CH 15: Oscillations

Chapter 15: Oscillations
Chapter 16: Waves I
Chapter 17: Waves II

All very closely related:
Oscillating charges generate electro-magnetic waves
Electro-magnetic waves force charges to oscillate

Motion of a boat on a lake generates water waves
Water waves move boat around

...

Also related to circular motion.

CH 15: Oscillations

Oscillations and waves are everywhere!

Clocks:

- The time standard is an oscillator
- Your wrist watch contains an oscillator

Radio, TV, Talking, Listening, Seeing, ... :

- The submitted waves are generated by oscillators
- The receiver is an oscillator driven by the received waves

Heat in or from a solid body:

- The random oscillatory motion of atoms in their lattice
- The radiation emitted from a hot body like our sun are waves

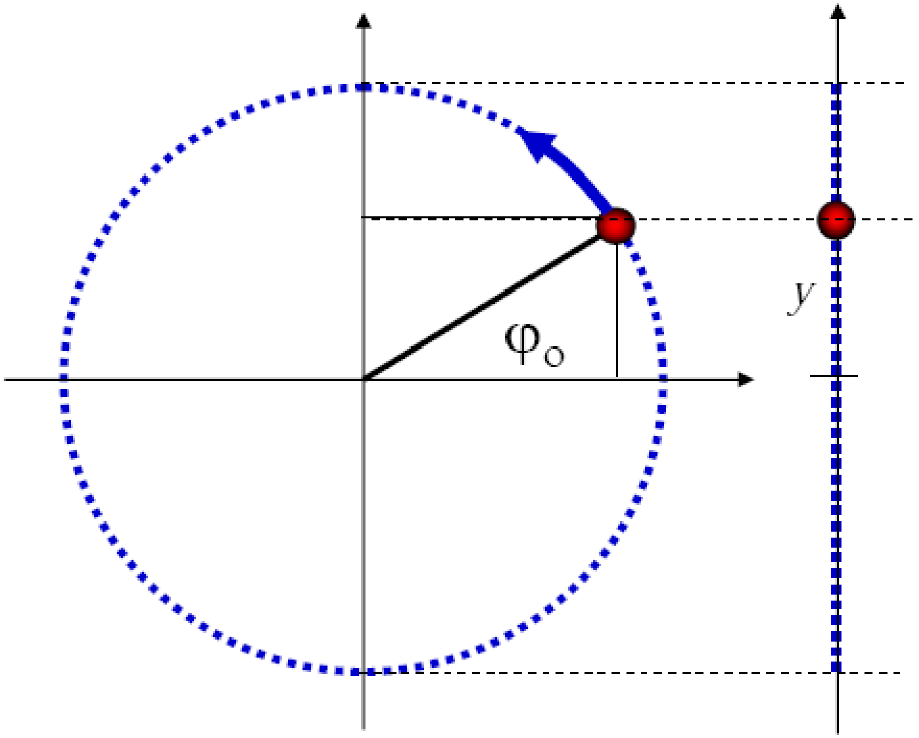
Structural failures:

- Vibrations of bridges, buildings, machines, ...
  caused by wind, earthquakes, engines, ...

- ...

The ultimate question for a physics student could be:
Do you really understand the harmonic oscillator?

Circular Motion → Simple Harmonic Motion *y(t)*



$$\varphi = \varphi_0 + \omega t$$

$$y = y_m \sin(\varphi)$$

$$T = \frac{2\pi}{\omega} \quad \rightarrow \quad \text{period}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \rightarrow \quad \text{frequency}$$

$$y = y_m \sin(\omega t + \varphi_0) \quad \rightarrow \quad \text{simple harmonic motion}$$

$\varphi \rightarrow$ phase

$\varphi_0 \rightarrow$ initial phase (phase const)

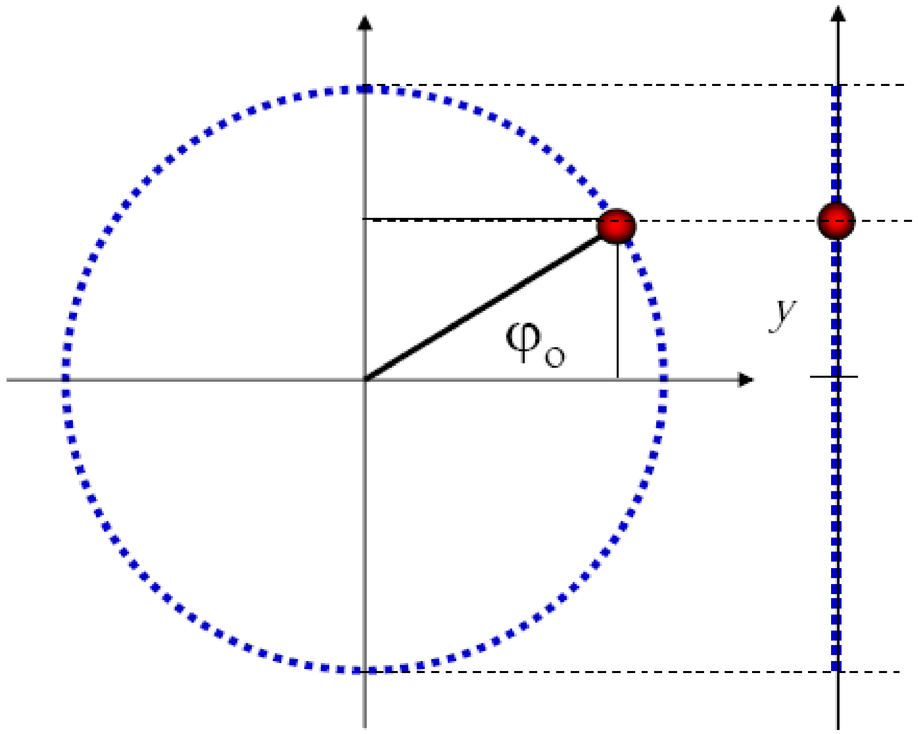
$\omega \rightarrow$ angular frequency ($\omega = 2\pi f$)

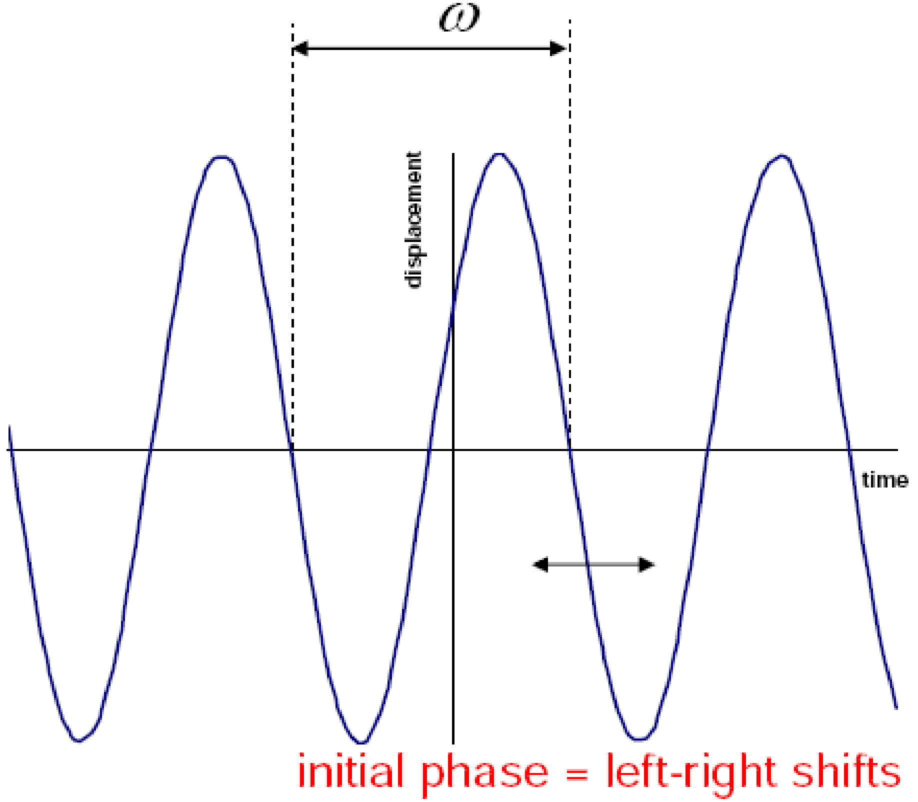
$y_m \rightarrow$ amplitude

Circular Motion → Simple Harmonic Motion *y(t)*

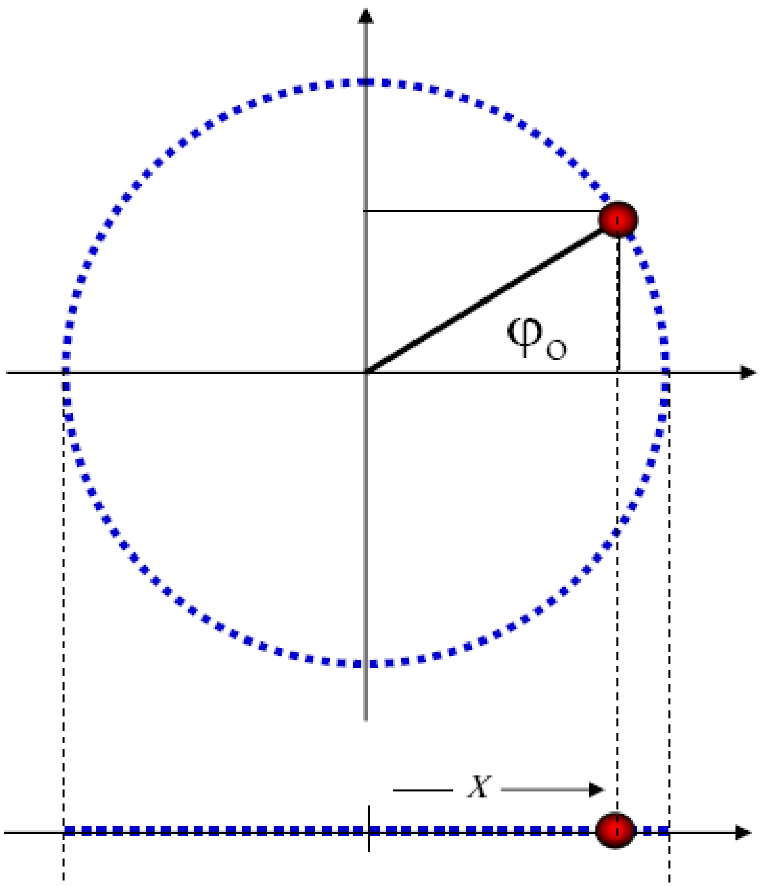
$$y = y_m \sin(\omega t + \varphi_0)$$

$$T = \frac{2\pi}{\omega}$$





Circular Motion → Simple Harmonic Motion *x(t)*



$$\varphi = \varphi_0 + \omega t$$

$$x = x_m \cos(\varphi)$$

$$T = \frac{2\pi}{\omega} \quad \rightarrow \quad \text{period}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \rightarrow \quad \text{frequency}$$

$$x = x_m \cos(\omega t + \varphi_0) \quad \rightarrow$$ simple harmonic motion

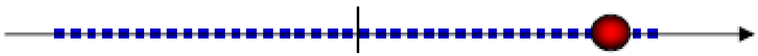
φ → phase

φ_0 → initial phase (phase const)

ω → angular frequency ($\omega = 2\pi f$)

x_m → amplitude

Simple Harmonic Motion *x(t)*



displacement

$$x(t) = x_m \cos(\omega t + \varphi_0), \qquad T = \frac{2\pi}{\omega}$$

velocity

$$v(t) = \frac{dx}{dt} = -(x_m \omega) \cdot \sin(\omega t + \varphi_0)$$

acceleration

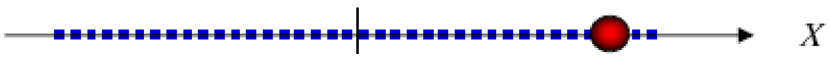
$$a(t) = \frac{dv}{dt} = (x_m \omega) \cdot (-\omega) \cdot \cos(\omega t + \varphi_0) = -\omega^2 \cdot x_m \cos(\omega t + \varphi_0)$$

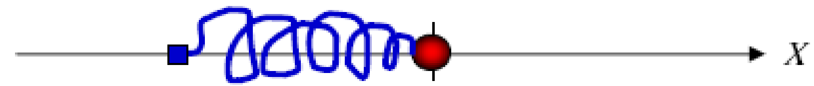
$$\boxed{\frac{d^2 x(t)}{dt^2} = a(t) = -\omega^2 \cdot x(t)}$$

2nd order differential equation for x(t)!

Simple Harmonic Motion *x(t)*

(our friend the mass on a spring)





$$x(t) = x_m \cos(\omega t + \varphi_0), \qquad T = \frac{2\pi}{\omega}$$

$$a(t) = -\omega^2 \cdot x(t)$$

$$F = ma$$

$$F = -m\omega^2 \cdot x(t)$$

$$F = -k \cdot x(t)$$

$$m\omega^2 = k$$

$$\omega = \sqrt{\frac{k}{m}}, \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

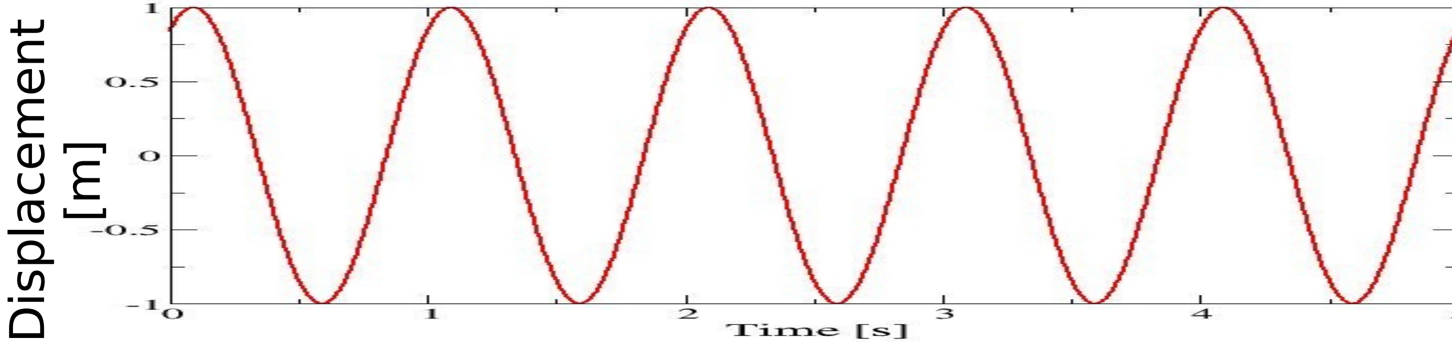
$$x(t) = x_m \cos(\omega t + \varphi_0)$$

Force proportional to displacement!

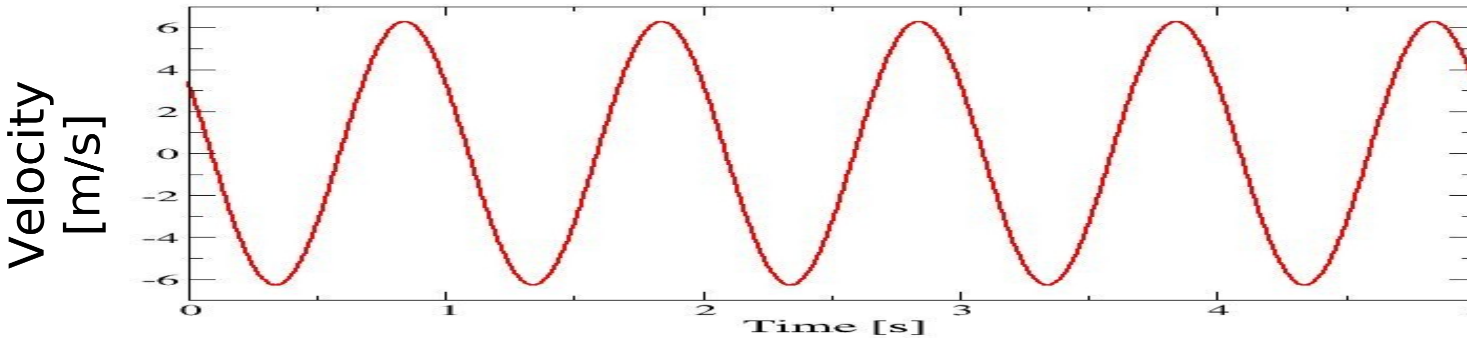
Mass on a spring is **one** (of many) examples for SHM

Single Harmonic Motion

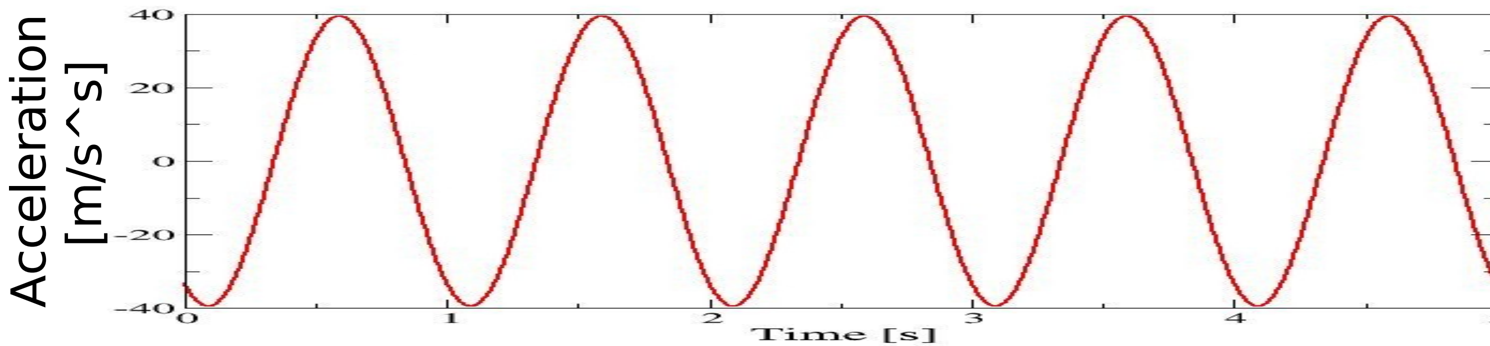
$$x(t) \sim \cos(2\pi\frac{t}{T}+1)$$



$$v(t) \sim \sin(2\pi\frac{t}{T}+1)$$

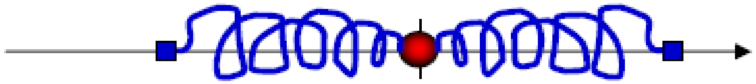


$$a(t) \sim -\cos(2\pi\frac{t}{T}+1)$$



Example 1

An object of mass m is attached to two springs with Hooke's constants k_1 and k_2. Find the period of oscillations.



x=0 is the rest point where **$|F_1| = k_1 d_1$** **$|F_2| = k_2 d_2$** **$F_2 = -F_1$**

Spring 1 and 2 pull with the same force in opposite directions:

If we move the mass to a new position x:

$|F_1| = k_1(d_1+x)$ pulls now more than **$|F_2| = k_2(d_2-x)$**

The new net Force:

$F_{net} = |F_1| - |F_2| = k_1(d_1+x) - k_2(d_2-x) = (k_1+k_2)x$ **$T = 2\pi\sqrt{\frac{m}{(k_1+k_2)}}$**

Example 2

An mass m=1kg oscillates with SH motion according to $x(t)=2.0\cos[(\pi/2)t+\pi/4]$ meters.

1. what are the displacement, velocity, acceleration at t=2s?
2. what are phase angle, frequency f, and period T of motion?
3. what is the kinetic energy as a function of time?
4. what is the total energy as a function of time?

$x(t)=2.0\cos[(\pi/2)t+\pi/4]$

$v(t)= -\pi\sin[(\pi/2)t+\pi/4]$

$a(t)= -0.5\pi^2\cos[(\pi/2)t+\pi/4]$

$x(t=2s) = 2.0\,(-0.5)^{1/2} = -2^{1/2}$

$v(t=2s) = \pi\,(0.5)^{1/2}$

$a(t=2s) = \pi^2/8^{1/2}$

Phase angle: $\pi/4$

Frequency: $f= (\pi/2) / (2\pi)$Hz = 0.25Hz

Period $T = 1/f = 4$s

Example 2

An mass m=1kg oscillates with SH motion according to
$x(t)=2.0\cos[(\pi/2)t+\pi/4]$ meters.

1. what are the displacement, velocity, acceleration at t=2s?
2. what are phase angle, frequency f, and period T of motion?
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4. what is the total energy as a function of time?

$$K = 0.5mv^2 = 0.5 * 1 * \pi^2 * \sin^2[(\pi/2)t+\pi/4] \text{ J}$$

For a force F=-kx, the potential is $U=0.5kx^2$

$$U = 0.5kx^2 = 0.5 * k * 4.0 * \cos^2[(\pi/2)t+\pi/4]$$

Use: $k = 4\pi^2 m/T^2 = \pi^2/4$ $\quad U=0.5*1*\pi^2 * \cos^2[(\pi/2)t+\pi/4] \text{ J}$

$$K+U = 0.5 * 1 * \pi^2 \text{ J} = \text{const.}$$

Potential Energy and SHM

Simple harmonic motion requires a force proportional to a coordinate (for example: x)
Example:

$$F=-kx$$

Such a force has a potential of

$$U=0.5kx^2$$

Remember: A stable equilibrium position is where the potential energy has a minimum.
The potential energy around that minimum can always be approximated by a parabola: $U \sim x^2$
Leads always to SHM for small displacements

Example 3

15.2: A particle of mass $m=1.0 \times 10^{-20}$kg is oscillating with simple harmonic motion with a period of 1.0×10^{-5}s and a maximum speed of 1.0×10^{3}m/s.

Calculate the:

a) angular frequency

b) maximum displacement

c) potential energy as a function of the displacement

to a) $f = 1/T = \omega/2\pi$

→ $\omega = 2\pi/T = 6.28\times10^{5}$rad/s

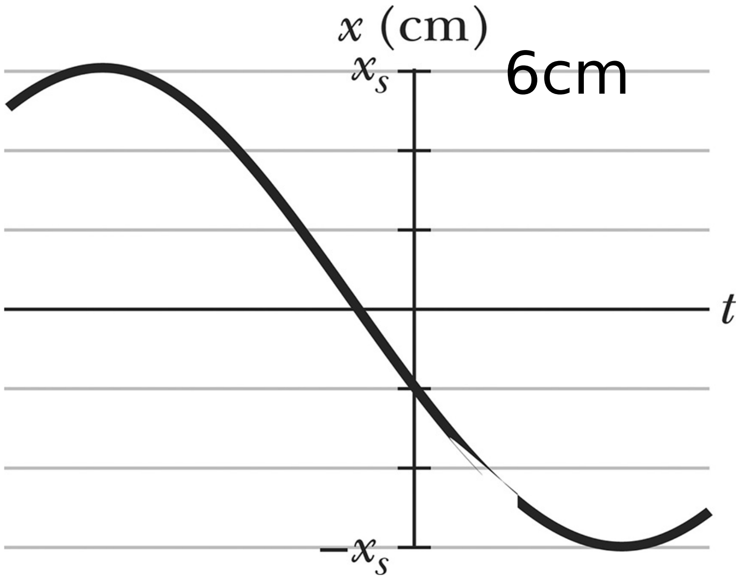
to b) $v(t) = v_{max}\cos(\omega t) = x_{max}\omega \cos(\omega t)$

→ $x_{max} = v_{max}/\omega = 1.59$mm

to c) $\omega = (k/m)^{1/2}$ → $k = \omega^2 m = 4\times10^{-9}$N/m → $U=0.5kx^2 \sim 2\times10^{-9}x^2$

Example 4

Problem 15.10: What is the phase constant for the harmonic oscillator with the position x(t) given in the Figure below if the displacement has the form $x(t)=x_m\cos(\omega t+\phi)$?



x_m = 6cm

x(0)=-2cm

=> $\cos(\phi)$ = -1/3

=> ϕ = 1.91rad