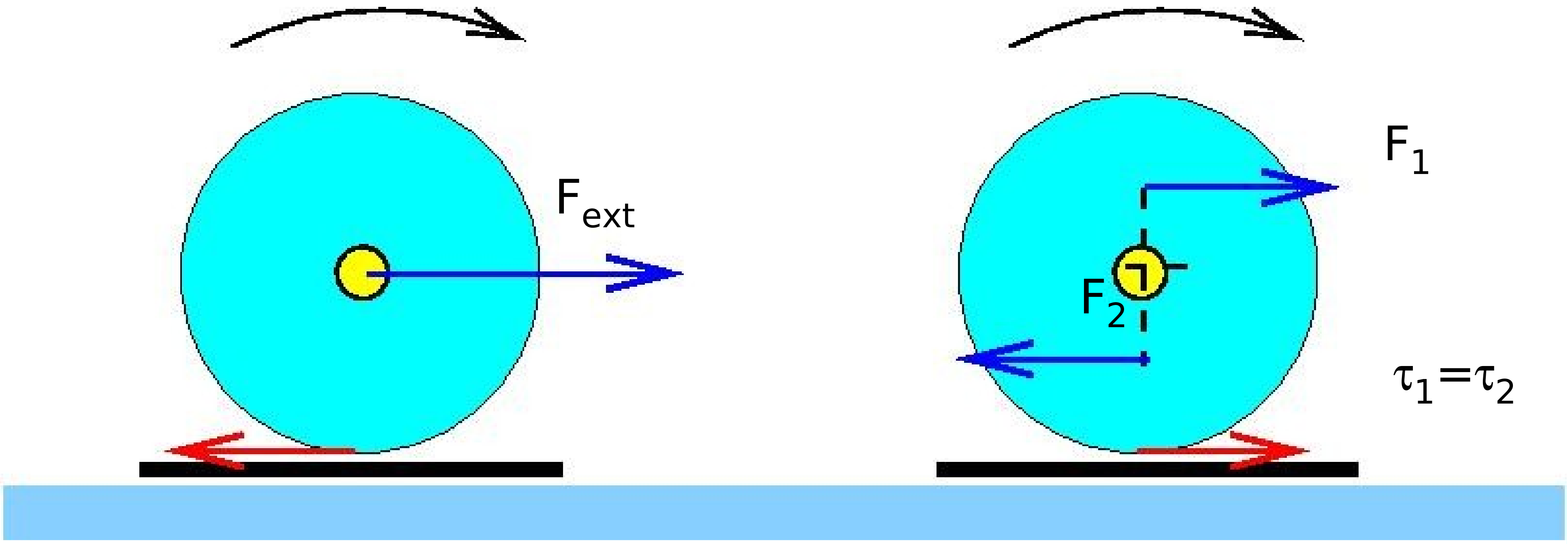
- An **external torque** (generated here by two forces ($F_1+F_2=0$)) is applied to the wheel and starts to rotate it. The static friction prevents the wheel from slipping and pushes the wheel forward.

If the wheel rotates w/o slipping: static friction (most of our cases)
If the wheel rotates with slipping: kinetic friction

Example: Riding a bike or driving a car.

Direction of the friction points **forward** and is now the net force which provides the acceleration of the center of mass!

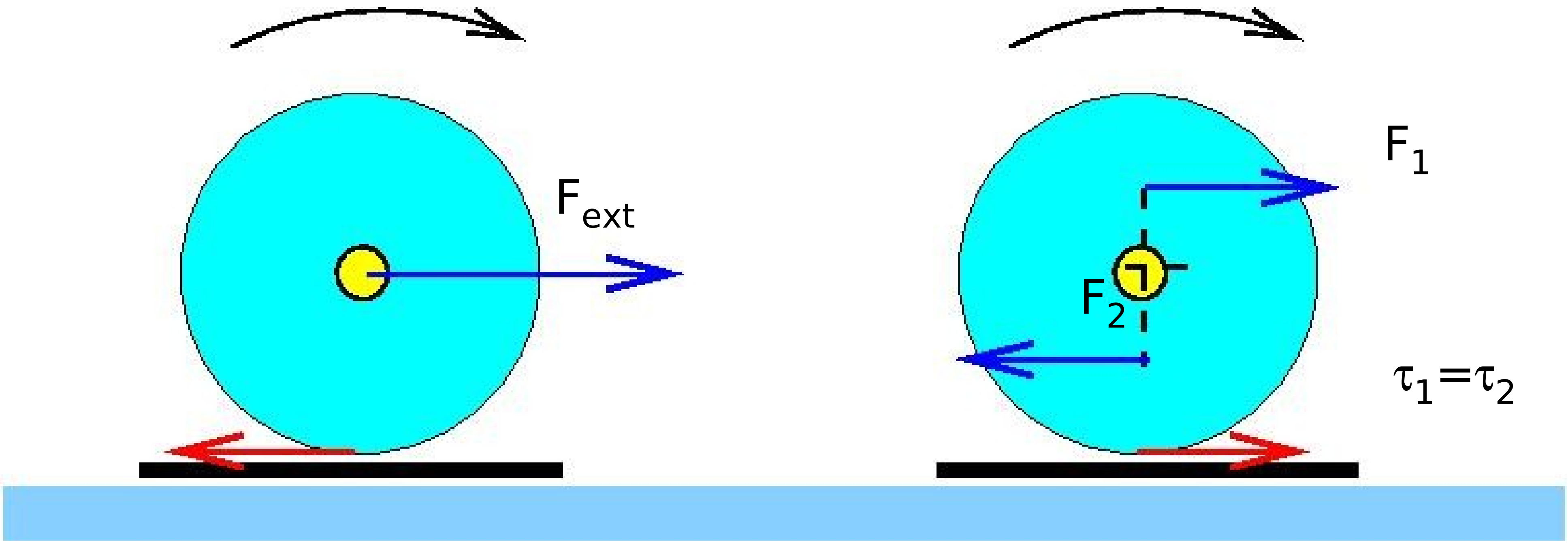
Rolling and friction II: with external forces or torques



To the direction of the frictional force: Imagine the wheel is sitting initially on a mat which sits on an ice field.

• When you push the wheel with an external force (no torque) the wheel will not rotate but will fold the mat up in front of it. Apparently, the force the wheel applies to the mat is going forward. **Actio = Reactio:** The frictional force the mat applies to the wheel is going backwards!

Rolling and friction II: with external forces or torques

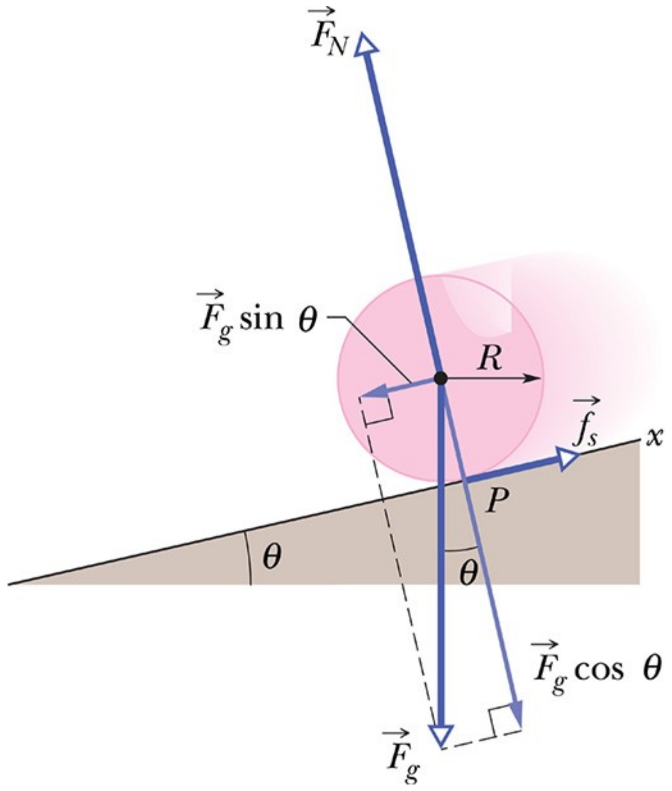


To the direction of the frictional force: Imagine the wheel is sitting initially on a mat which sits on an ice field.

• When you torque the wheel with an external torque, the wheel will rotate but now move. Instead it will fold the mat up behind the wheel. Apparently, the force the wheel applies to the mat is going backwards.

Actio = Reactio: The frictional force the mat applies to the wheel is going forward!

Example: Rolling down a ramp



No slidding!

Forces:

Gravity F_g = -Mg

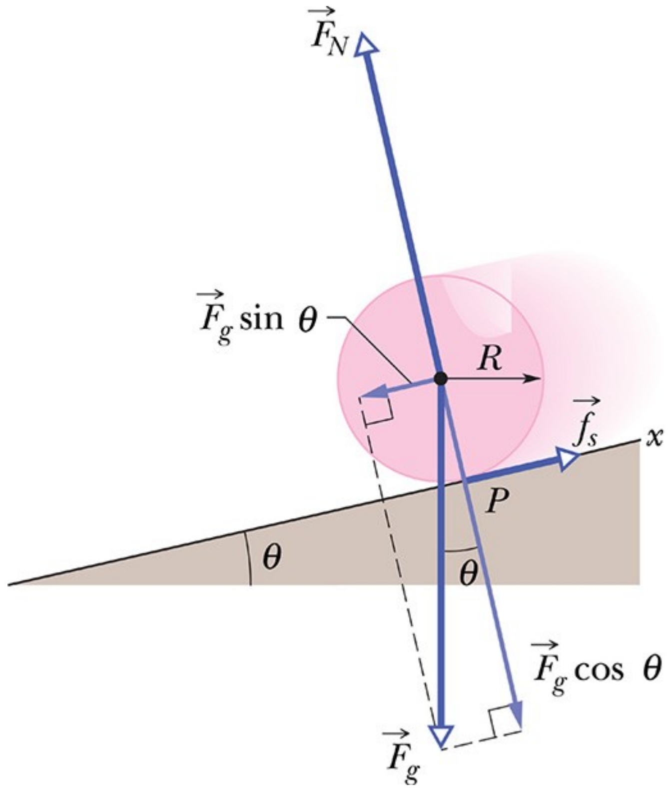
has two components:

- -Mg sinθ working on the center of mass
- -Mg cosθ Normal force creating friction (not at the max value!)

Torques:

- frictional force f_s has lever arm R to provide torque

Example: Rolling down a ramp



No slidding!

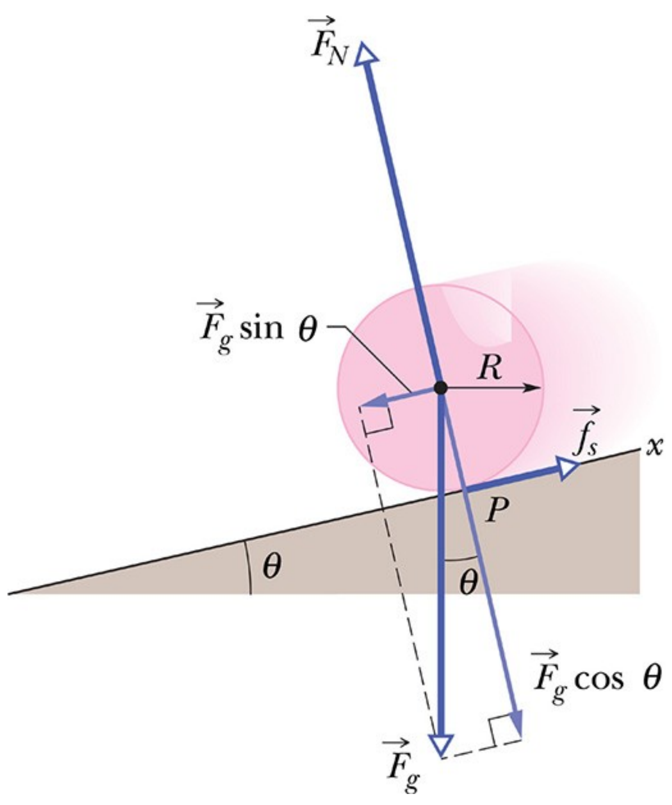
Forces:
Gravity $F_g = -Mg$
has two components:

- $-Mg \sin\theta$ working on the center of mass
- $-Mg \cos\theta$ Normal force creating friction f_s (not at the max value!)

Linear motion: $Ma_{com,x} = f_s - Mg \sin\theta$ Unknowns: $a_{com,x}$, f_s

Note the error in the drawing: fs < Mgsinθ

Example: Rolling down a ramp



No slidding!

Torques:

- frictional force f_s has lever arm R to provide torque

$$Rf_s = I_{com}\alpha$$

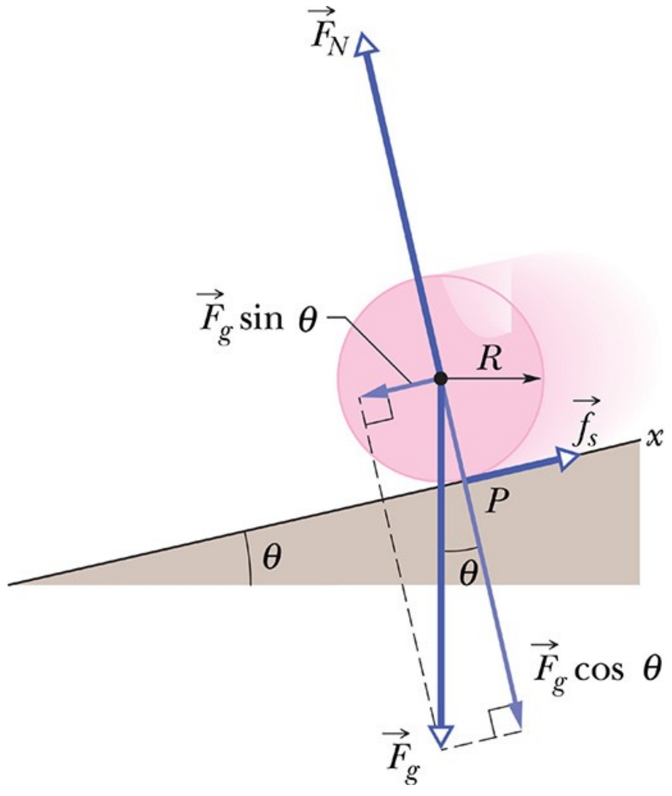
Again two unknowns: f_s, α

Geometry (no slipping):

$$a_{com,x} = -R\alpha$$

Geometry: - sign because $a_{com,x}$ is <u>negative</u> while α is positive

Example: Rolling down a ramp



$Rf_s = I_{com}\alpha$ $a_{com,x} = -R\alpha$

$f_s = -I_{com}a_{com}/R^2$

into

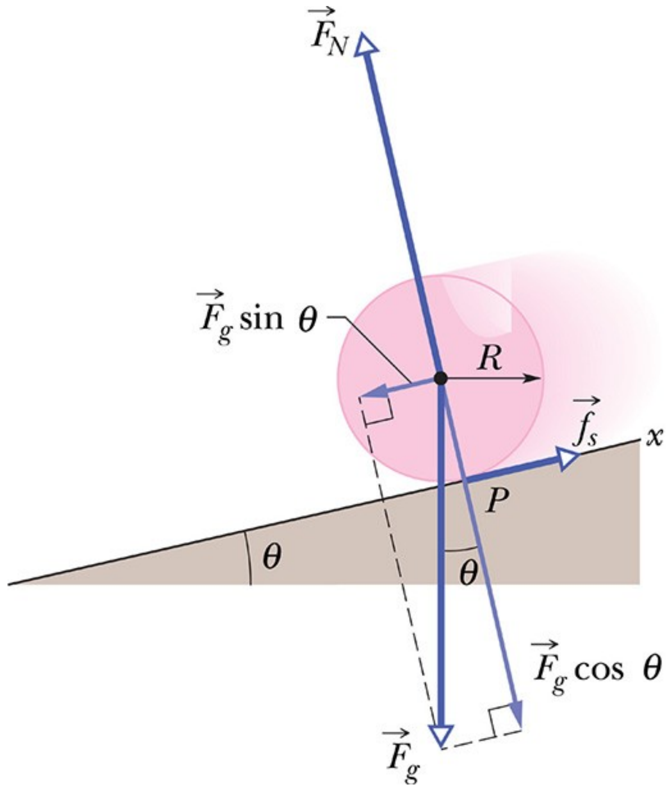
$Ma_{com,x} = f_s - Mg\sin\theta$

$Ma_{com,x} = -I_{com}a_{com}/R^2 - Mg\sin\theta$

Solve for $a_{com,x} = - g\sin\theta/(1+I_{com}/MR^2)$

The acceleration of any body rolling an incline w/o slipping

Example: Rolling down a ramp



$$f_s = -I_{com}a_{com}/R^2$$

$$a_{com,x} = - \; g \sin\theta/(1+I_{com}/MR^2)$$

The acceleration decreases with increasing moment of inertia.

HITT (only a test)

Checkpoint 2: Disks A and B are identical and roll across a floor with equal speeds. Then disk A rolls up an incline, reaching a maximum height h, and disk B moves up an identical incline except that it is frictionless. Is the maximum height reached by disk B greater than, less than, or equal to h?

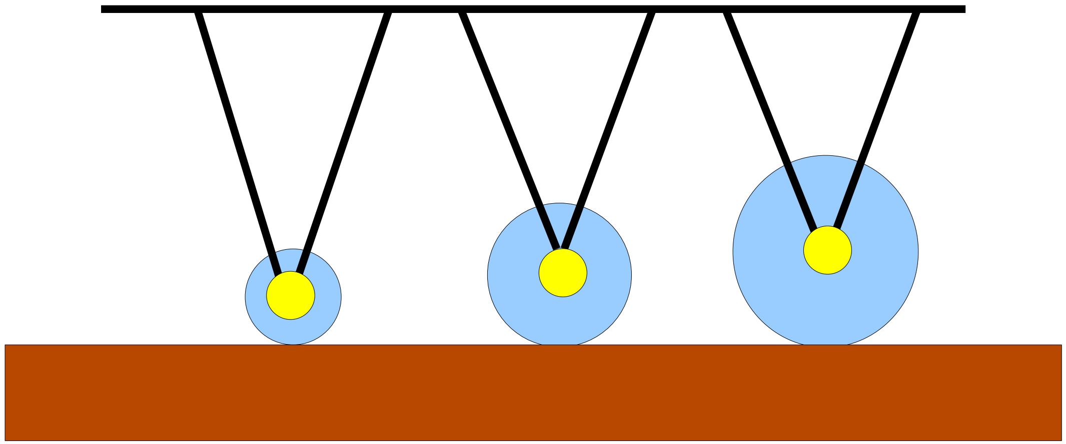
A: Greater

B: Less

C: Equal

HITT

Assume three wheels having diameters of 10, 20, and 30cm. The axis of the three wheels are connected as shown in the drawing:



When this 'trike' moves to the right, what is the ratio between the linear speeds at the very top of each wheel starting with the 10cm wheel?

A: 9:4:1

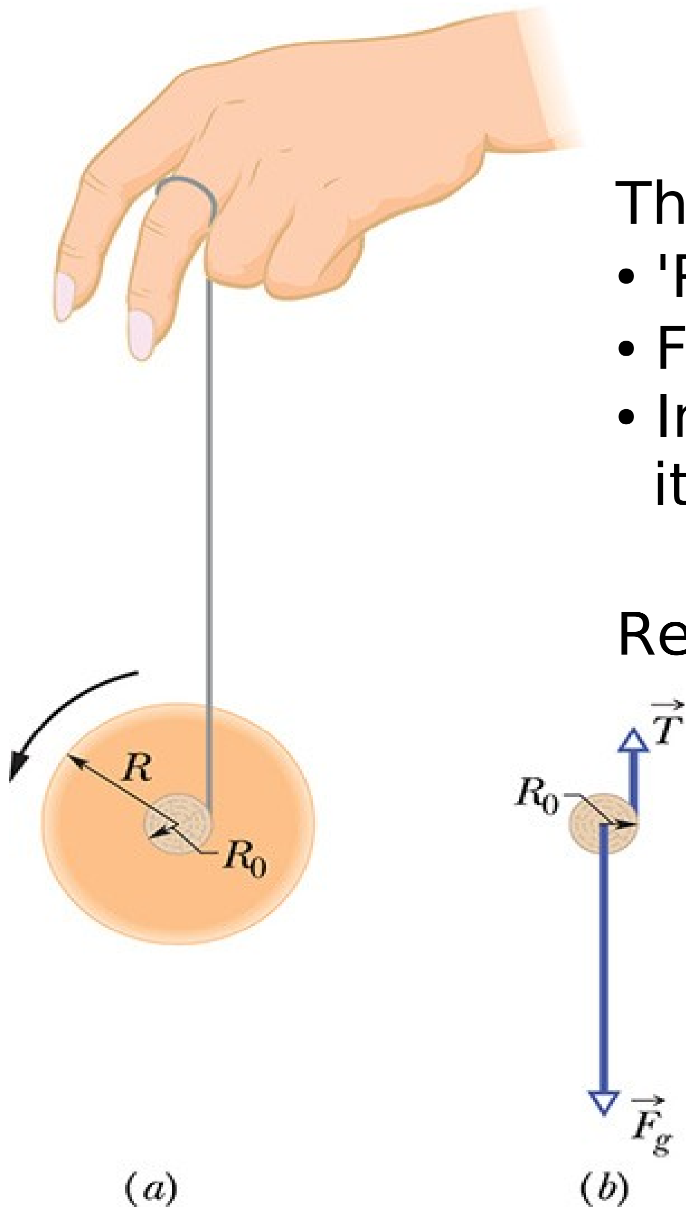
B: 3:2:1

C: 1:1:1

D: 1:2:3

E: 1:4:9

Example: Rolling down a ramp



The Yo-Yo:

- 'Ramp' has now angle θ =90deg
- Frictional force is now tension in string
- Instead of rolling on the outer surface it roles on the inner surface (Radius R_0)

Rest is the same.

$$a_{com,x} = -\ g/(1+I_{com}/MR_0^2)$$

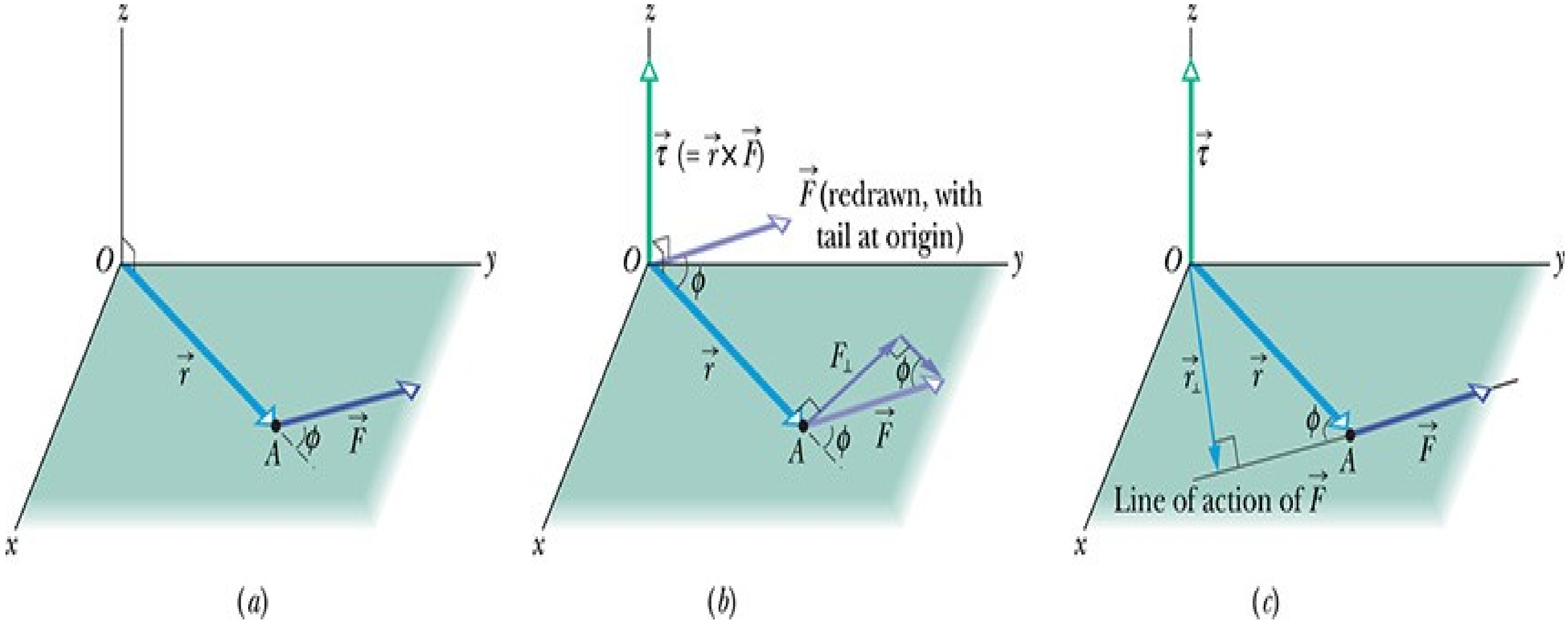
Torque Revisited

So far, we discussed torques for
- Rigid bodies
- Point mass on a string

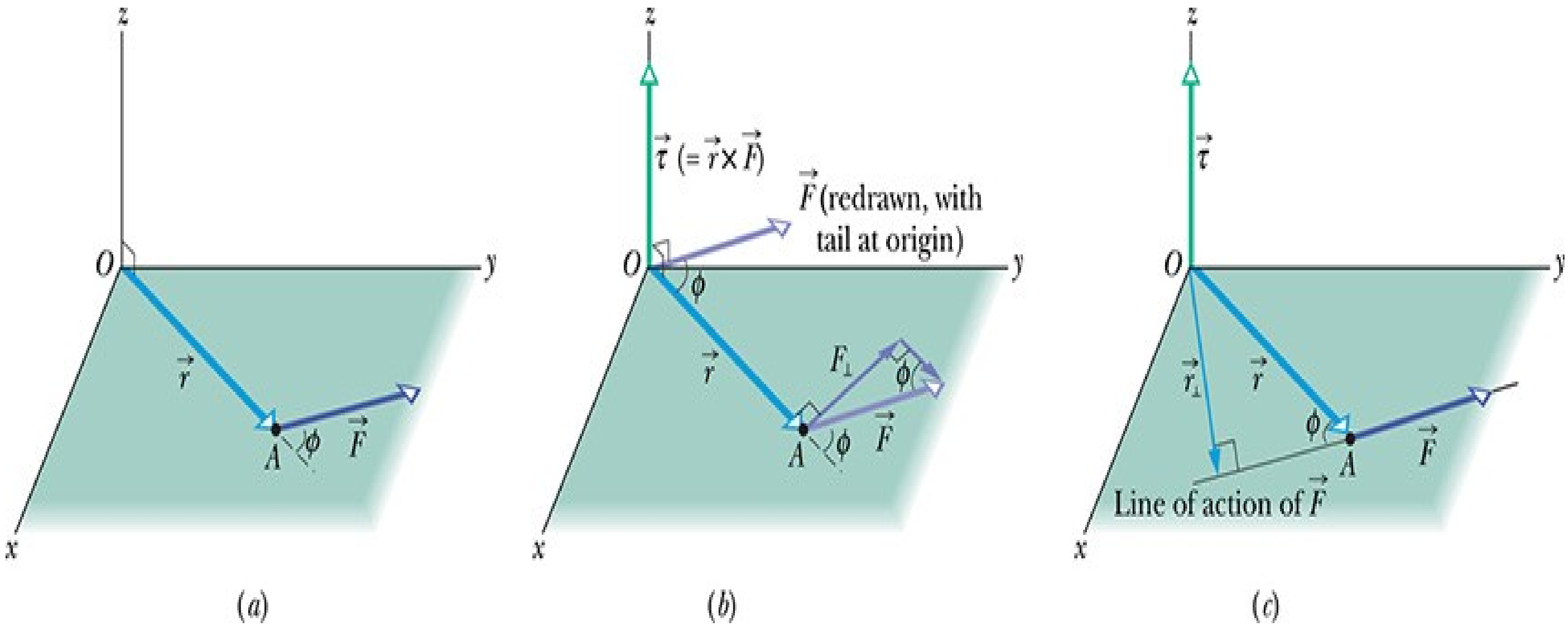
But the definition can also be used for point masses which are not restricted by the rigidity of the body or by a string.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$\vec{\tau}$ is starts at the origin and is perpendicular to the $\vec{r}$-$\vec{F}$ plane. (Right-hand rule)



Torque Revisited



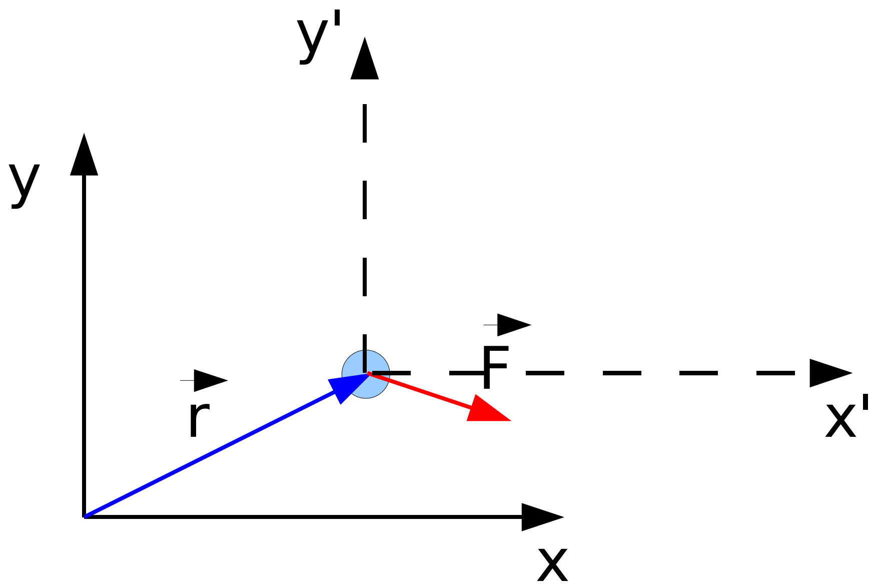
$\vec{\tau} = \vec{r} \times \vec{F}$ Magnitude: $\tau = r\,F\sin\phi = r\,F_\perp = r_\perp F$

$F_\perp$: The component of F perpendicular to the distance vector $\vec{r}$

$r_\perp$: The perpendicular distance between O and the line of action

Torque Revisited

Torque depends on coordinate system!

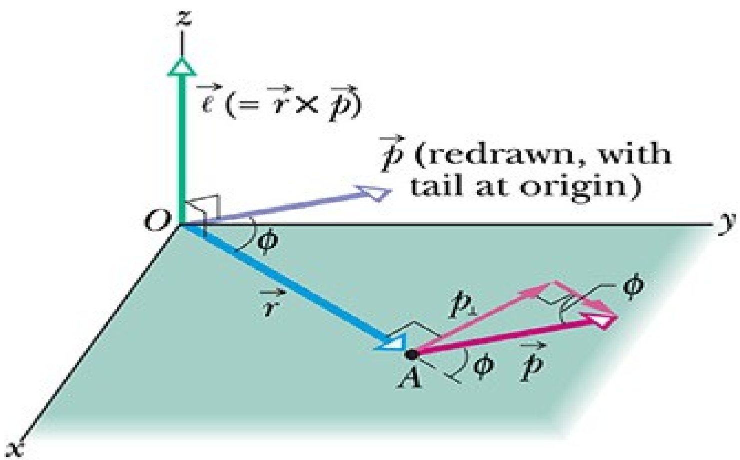


$\vec{\tau} = \vec{r} \times \vec{F} \neq 0$ in x-y system

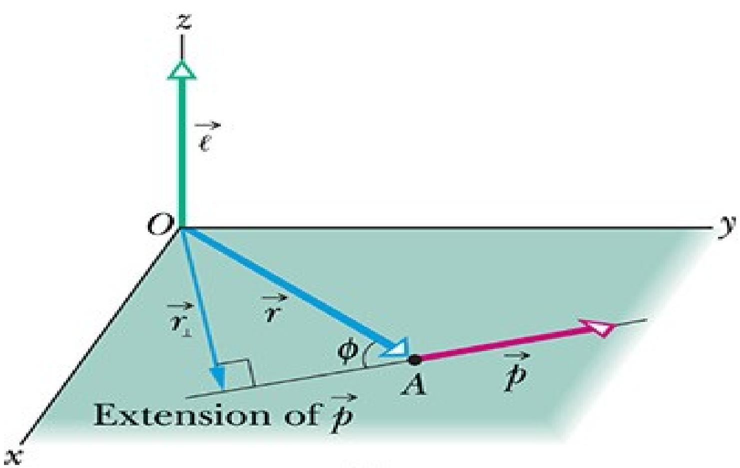
$\vec{\tau} = \vec{r} \times \vec{F} = 0$ in x'-y' system
because $\vec{r} = 0$

Note that, if the particle has an initial velocity which is not parallel to F, the moment the particle passes the origin, it will start to see a torque in the x'-y' system.
In that case, F would force the particle into a circular and translational motion until they are parallel.

Angular Momentum



(a)



(b)

Recall linear momentum $\vec{p}$ and the principle of conservation of linear momentum if we don't have net-forces (e.g. collisions)

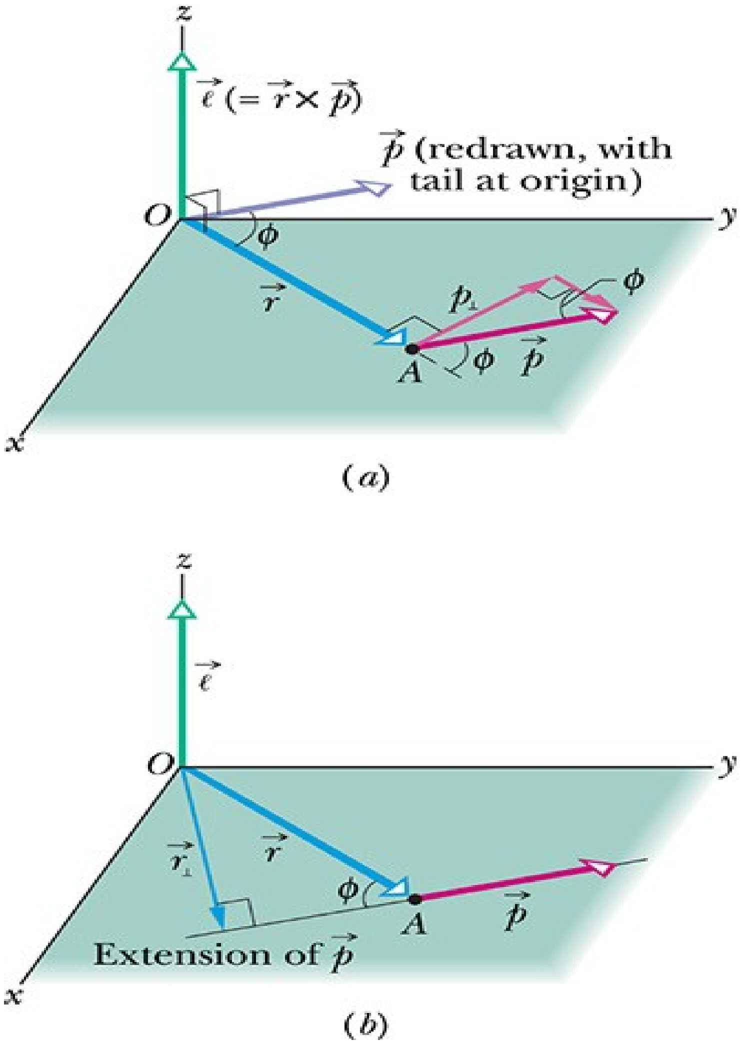
Its angular counterpart is:
Angular Momentum

The angular momentum for a single particle is:

$$\vec{l} = \vec{r} \times \vec{p} = m\,(\vec{r} \times \vec{v})$$

What a surprise ...

Angular Momentum



The angular momentum for a single particle is:

$$\vec{l} = \vec{r} \times \vec{p} = m\,(\vec{r} \times \vec{v})$$

Similar to the torque, the magnitude is:

$$l = rp_\perp = rmv_\perp = r_\perp p = r_\perp mv$$

or

$$l = r\,m\,v\,\sin\phi$$

Newton's Second Law in Angular Form

$$\frac{d}{dt}\vec{l} = \frac{d}{dt}\vec{r} \times \vec{p} = \frac{d}{dt} m\,(\vec{r} \times \vec{v})$$

$$= m\,(\vec{r} \times \vec{a}) + m\,(\vec{v} \times \vec{v})$$

$$= 0$$

$$= \vec{r} \times m\vec{a} = \vec{r} \times \vec{F}_{net} = \vec{\tau}_{net}$$

$$\frac{d}{dt}\vec{l} = \vec{\tau}_{net}$$

Example

Problem 34: A particle is acted on by two torques about the origin: $\vec{\tau}_1$ has a magnitude of 2.0Nm and is directed in the positive x-direction, and $\vec{\tau}_2$ has a magnitude of 4.0Nm and is directed in the negative direction of the y-axis.
Find dl/dt around the origin.

Solution:

$$d\vec{l}/dt = (2.0i + 0j + 0k)\ \text{Nm} + (0i - 4j + 0k)\ \text{Nm}$$

Magnitude: $|dl/dt| = (2.0^2 + (-4.0)^2)^{1/2} = 4.5\text{Nm}$

Angle: $\theta = \text{atan}(-4.0/2.0) = -63^o$