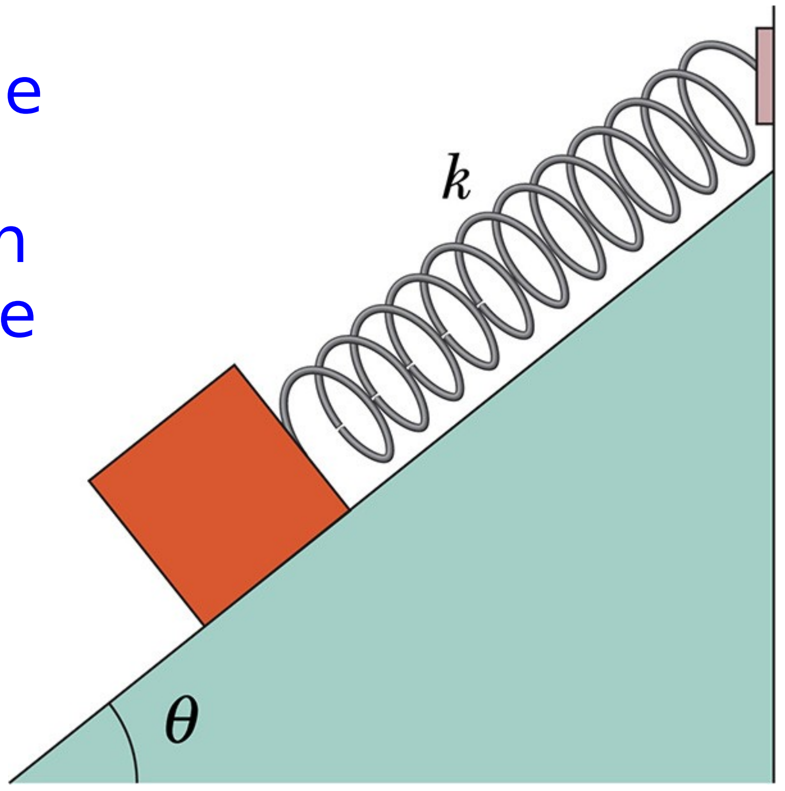
Example

Problem 15.25: In the Figure below, a block weighting 14.0N, which can slide without friction on an incline at angle θ=40deg, is connected to the top of the incline by a massless spring of unstretched length of 45cm and a k=120N/m.

a) How far from the top of the incline is the block's equilibrium position?

b) If the block is pulled slightly down the incline and released, what is the period of the resulting oscillation?



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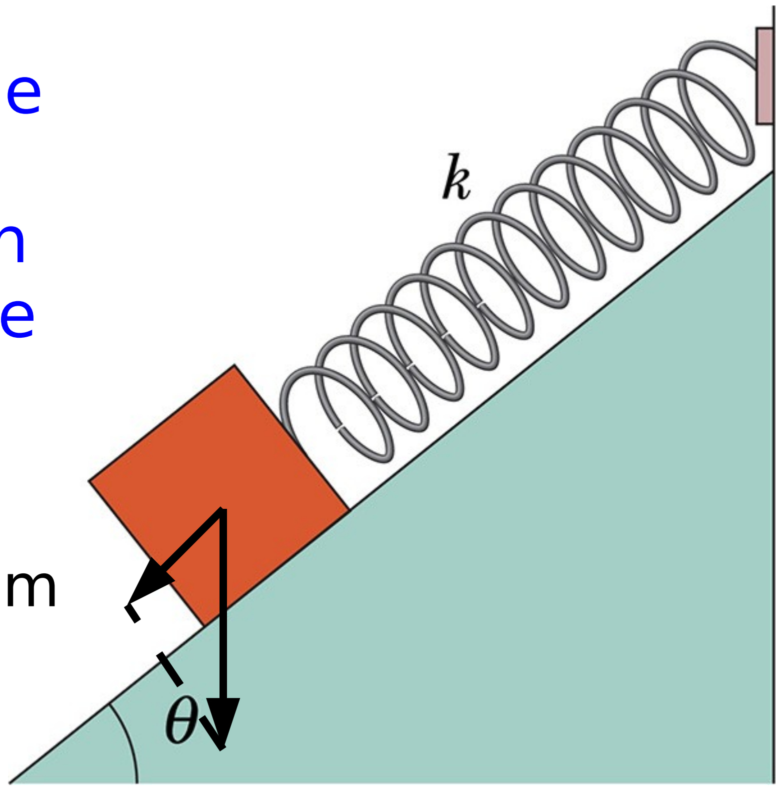


to a) parallel to incline: $F_G \sin\theta = F_S$

$mg\sin\theta = kx_0$ ⇨ $x_0 = mg\sin\theta / k = 7.5\text{cm}$

to b) Any additional length Δx of the spring will create a net force $\sim k\Delta x$

The force is proportional to the displacement: $T = 2\pi(m/k)^{1/2} = 0.686\text{s}$

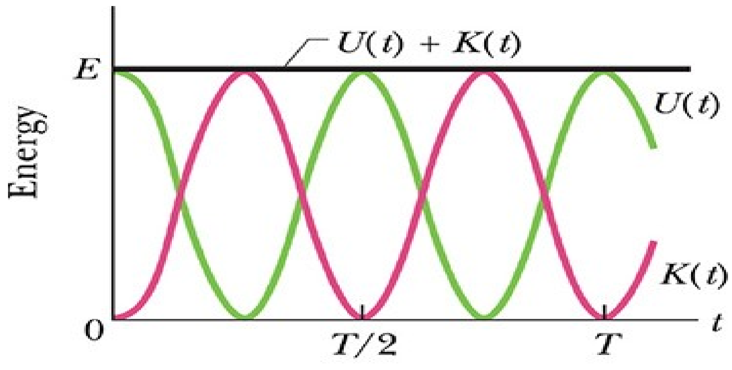
Energy in Simple Harmonic Motion

$F = -k \cdot x$ →

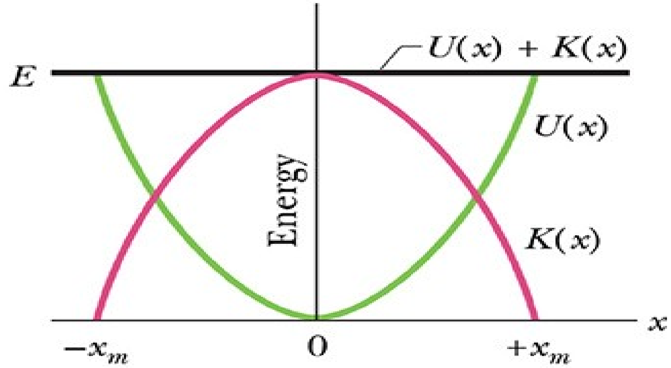
$$x = x_m \sin(\omega t + \varphi_0), \quad \text{where} \quad \omega = \sqrt{k/m}$$
$$v = \omega x_m \cos(\omega t + \varphi_0)$$

$$U(t) = \frac{1}{2} k \cdot x(t)^2$$
$$K(t) = \frac{1}{2} m \cdot v(t)^2$$

$$U(t) + K(t) = \frac{1}{2} k \cdot x(t)^2 + \frac{1}{2} m \cdot v(t)^2$$
$$= \frac{1}{2} k x_m^2 \sin^2(\omega t + \varphi_0) + \frac{1}{2} m\omega^2 x_m^2 \cos^2(\omega t + \varphi_0)$$
$$= \frac{1}{2} k x_m^2 \left(\sin^2(\omega t + \varphi_0) + \cos^2(\omega t + \varphi_0)\right) = \frac{1}{2} k x_m^2$$
$$= \frac{1}{2} m v_m^2$$



(a)



(b)

Different Simple Harmonic Oscillator

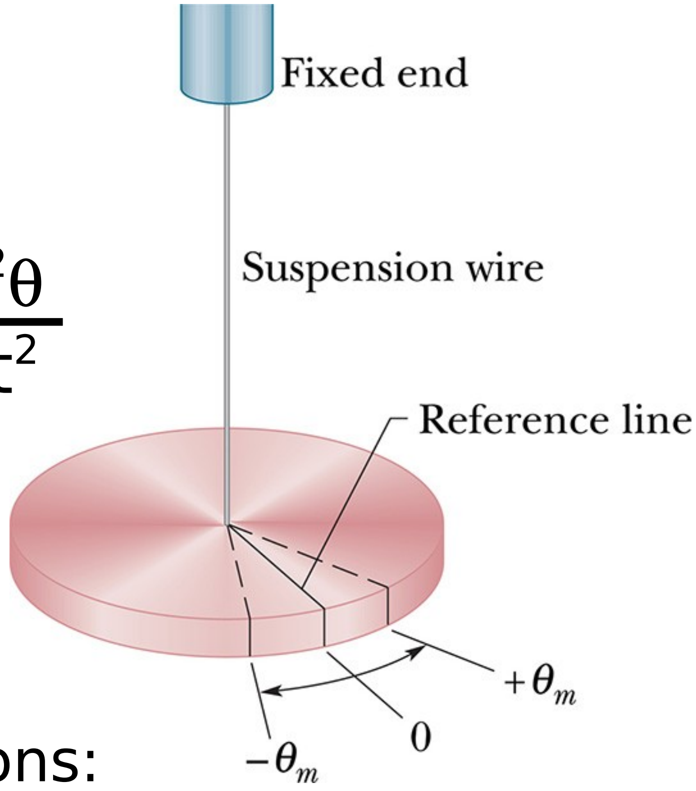
Examples of Simple Harmonic Oscillator:

- Torsion pendulum
- Simple pendulum
- Atom in potential from host crystal
- Elastic springs
  - Spring
  - Vibrating tuning fork
  - Airplane wings
- Crystal oscillator
- Electric resonance circuits (Charges)
- ...

Every time the system is near a local minimum of a potential energy curve (near a stable equilibrium point)

An Angular Simple Harmonic Oscillator

Torsion pendulum



Torques instead of forces:

$$\tau = -\kappa\theta = I\alpha = I\,\ddot{\theta} = I\,\frac{d^2\theta}{dt^2}$$

κ: torsion constant

$-\kappa\theta = I\alpha \iff kx=ma$

Similar equations:

Will also show simple harmonic oscillations with

$T=2\pi(I/\kappa)^{1/2}$

In general: When we have α =-(positive constant)θ

or τ = -(positive constant)θ, we have an angular SHO

Attention:

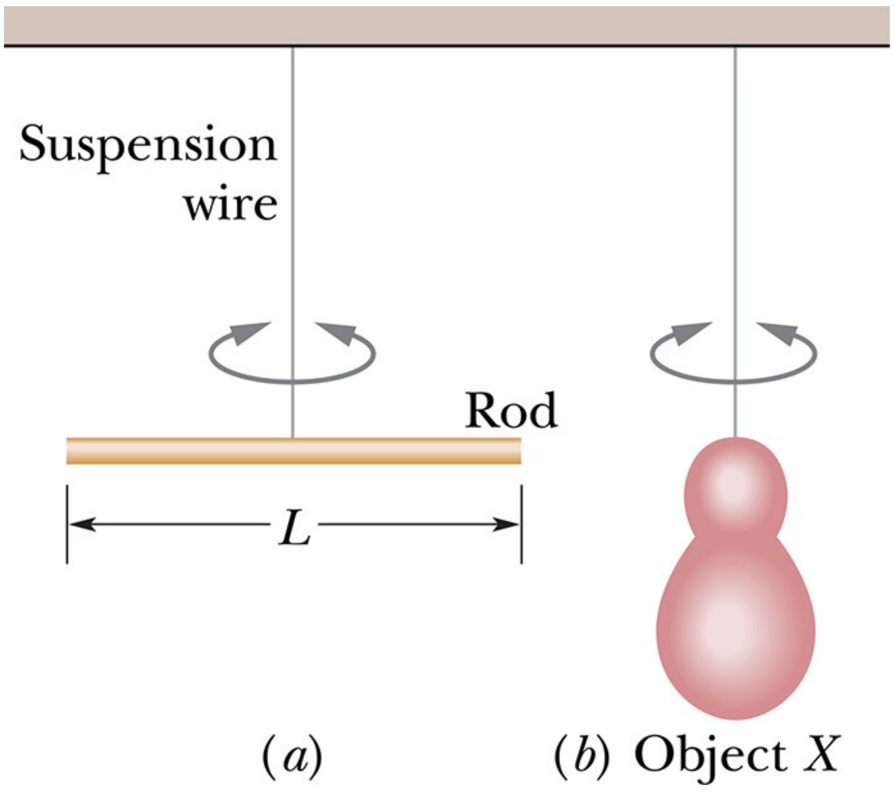
$$\Omega = \frac{d\theta}{dt} \neq \omega$$

Ω : Angular velocity (= ω in Ch10)

ω : Angular frequency

$\omega = 2\pi f$

Application of torsion pendulum



$$I_b = I_a \frac{T_b^2}{T_a^2}$$

Measuring the rotational inertia of an irregularly shaped object:

Period:

$T = 2\pi(I/\kappa)^{1/2}$

depends on I and κ

- measure T_a for object with known rotational inertia I_a --> gives κ

- measure T_b for new object --> gives rotational inertia I_b

Example: Torsion pendulum

The balance wheel of an old fashioned watch oscillates with an angular amplitude π rad and a period of 0.50s. Find
(a) the maximum angular speed of the wheel,
(b) the angular speed at displacement $\pi/2$ rad,
(c) the magnitude of the angular acceleration at displacement $\pi/4$ rad

Based on initial information: $\omega = 2\pi/T = 4\pi$ rad/s

→ $\theta(t) = \pi \cos(\omega t)$

Angular speed: $\dot{\theta}(t) = -\pi\omega \sin(\omega t) = -4\pi^2 \sin(\omega t) = \Omega$

maximum angular speed: $\Omega = 4\pi^2$

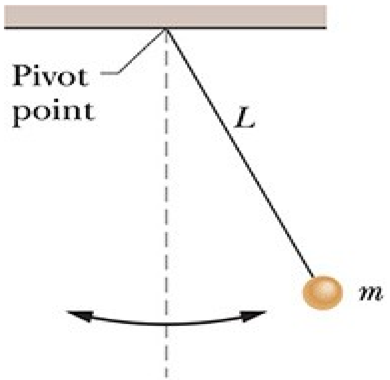
angular speed at $\theta(t_0) = \pi/2$ → $\cos(\omega t_0) = 1/2$ → $\omega t_0 = 60\text{deg}$

→ $\Omega(t_0) = 4\pi^2 \sin(\omega t_0) =$ 34.2rad/s

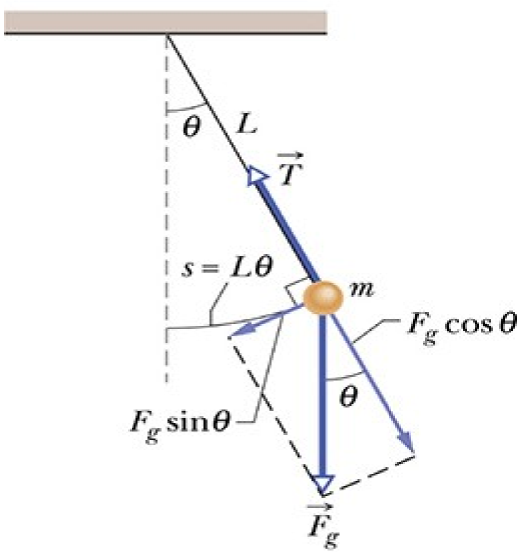
Angular acceleration: $\ddot{\theta}(t) = -\pi\omega^2 \cos(\omega t) = -16\pi^3 \cos(\omega t) = \alpha$

Angular acceleration at $\theta(t_0) = \pi/4$ → $\cos(\omega t_0) = 1/4$ → $|\alpha(t)| = 4\pi^3$

Pendulums



(a)



(b)

Simple (or Idealized) Pendulum:
- massless, unstretchable string
- mass at the end
- small angles: $\sin\theta \sim \theta$

$$\tau = -Lmg\sin\theta \simeq -Lmg\theta$$

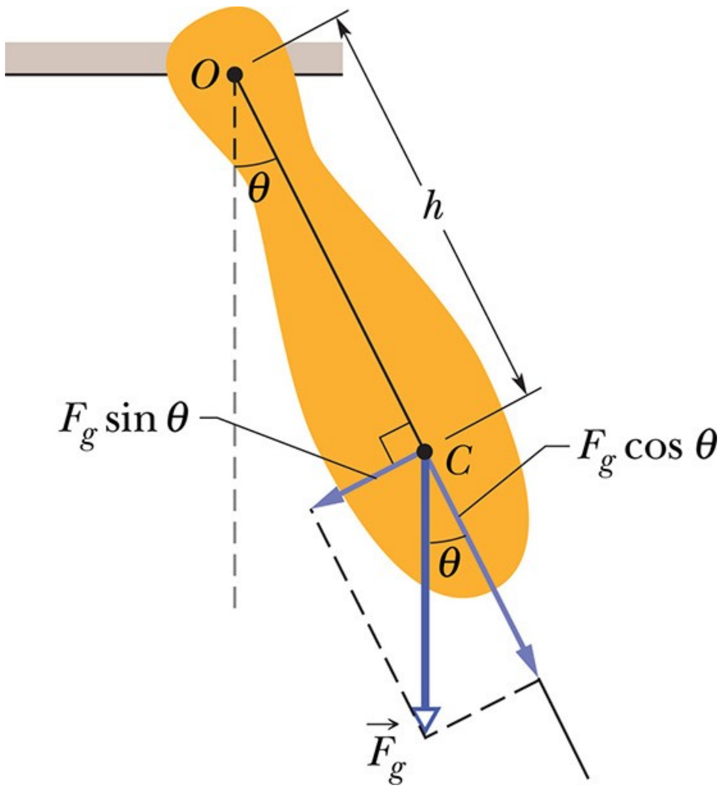
$$\tau = I\alpha = I\ddot{\theta}$$

Gives: $\alpha = -\dfrac{mgL}{I}\,\theta \;\Rightarrow\; \omega = \sqrt{\dfrac{mgL}{I}}$

Ideal Pendulum:
$I = mL^2 \;\Rightarrow\; \omega = \sqrt{\dfrac{g}{L}}$

Period of the ideal pendulum: $T = 2\pi\sqrt{\dfrac{L}{g}}$

Pendulums



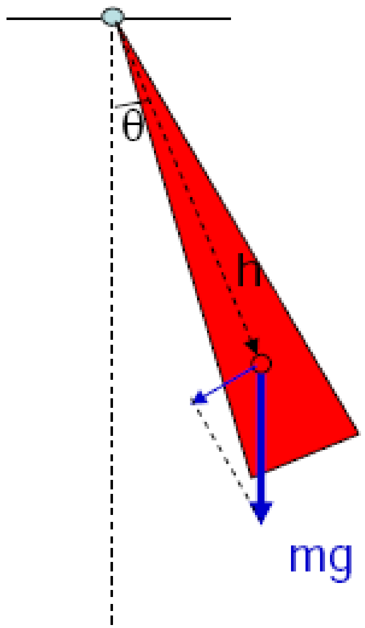
Physical Pendulum:
- complicated mass distribution
- rotational inertia
- small angles

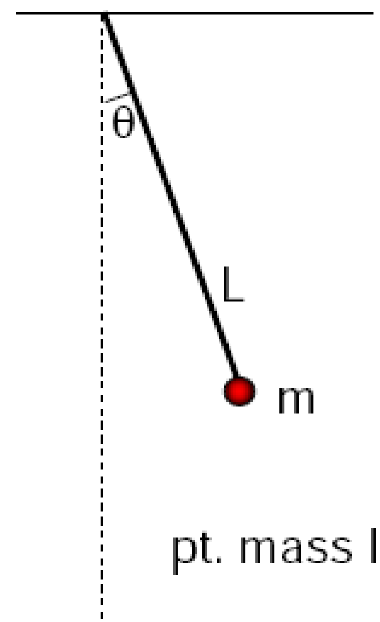
Physical pendulum:

$$\omega = \sqrt{\frac{mgh}{I}} \qquad T = 2\pi\sqrt{\frac{I}{mgh}}$$

h: distance between pivot point and center of mass

Center of oscillation of a physical pendulum



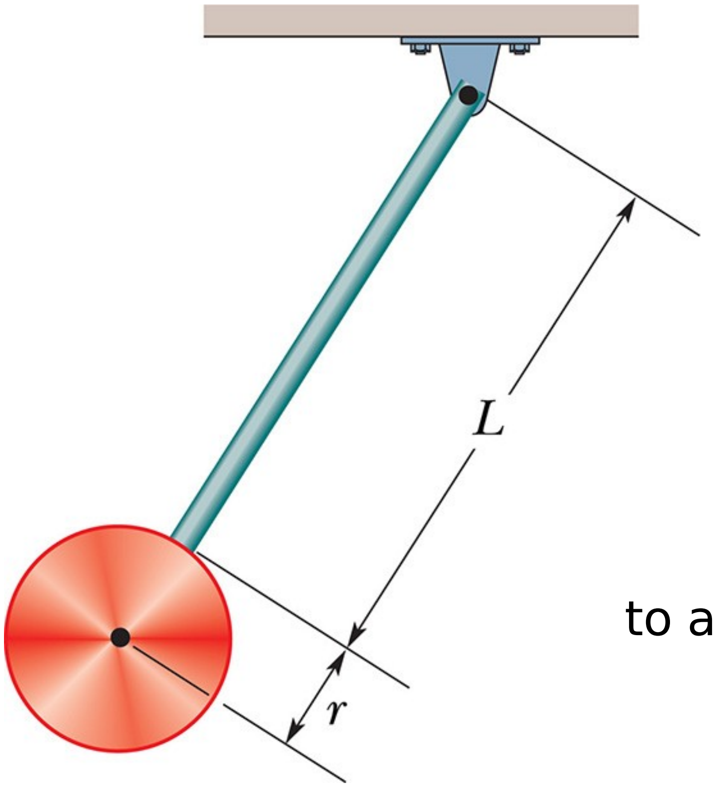


pt. mass $I=mL^2$

this is called a "simple pendulum"!

$$T = 2\pi\sqrt{\frac{I}{mgh}} \quad \Leftrightarrow \quad T = 2\pi\sqrt{\frac{L}{g}}$$

Pendulums



The pendulum consists of a uniform disk with radius r=10.0cm and mass M=500g attached to a uniform rod with length L=0.5m and mass m=270g.

a) Calculate the rotational inertia of the pendulum about the pivot point.

b) What is the distance between the pivot point and the center of mass of the pendulum?

c) Calculate the period of oscillation.

to a) I of the disk: $I_D=0.5Mr^2+M(L+r)^2$

I of the rod: $I_R=mL^2/12 + m(L/2)^2= mL^2/3$

I of the pendulum: $I_P= I_D+ I_R = 0.205\text{kgm}^2$

to b) COM of the disk: $l_D=(L+r) = 0.6\text{m}$

COM of the rod: $l_R=L/2 = 0.25\text{m}$

COM of the pendulum: $l_P= (Ml_D+ ml_R)/(M+m)= 0.477\text{m}$

to c)

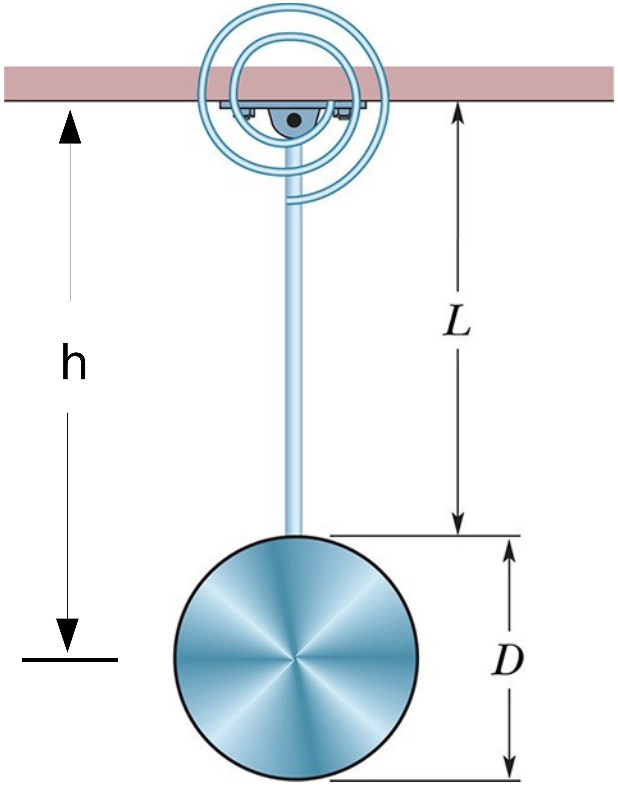
use a) $I_P= 0.205\text{kgm}^2$

use b) $h = 0.477\text{m}$

in: $$T = 2\pi \sqrt{\frac{I}{(M+m)gh}} = 1.5\text{s}$$

Problem Solving

Problem 15.56:



A 2.5kg disk of diameter D=42.0cm is supported by a rod of length L=76.0cm and negligible mass that is pivoted at its end.

a) With the massless torsion spring unconnected, what is the period of the oscillation?

The moment of inertia of the disk supported by the rod is:

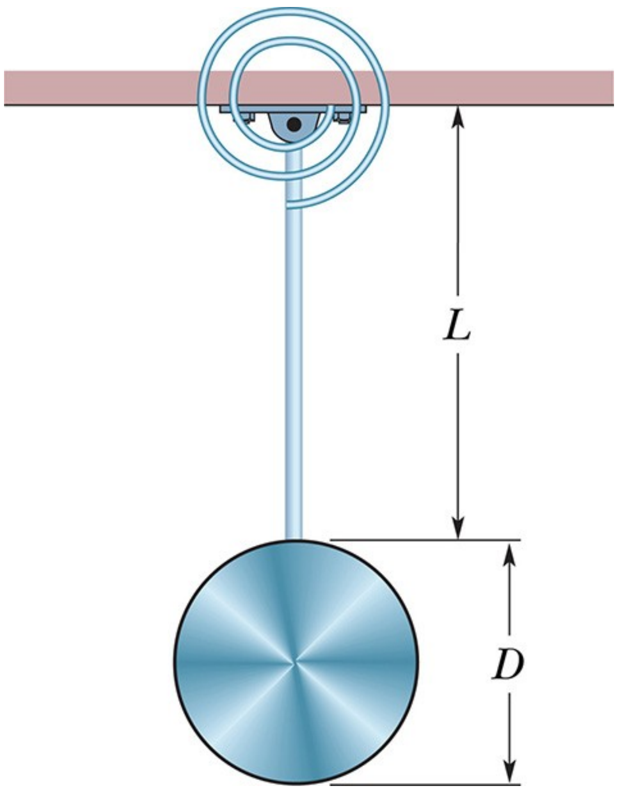
Parallel axis: h=L+D/2=0.97cm

$$I = \frac{1}{2}MR^2 + Mh^2 = 2.41 kgm^2$$

w/o the torsion spring: $T = 2\pi\sqrt{\frac{I}{mgh}} = 2.0s$

Problem Solving

Problem 15.56:



b) With the torsion spring connected, the rod is vertical at equilibrium. What is the torsion constant of the spring if the period of the oscillaton has been decreased by 0.50s?

T_{new} = 1.50s

The additional torsion spring will add a second torque which is also proportional to the angle: $\tau = -\kappa\,\theta$

The original torsion created by gravity is $\tau = mgh\,\theta$

$$\tau_{net} = -(hmg+\kappa)\theta \;\Rightarrow\; T = 2\pi\sqrt{\frac{I}{mgh+\kappa}}$$

Solve for $k = 4\pi^2 I/T^2_{new} - mgh = 18.5$ Nm/rad

HITT

In a simple harmonic oscillator, at one time t_0 the absolute value of the displacement is $|x(t_0)| = 4$cm and the absolute value of the acceleration is $|a(t_0)|=(4\pi^2)16$cm/s^2.
What is the period of the oscillator?

A: $T = 3\pi$ s

B: T = 0.5 s

C: $T = 9\pi$ s

D: T = 6 s

E: T = 0.33 s

We know $\omega^2=a/x = 4\pi^2\, 16/4$
--> $\omega=4\pi$ rad/s

$T = 2\pi/\omega = 1/2$s