Announcements

Exam 3 Review:
Sunday, December 7th 4:00pm-6:00pm in 1001 NPB (here)

Teacher Evaluations:
Wednesday, December 10th in class. In addition to the evaluations the students will be asked 3 HITT questions about the class. If they come and do the evaluations and answer the 3 questions they will get 6 HITT points! (And all possible answers are 'correct')

Superposition principle

Formal: Let $y_1(x,t)$ and $y_2(x,t)$ be solutions of the wave equation. Then

$$y'(x,t) = c_1 y_1(x,t) + c_2 y_2(x,t)$$

is also a solution.

Proof:

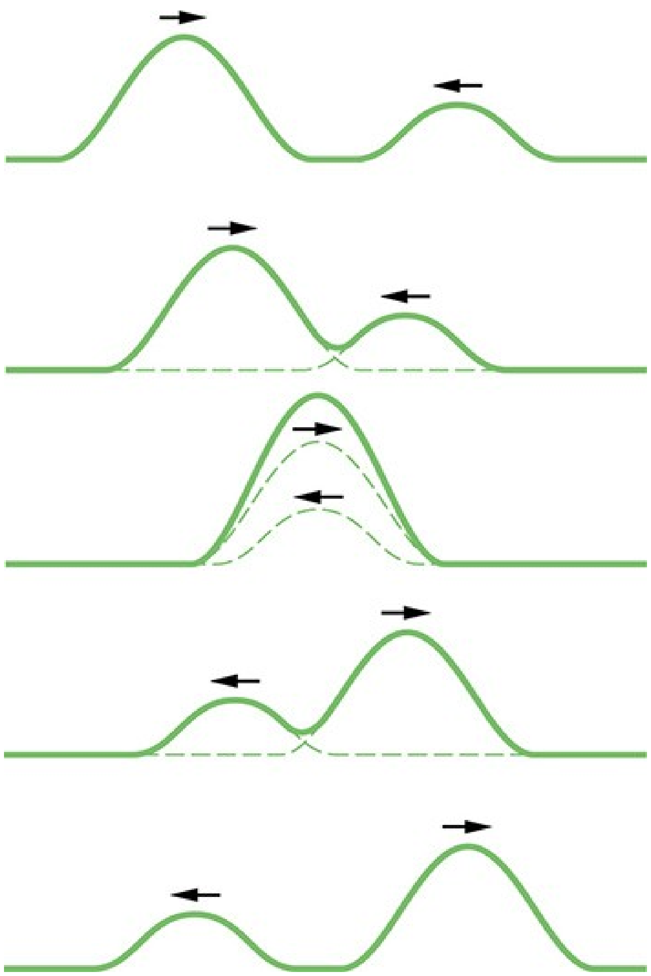
$$\frac{d^2 y'}{dx^2} = c_1 \frac{d^2 y_1}{dx^2} + c_2 \frac{d^2 y_2}{dx^2} = \frac{c_1}{v^2}\frac{d^2 y_1}{dt^2} + \frac{c_2}{v^2}\frac{d^2 y_2}{dt^2} = \frac{1}{v_2}\frac{d^2(c_1 y_1 + c_2 y_2)}{dt^2} = \frac{1}{v^2}\frac{d^2 y'}{dt^2}$$

What are the consequences?

If two waves travel in the same medium (along the same string), they add algebraically to a resultant net wave.

They travel independent of each other, don't influence each. In other words: The can be treated completely independent from each other (assuming that all approximations we used to derive the wave equation are still valid).

Superposition principle



Example:
Two waves propagating in opposite directions:

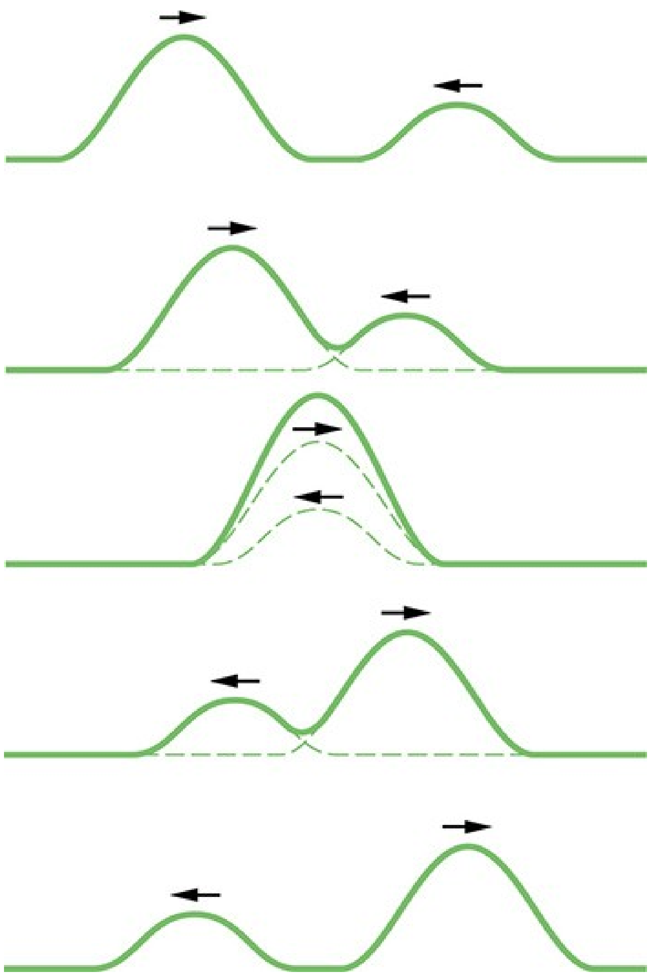
$$y_1(x,t) = y_{1m} h(x - vt)$$

$$y_2(x,t) = y_{2m} h(x + vt)$$

$$y'(x,t) = y_{1m} h(x - vt) + y_{2m} h(x + vt)$$

Note: This is not a single wave because it has two different velocities

Superposition principle



Waves travel

- independent of each other
- through each other without being distorted

This is the reason why it is useful and possible to describe any wave pulse as a linear combination of harmonic (sin-, cos-) waves.
Then figure out what each harmonic wave does and finally add them all back up again.

Interference

Interference of waves

- is superposition principle in action.
- is everywhere but we will discuss only the most fundamental examples

Interference between two waves of same frequency, amplitude, and propagation direction:

$$y_1(x,t) = y_m \sin(kx - \omega t) \qquad y_2(x,t) = y_m \sin(kx - \omega t + \phi)$$

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \left[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)\right]$$

$$\text{use:} \qquad \sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$y'(x,t) = \underbrace{\left[2y_m \cos\frac{\phi}{2}\right]}_{\text{New Amplitude}} \sin\left(kx - \omega t + \frac{1}{2}\phi\right)$$

New wave with new amplitude and new initial phase

Interference

Interference between two waves of same frequency, amplitude, and propagation direction:

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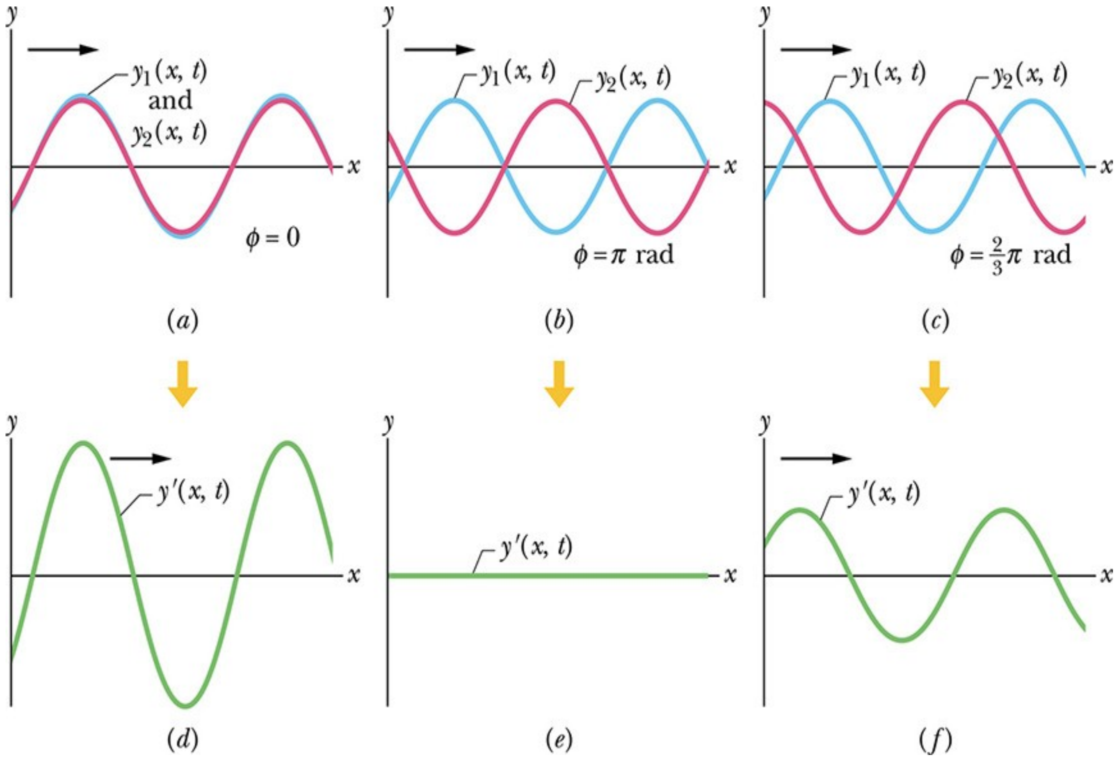


Both waves in phase (ϕ=N2π):
- Maximum Amplitude $2y_m$
- Constructive interference

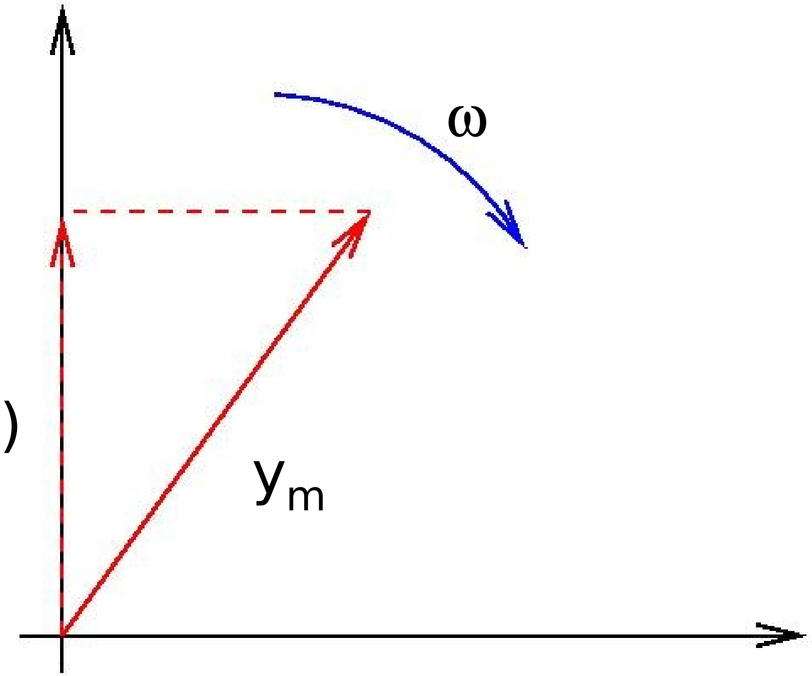
Both waves 180deg out of phase (ϕ=(2N+1)π):
- Amplitude is 0
- Destructive Interference

Phase somewhere between:
- Amplitude between 0 and $2y_m$
- Intermediate Interference

Phasors

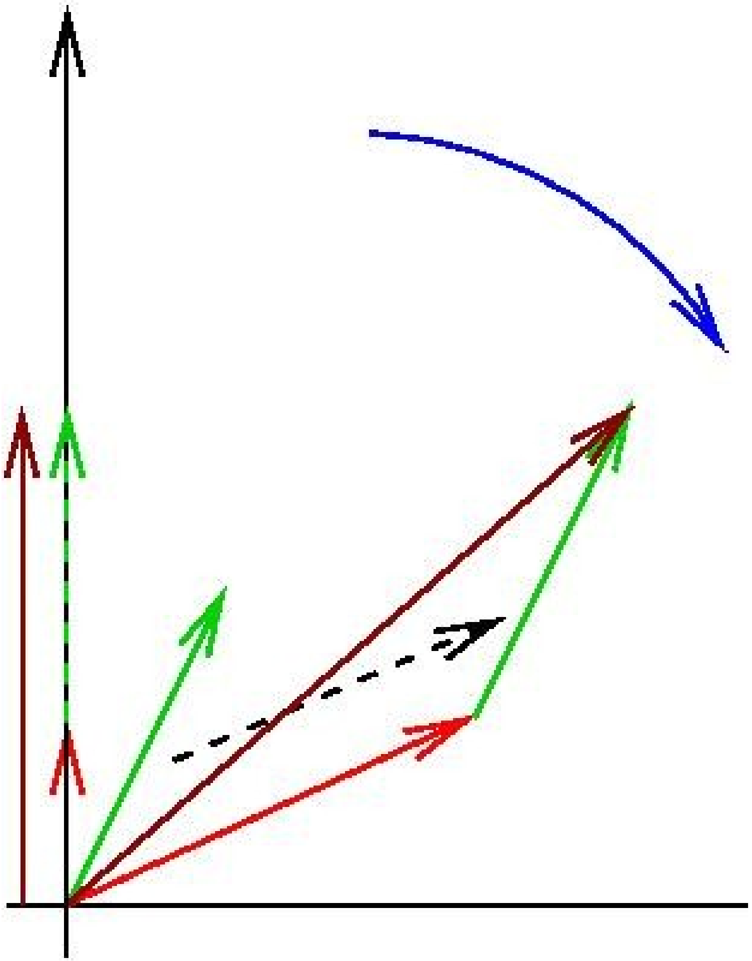
Use the connection between circular motion
and sin-, cos- functions:

$$y(x,t) = y_m \sin(kx - \omega t)$$



The phasor has a length of y_m which rotates with ω in the x-y plane.
The wave is then represented by the projection
of the phasor on the y-axis.

Phasors



$$y_1(x,t) = y_{1m}\sin(kx - \omega t + \phi_1)$$

$$y_2(x,t) = y_{2m}\sin(kx - \omega t + \phi_2)$$

$$y'(x,t) = y_1(x,t) + y_2(x,t) = \ldots$$

Geometric addition of vectors

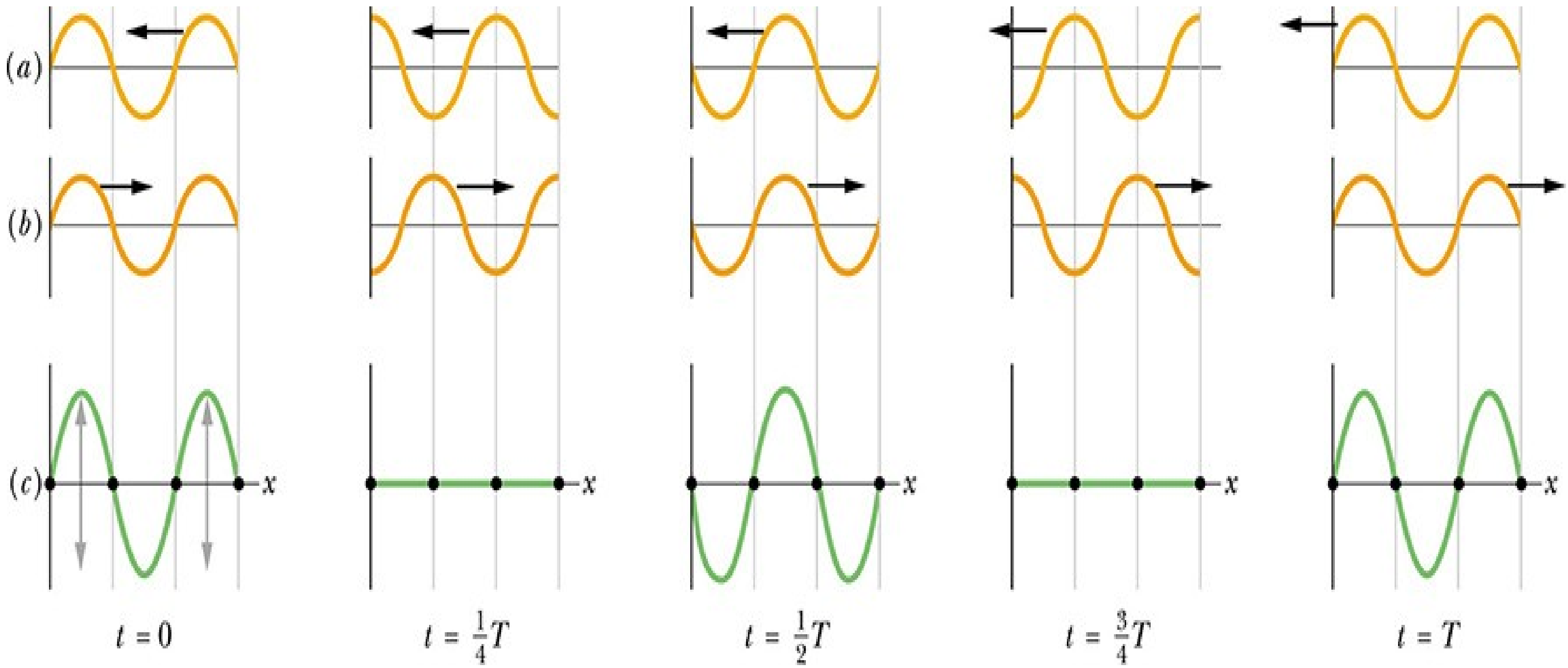
Phasor diagrams are great to visualize what is going on. They can be applied to virtually all cases where you have to add waves. The math doesn't get easier, but the diagrams can guide you through it.

Standing Waves

Interference between two waves of same frequency, same amplitude, but opposite propagation direction:

$$y_1(x,t) = y_m \sin(kx - \omega t) \qquad y_2(x,t) = y_m \sin(kx + \omega t)$$

$$y'(x,t) = y_1(x,t) + y_2(x,t) = \underbrace{2y_m \sin kx}_{\text{Amplitude}} \cos \omega t$$



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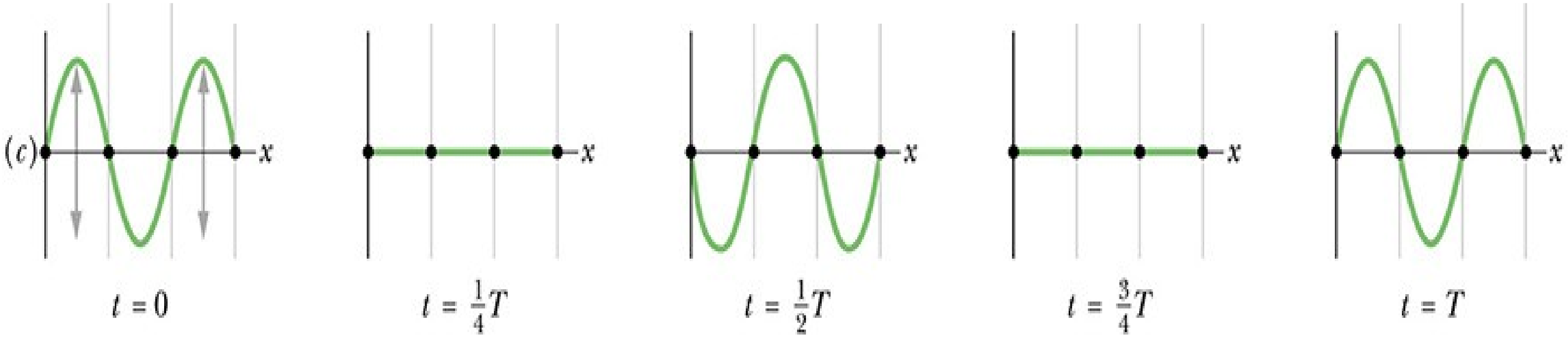


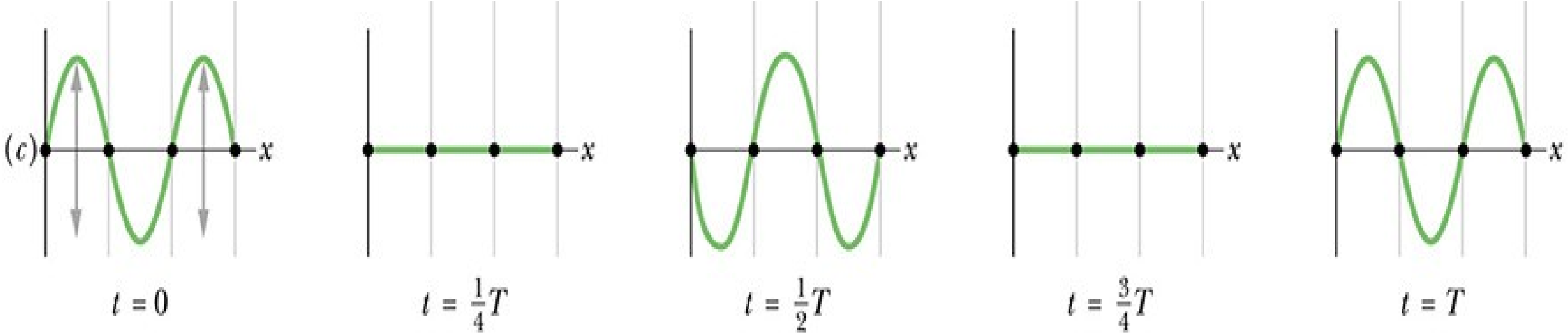
Standing wave: Nodes (zero amplitude) where

$kx=n\pi$ $n=0,1,2,\ldots$ or   at $x = n\,\lambda/2$

Standing wave: Maximum amplitude (Anti-nodes) where

$kx=(n+0.5)\pi$ $n=0,1,2,\ldots$ or   at $x = (n+0.5)\,\lambda/2$

Standing Waves



Energy:

- Potential elastic energy maximal at nodes when amplitude (away from knots) is maximum
- Kinetic energy is maximal at anti-nodes when amplitude is zero

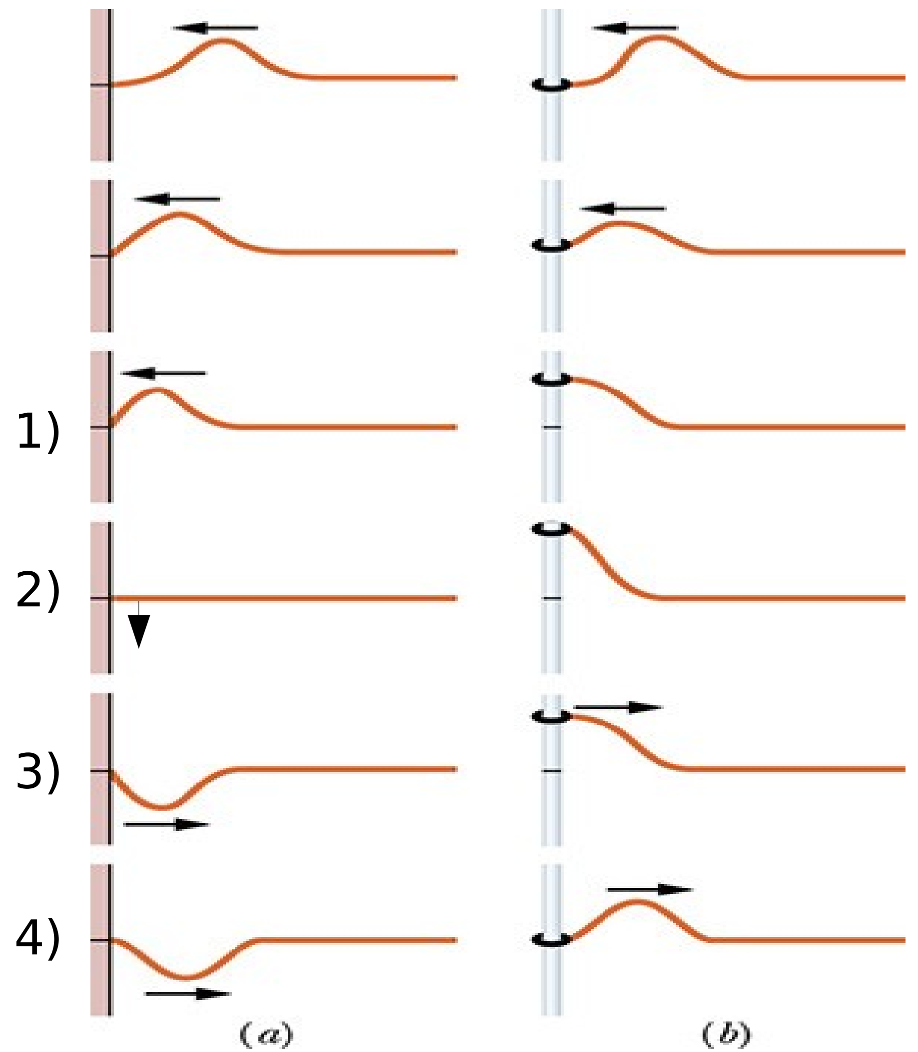
Standing Waves

Reflections at a boundary:

If we fix the string at the other end, the wave pulse will:

1) stretch the string and store elastic energy in the string
2) the string will move back and the energy will go into kinetic energy, moves through zero and
3) stretch the string now in the opposite direction and
4) then travels backwards

Creates a node at the end



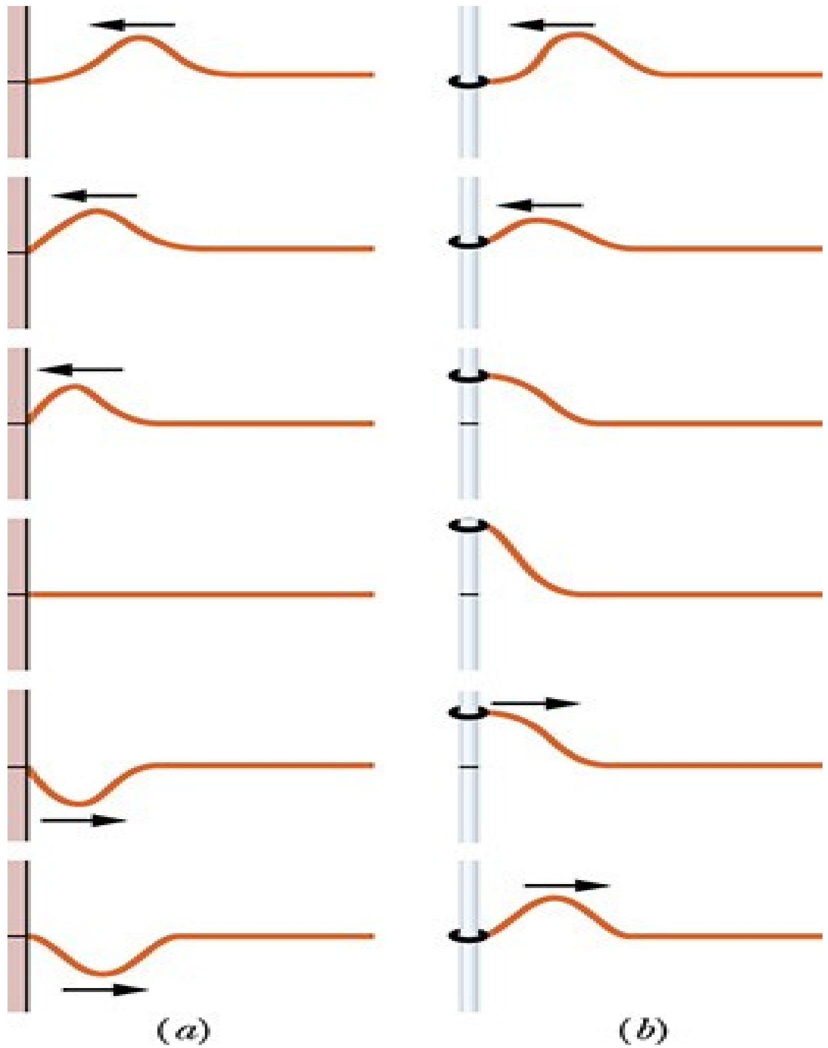
Standing Waves

Reflections at a boundary:

If the other end of the string is free to move up and down (but not forward or backward)

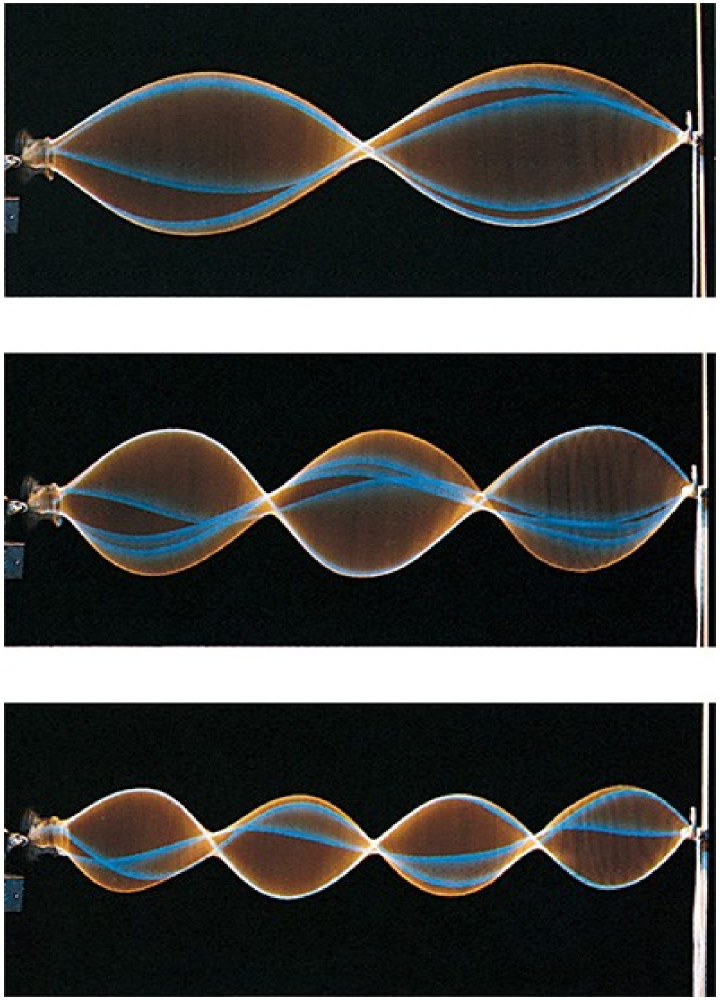
1) moves the string up and stretches the string in the process
2) the elastic force is then pulling the string back and accelerates the end downwards
3) this generates a 'new' wave traveling backwards

Creates an anti-node at the end



Both situations can create standing waves

Standing Waves

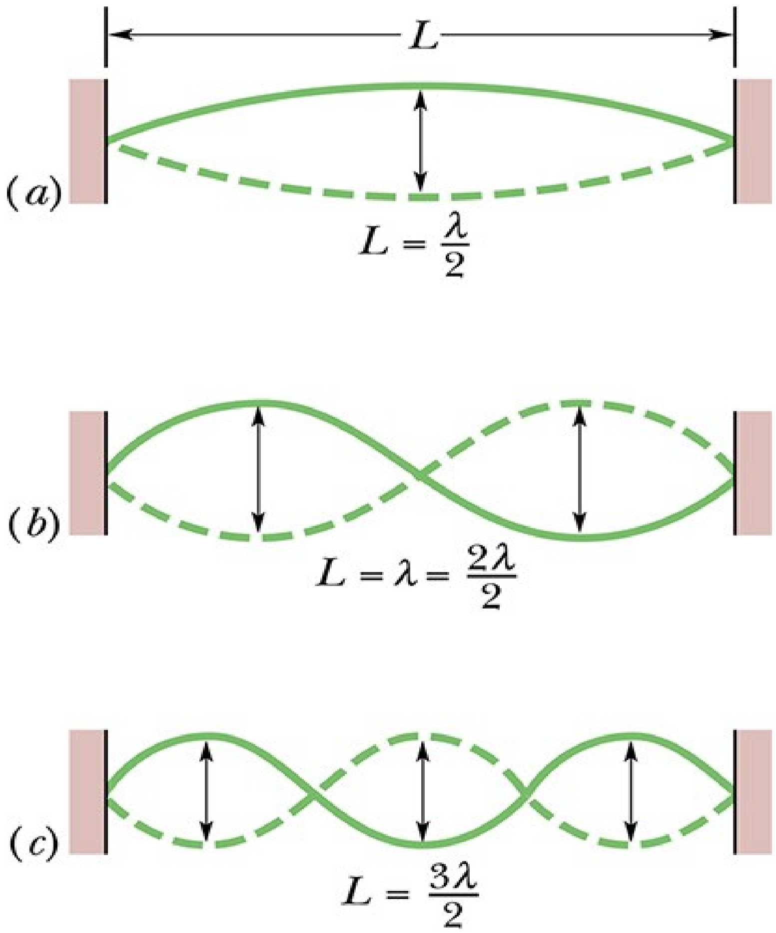


Standing waves and resonances:

- The end points of a string are fixed
- The string is excited at one end by a sinusoidal wave
- The wave travels along the string and reflects at the back end
- The wave will then bounce back and forth between the end points
- Different parts of the wave interfere with each other
- If the conditions (length of string, frequency of excitation, wavelength) are just right, this interference is constant in space:

Standing waves

Standing Waves



Standing waves:

- The clamps enforce knots at the ends.
- Waves with a very specific set of wavelengths fit into this pattern

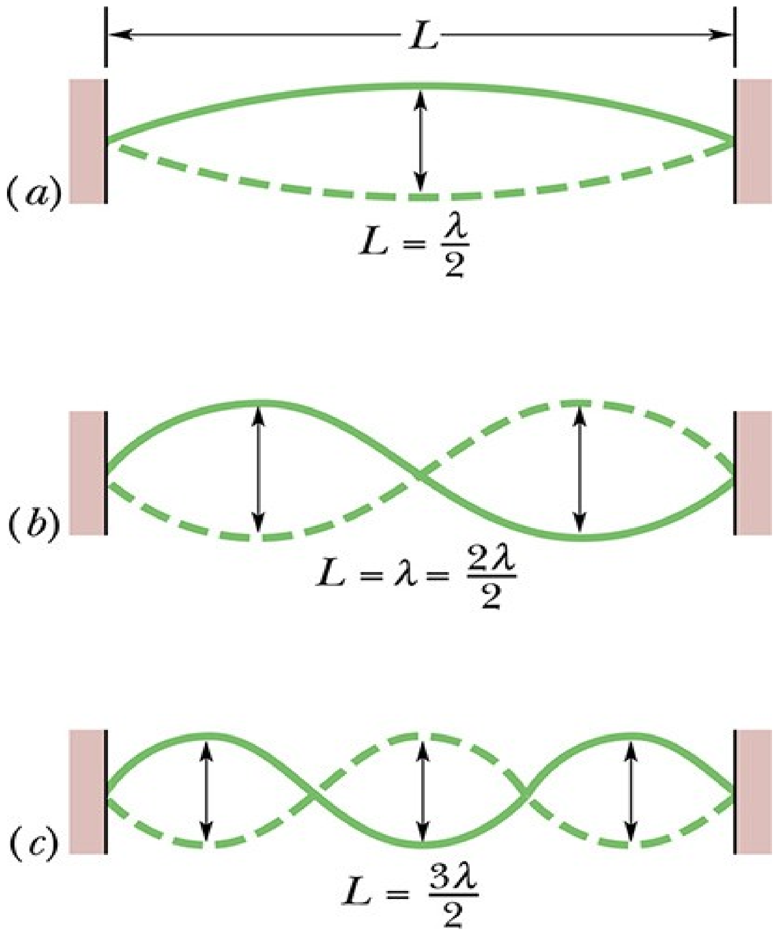
$$\lambda = \frac{2L}{n} \qquad \text{for } n = 1, 2, 3, \ldots$$

- These waves will interfere constructively at the anti-nodes and destructively at the nodes

$$f = \frac{v}{\lambda} = n\frac{v}{2L} \qquad \text{for } n = 1, 2, 3, \ldots$$

n = 1: fundamental
n = 2: second harmonic
n = 3: third harmonic (and so on)

Standing Waves



Standing waves:

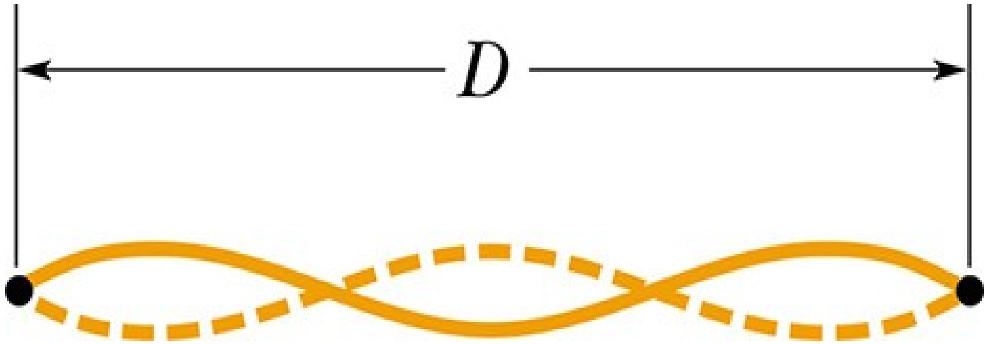
- Critical when a system has some kind of 'positive feedback'

Example:

- People walking over a bridge with resonance frequency similar to walking frequency:
  - The bridge will start to shake at that frequency
  - The folks on the bridge will then start to coordinate their walking with the shaking of the bridge to maintain their balance
  - This increases the shaking which forces the people to even coordinate their walking furthermore

Standing Waves

Example: A nylon guitar string has a length of 90.0cm, a linear density of 7.2g/m and is under a tension of 150N. The string is oscillating in the standing wave pattern shown in the figure.
What is the
a) speed
b) wavelength
c) frequency
of the traveling waves which generate the standing wave.



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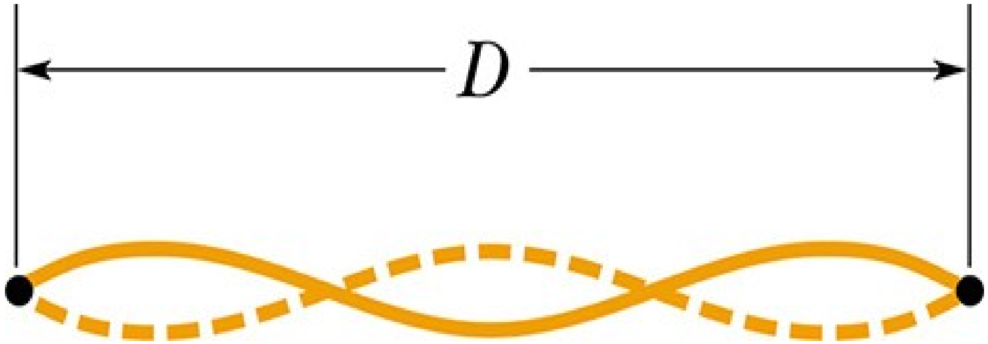

Speed: $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{150}{7.2 \cdot 10^{-3}}} \frac{\mathrm{m}}{\mathrm{s}} = 144 \frac{\mathrm{m}}{\mathrm{s}}$

Wavelength: $\lambda = \frac{2}{3} D = 0.6\,\mathrm{m}$ just geometry

Frequency: $f = \frac{v}{\lambda} = \frac{144}{0.6} = 241\,\mathrm{Hz}$

HITT 1

Which of the following expressions describes a standing wave, a wave traveling into the positive direction, a wave traveling into the negative direction?

$a(x,t) = 3 \sin(3x-\omega t)$ $b(x,t) = 3 \sin(3x) \cos(\omega t)$ $c(x,t) = 3 \cos(3x+\omega t)$

	standing	positive	negative
A	a	b	c
B	b	c	a
C	c	a	b
D	b	a	c
E	c	b	a

HITT 2

Which of the following expressions describes a standing wave, a wave traveling into the positive direction, a wave traveling into the negative direction?

$a(x,t) = 3 \sin(3x-\omega t)$ $b(x,t) = 3 \cos(3x+\omega t)$ $c(x,t) = 3 \sin(3x) \cos(\omega t)$

	standing	positive	negative
A	a	b	c
B	b	c	a
C	c	a	b
D	b	a	c
E	c	b	a

Interference

Interference between two waves of different frequency, but same amplitude, and propagation direction:

$$y_1(x,t) = y_m \sin(k_1 x - \omega_1 t) \qquad y_2(x,t) = y_m \sin(k_2 x - \omega_2 t)$$

use: $$\Delta k = \frac{k_1 - k_2}{2} \qquad \Delta\omega = \frac{\omega_1 - \omega_2}{2} \qquad \bar{k} = \frac{k_1 + k_2}{2} \qquad \bar{\omega} = \frac{\omega_1 + \omega_2}{2}$$

$$y'(x,t) = y_1(x,t) + y_2(x,t) = 2y_m \left[\cos(\Delta k x - \Delta\omega t)\sin\left(\bar{k}x - \bar{\omega}t\right)\right]$$