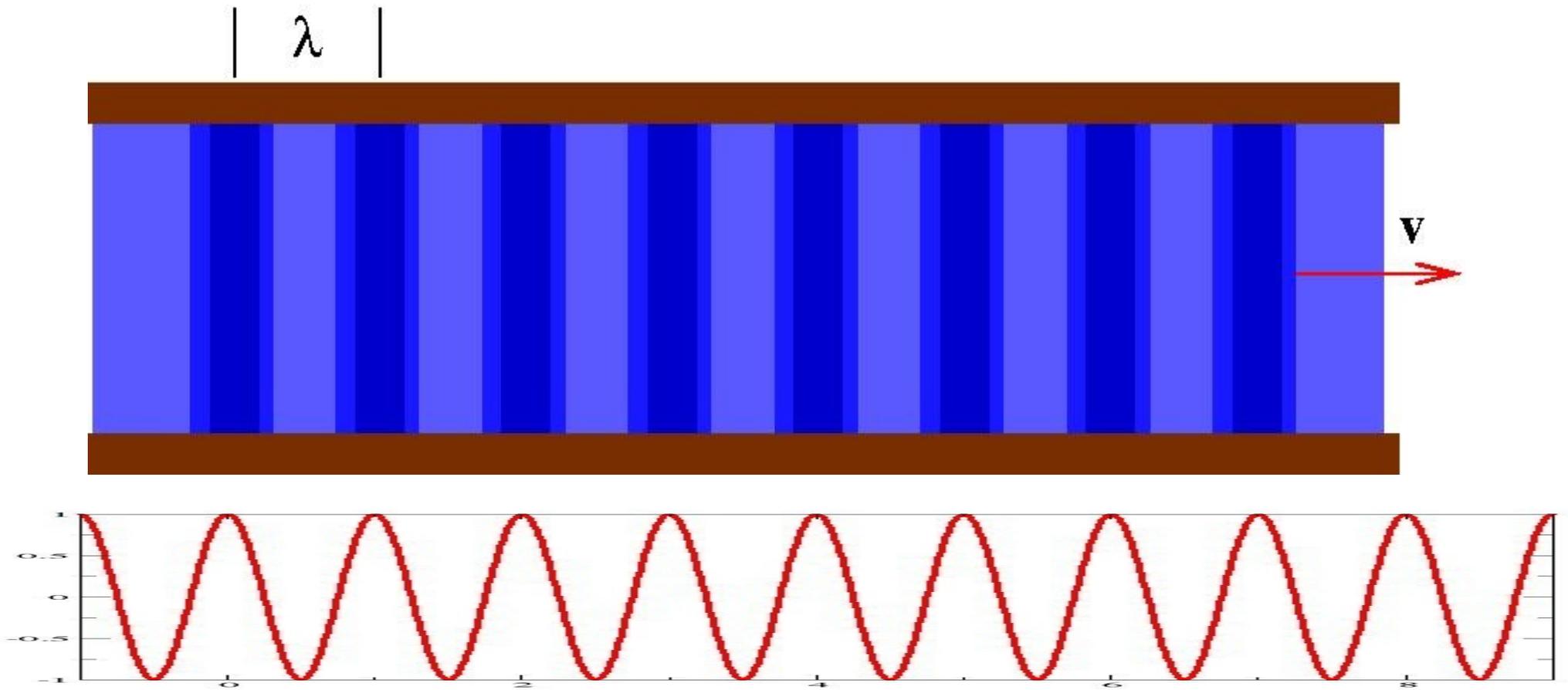


Chapter 17: Waves II

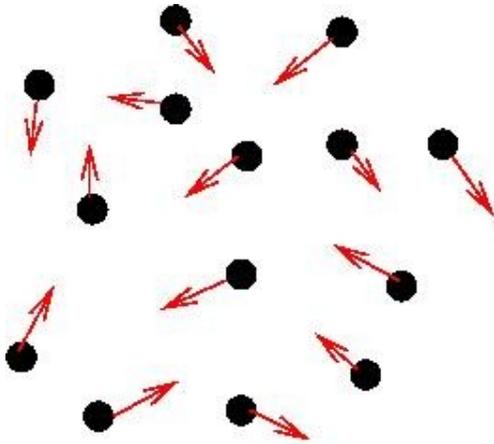
Sound waves are one example of Longitudinal Waves



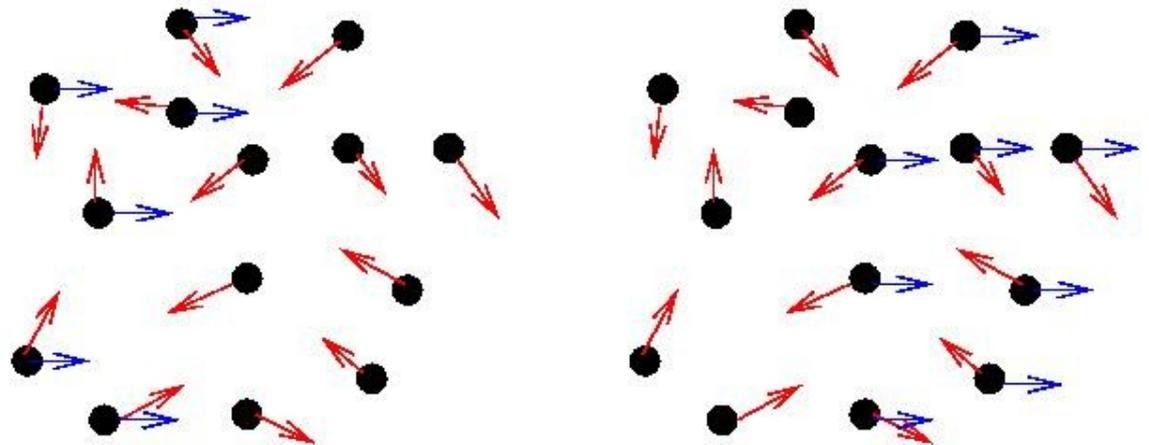
Sound waves are pressure waves: Oscillations in air pressure and air density

Chapter 17: Waves II

Before we can understand pressure waves in detail, we need to understand what happens in a gas (or liquid).

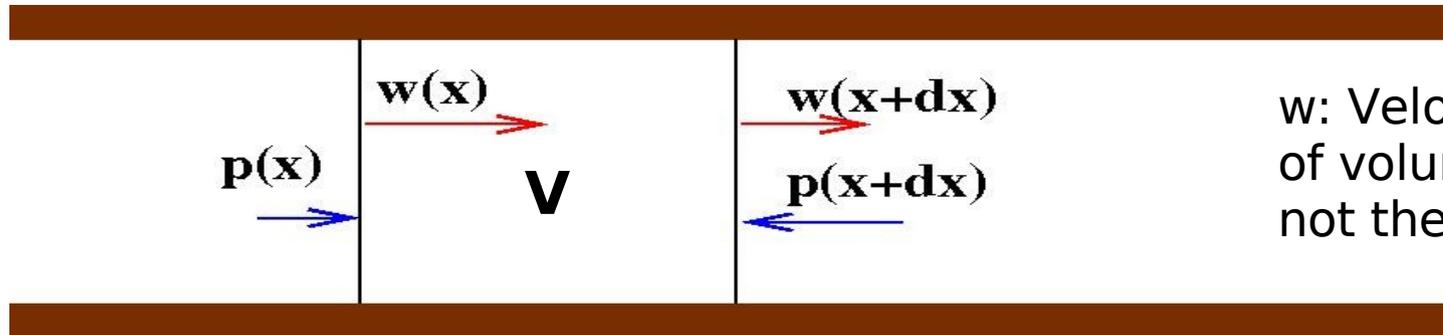


w/o waves: Gas molecules move around randomly. Collide every $\sim 100\text{nm}$. These collisions create the static and homogeneous pressure.

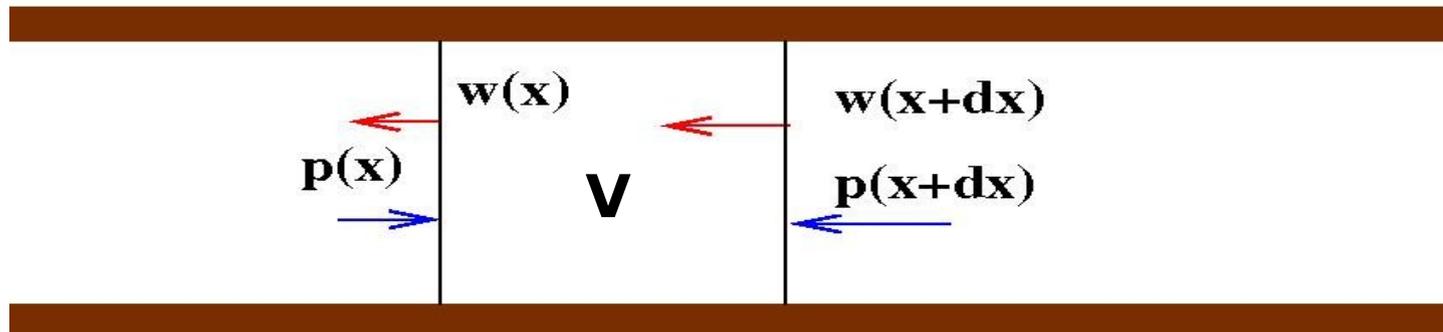
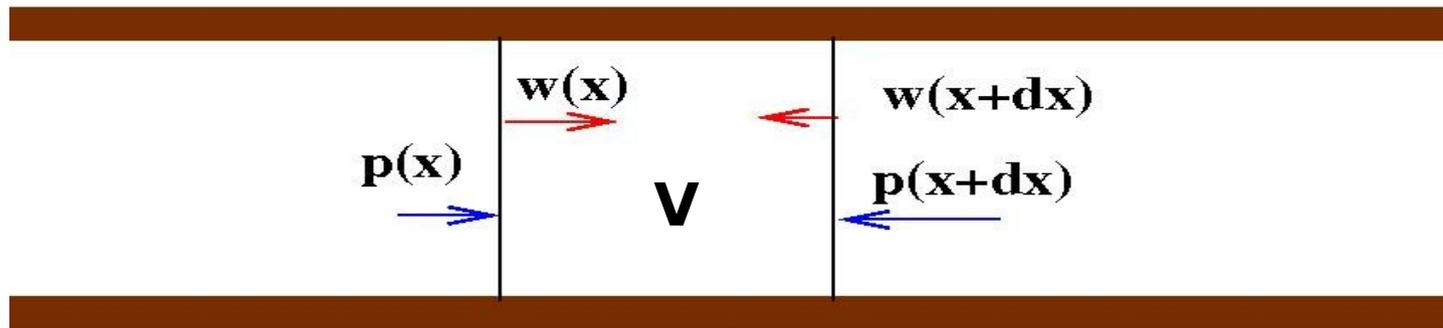


A pressure wave coming from one side adds a velocity component to all gas molecules in an area which points into one specific direction. Now you have more collisions where one gas molecule comes from a specific direction. This compresses the gas in one area and pushes the neighboring gas molecules to move into that direction and pick up a velocity component into that direction.

Chapter 17: Waves II



w : Velocity of 'walls' of volume element V , not the wave velocity!



Chapter 17: Waves II

Variations of pressure along x-axis \Rightarrow Forces on volume element $V=A\Delta x$

$$F = -A\Delta x \frac{\Delta p}{\Delta x} \quad \Rightarrow \quad a = \frac{dw}{dt} = - \frac{A\Delta x \frac{\Delta p}{\Delta x}}{\underbrace{\rho A \Delta x}_{=m}} = - \frac{1}{\rho} \frac{\Delta p}{\Delta x}$$

Left side of V moves with w, right side with $w+(dw/dx)\Delta x$

After time dt, both sides moved by different amount \Rightarrow Changes Volume

$$\Delta V = A \frac{dw}{dx} \Delta x dt \quad \Rightarrow \quad \frac{\Delta V}{V} = \frac{A \frac{dw}{dx} \Delta x dt}{A \Delta x} = \frac{dw}{dx} dt$$

This creates a pressure change inside the volume element (B: Bulk modulus):

$$\Delta p = -B \frac{\Delta V}{V} = -B \frac{dw}{dx} dt \quad \Rightarrow \quad \frac{dp}{dt} = -B \frac{dw}{dx}$$

Chapter 17: Waves II

Two coupled differential equations for pressure and velocity:

$$\frac{dp}{dt} = -B \frac{dw}{dx}$$

$$\frac{dw}{dt} = -\frac{1}{\rho} \frac{dp}{dx}$$



$$\frac{d^2 p}{dt^2} = -B \frac{d}{dt} \left(\frac{dw}{dx} \right) = -B \frac{d}{dx} \left(\frac{dw}{dt} \right) = \frac{B}{\rho} \frac{d^2 p}{dx^2}$$

Wave equation for pressure



$$\frac{d^2 w}{dt^2} = -\frac{1}{\rho} \frac{d}{dt} \left(\frac{dp}{dx} \right) = -B \frac{d}{dx} \left(\frac{dp}{dt} \right) = \frac{B}{\rho} \frac{d^2 w}{dx^2}$$

Wave equation for velocity of V (not of the wave!)

Chapter 17: Waves II

$$B = -V \frac{\partial p}{\partial V} \quad \text{bulk modulus}$$

Speed of sound: $v = \sqrt{\frac{B}{\rho}}$ ρ : density

Compressibility:

The fractional change of volume of a material for a given pressure change.

Bulk modulus is 1/compressibility.

Chapter 17: Waves II

Harmonic waves: $p = \Delta p_0 \sin(kx - \omega t) \Rightarrow w = \frac{v\Delta p_0}{B} \sin(kx - \omega t) = \frac{ds}{dt}$

Displacement: $s = \frac{1}{\omega} \frac{v\Delta p_0}{B} \cos(kx - \omega t) = s_m \cos(kx - \omega t)$

$$\Rightarrow s_m = \frac{1}{\omega} \frac{v\Delta p_0}{v^2\rho} \Rightarrow \Delta p_0 = (\omega v\rho) s_m$$

Connects the amplitude of the motion with the amplitude of the pressure change

Pressure Change: This always understand as pressure difference compared with the average difference -> $p = p_0 + \Delta p_0 \sin(kx - \omega t)$

Chapter 17: Waves II

What are typical values?

The maximum pressure difference our ear can tolerate is about $28\text{Pa} = 28\text{N/m}^2$, the minimum difference a good ear can hear is $28\mu\text{Pa}$.

The normal air pressure is 10^5Pa !

What displacement does this correspond to?

Lets assume for a frequency of 1000Hz .

The density of air is 1.21kg/m^3 and the velocity of sound is 343m/s

$$s_m = \frac{\Delta p_0^{\max}}{v\rho\omega} = \frac{28\text{Pa}}{343\frac{\text{m}}{\text{s}} 1.21\frac{\text{kg}}{\text{m}^3} 2\pi \cdot 1000\text{Hz}} = 1.1 \times 10^{-5}\text{m} = 11\mu\text{m}$$

$$s_m = \frac{\Delta p_0^{\min}}{v\rho\omega} = \frac{28\mu\text{Pa}}{343\frac{\text{m}}{\text{s}} 1.21\frac{\text{kg}}{\text{m}^3} 2\pi \cdot 1000\text{Hz}} = 1.1 \times 10^{-11}\text{m} = 11\text{pm}$$

Chapter 17: Waves II

$$s_m = \frac{\Delta p_0^{max}}{v\rho\omega} = \frac{28\text{Pa}}{343\frac{\text{m}}{\text{s}} 1.21\frac{\text{kg}}{\text{m}^3} 2\pi \cdot 1000\text{Hz}} = 1.1 \times 10^{-5}\text{m} = 11\mu\text{m}$$

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6 orders of magnitude in terms of amplitude:
11μm smaller than the thickness of a piece of paper!
11pm is 1/10 of the radius of a small atom!

Why are our ears not even more sensitive?
If we would hear a bit better, we start to 'hear' the thermal motion of the gas atoms, a permanent white noise floor.

Chapter 17: Waves II

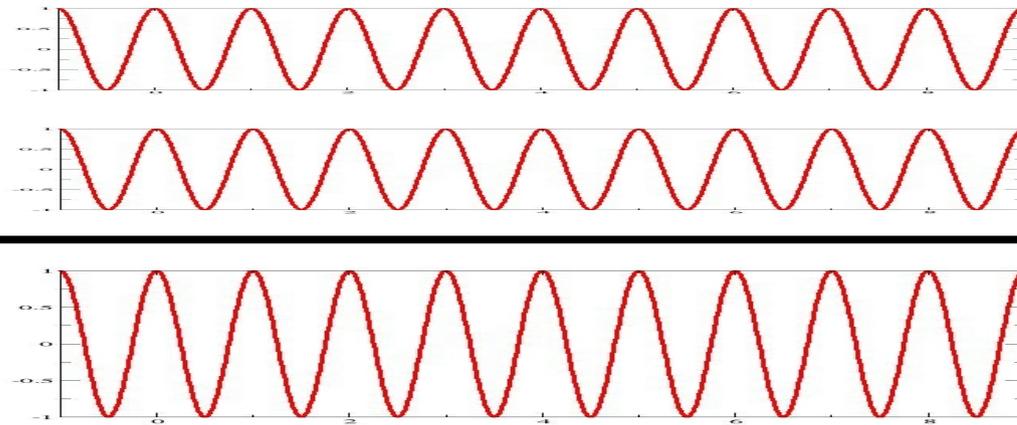
Interference:

Two waves interfere constructively if their pressure maxima are in phase:

Phase difference:

$$\Delta\phi = 2N\pi$$

+

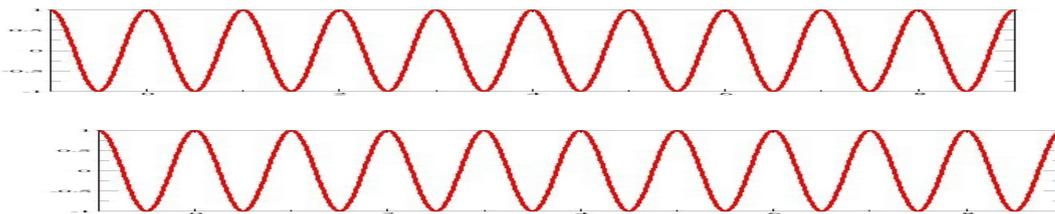


Two waves interfere destructively if their pressure maxima are 180deg out of phase:

Phase difference:

$$\Delta\phi = (2N+1)\pi$$

+



Chapter 17: Waves II

$$y_1(x,t) = y_m \sin(kx - \omega t) \quad y_2(x,t) = y_m \sin(kx - \omega t + \phi)$$

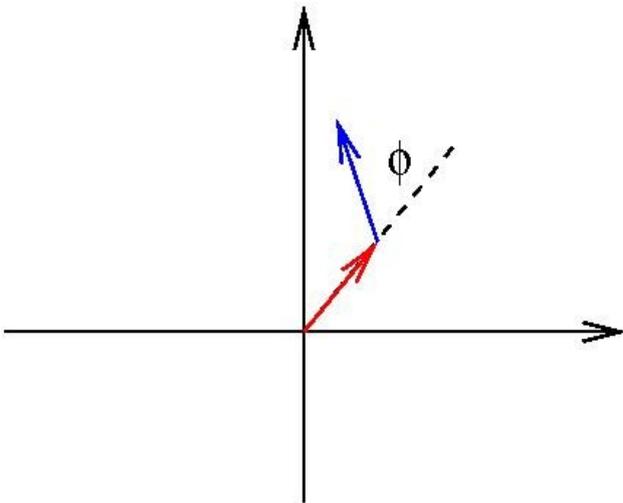
$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m [\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

use: $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

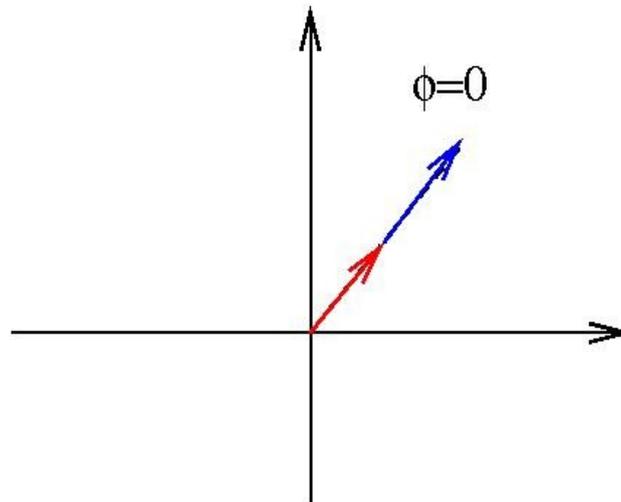
$$y'(x,t) = \underbrace{\left[2y_m \cos \frac{\phi}{2} \right]}_{\text{New Amplitude}} \sin \left(kx - \omega t + \frac{1}{2}\phi \right)$$

Phasor diagrams:

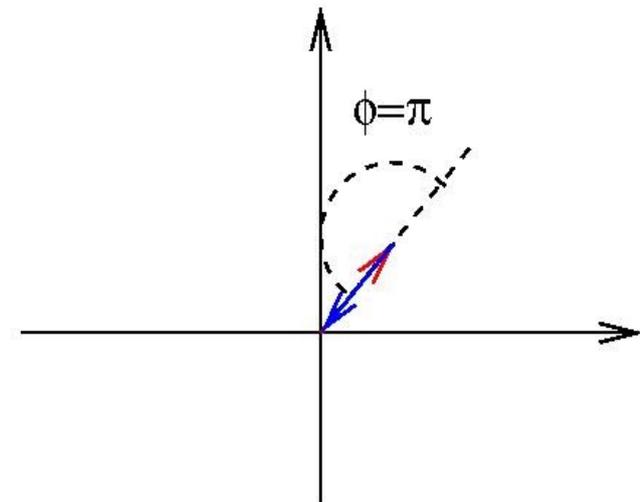
General case:



Constructive Interference

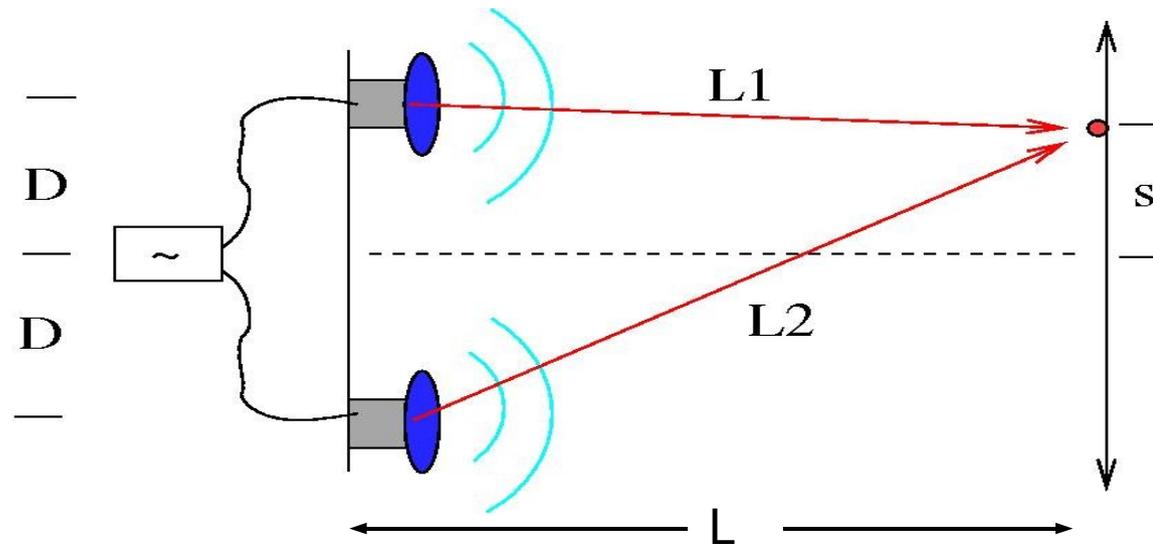


Destructive Interference



Chapter 17: Waves II

Interference:



2 loud speakers (assume point sources) generating identical signals at a frequency f . A microphone is measuring the amplitude of the resulting wave as a function of its position.

The amplitude will be high when the waves interfere constructively and low when they interfere destructively:

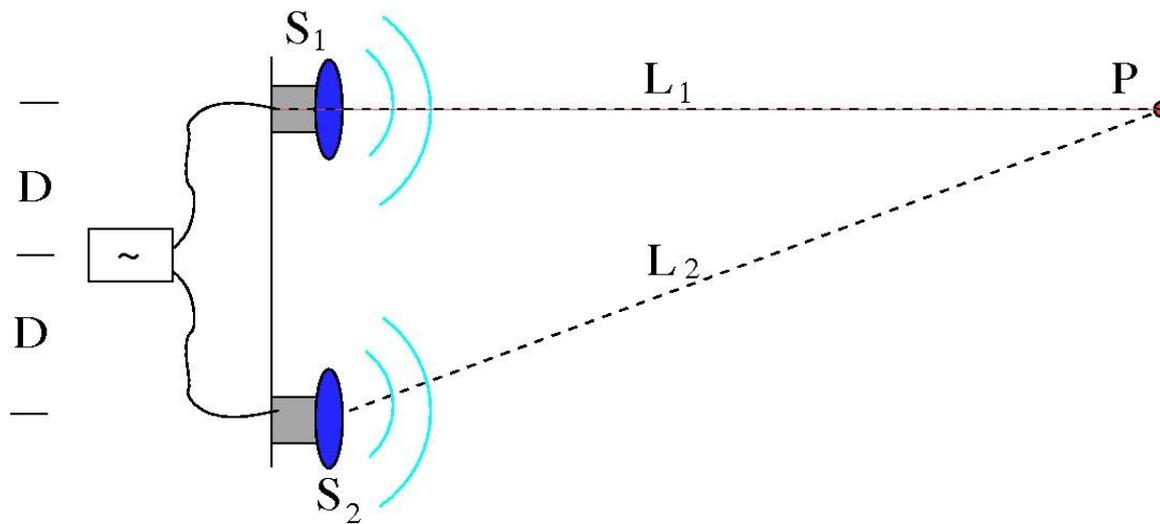
$$\Delta\phi = \frac{L_2 - L_1}{\lambda} 2\pi \quad L_2 = \sqrt{L^2 + (D + s)^2} \quad L_1 = \sqrt{L^2 + (D - s)^2}$$

$$\text{Constructive:} \quad \Delta\phi = 2N\pi \Leftrightarrow |L_2 - L_1| = N\lambda$$

$$\text{Destructive:} \quad \Delta\phi = (2N + 1)\pi \Leftrightarrow |L_2 - L_1| = (N + 0.5)\lambda$$

Chapter 17: Waves II

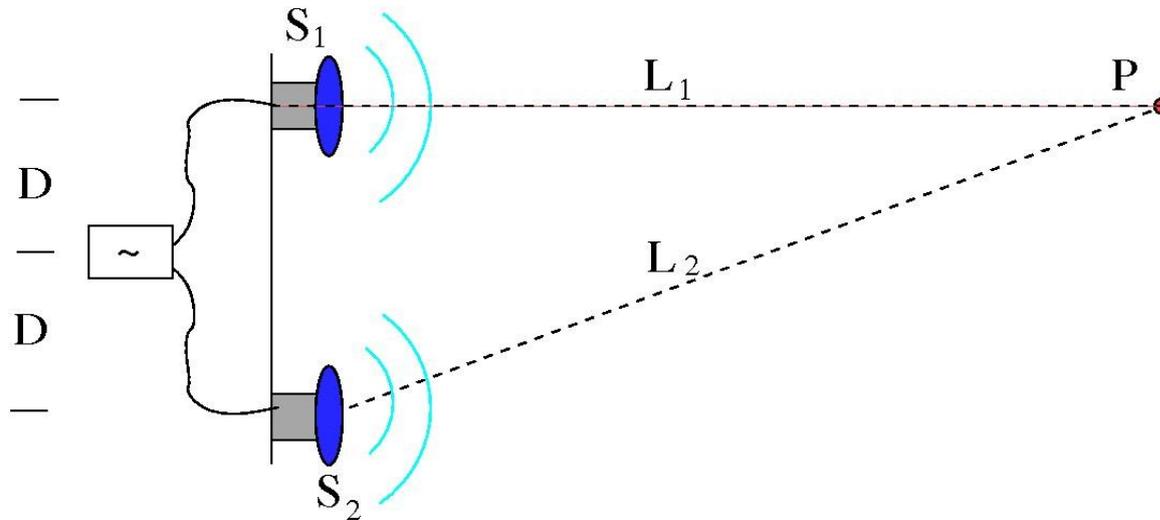
Example: Two point sources separated by 16m emit **coherent** sound waves with $\lambda=2\text{m}$. The waves are generated with the same amplitude and phase.



When P is far away what are

- the phase difference between the waves?
- the type of interference (constructive or destructive)?
- Now move P along the axis closer to the upper source. Does the phase difference increase or decrease?

Chapter 17: Waves II



When P is far away what are

- the phase difference between the waves?
- the type of interference (constructive or destructive)?
- Now move P along the axis closer to the upper source. Does the phase difference increase or decrease?

$$\text{a) } L_2 = \sqrt{L_1^2 + D^2} \stackrel{L \gg D}{\approx} L_1 + \frac{D^2}{2L_1} \approx L_1$$

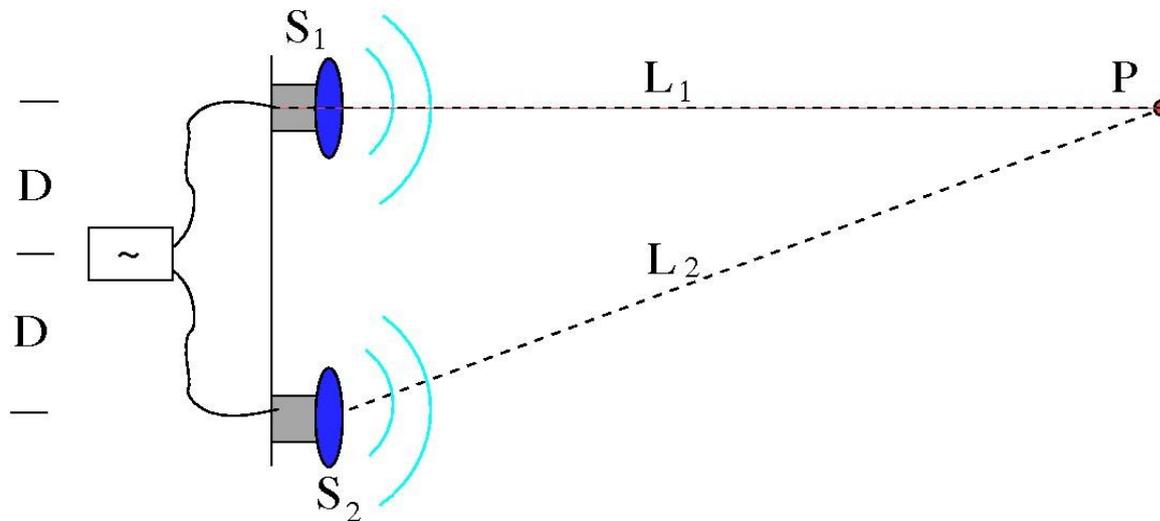
$$L_2 - L_1 \approx \frac{D^2}{2L_1} \approx 0 \quad \text{for very large } L_1 \Rightarrow \Delta\phi \approx 0$$

b) Constructive

c) Increases (can't get smaller than 0)

Chapter 17: Waves II

Example: Two point sources separated by 16m emit **coherent** sound waves with $\lambda=2\text{m}$. The waves are generated with the same amplitude and phase.



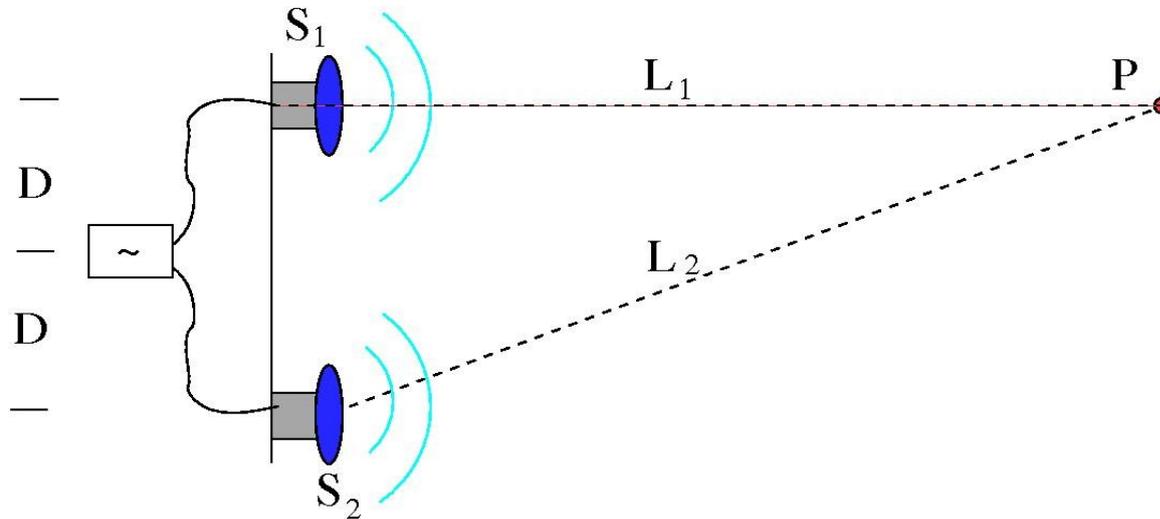
At what distances L_1 do the waves have a phase difference of
d) π e) 2π f) 3π

*Note that the book states “phase difference of d) 0.5λ e) λ f) 1.5λ ”
This is a length difference, the language is sometimes a little unprecise.

Obviously (?):

A phase difference of π (2π , 3π) happens when the length difference is 0.5λ (λ , 1.5λ)

Chapter 17: Waves II



General calculation:

$$L_2 - L_1 = \sqrt{L_1^2 + D^2} - L_1 = \zeta\lambda \quad \Rightarrow \quad L_1 = \frac{D^2}{2\zeta\lambda} - \frac{\zeta\lambda}{2}$$

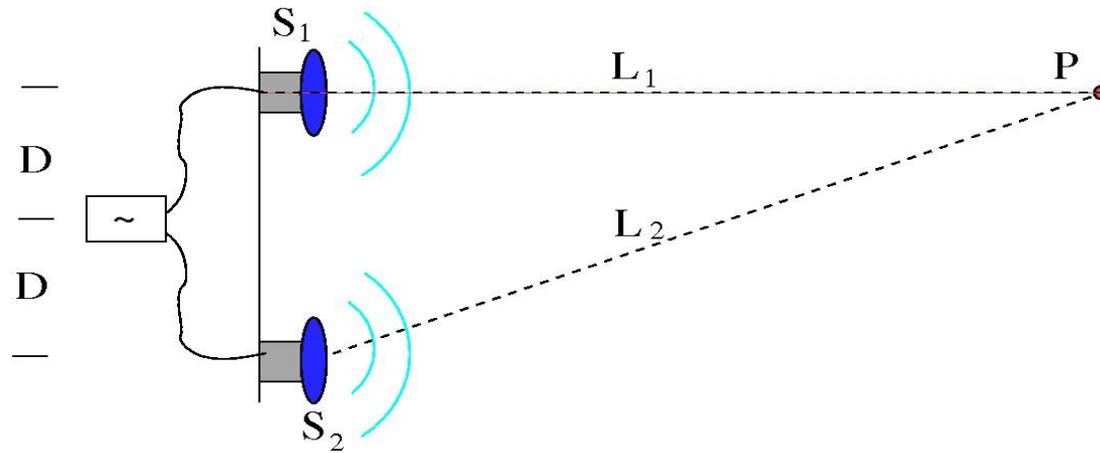
Individual solutions:

$$\phi = \pi \Leftrightarrow \Delta L = 0.5\lambda \Leftrightarrow \zeta = 0.5 \quad \Rightarrow \quad L_1 = 128 \text{ m}$$

$$\phi = 2\pi \Leftrightarrow \Delta L = \lambda \Leftrightarrow \zeta = 1 \quad \Rightarrow \quad L_1 = 63 \text{ m}$$

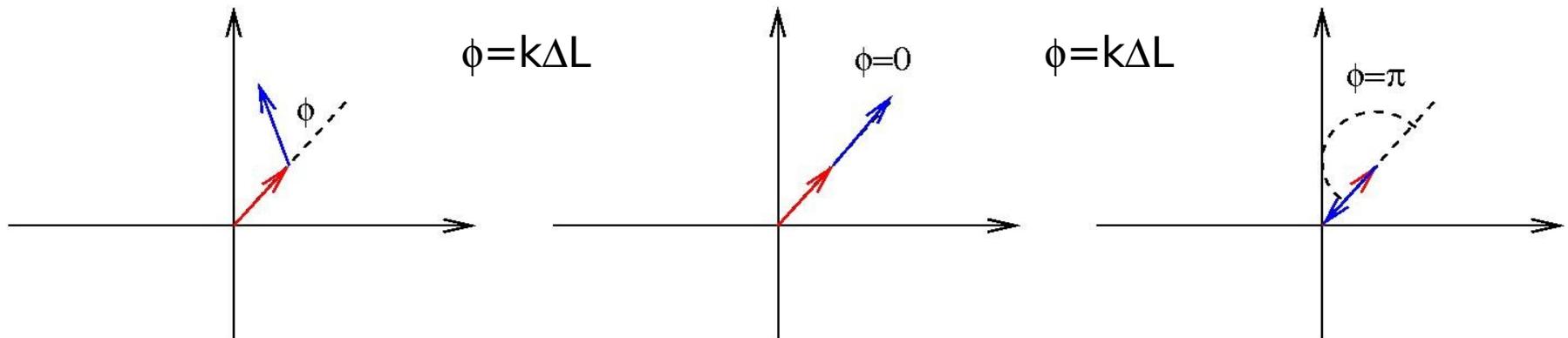
$$\phi = 3\pi \Leftrightarrow \Delta L = 1.5\lambda \Leftrightarrow \zeta = 1.5 \quad \Rightarrow \quad L_1 = 41.2 \text{ m}$$

Chapter 17: Waves II



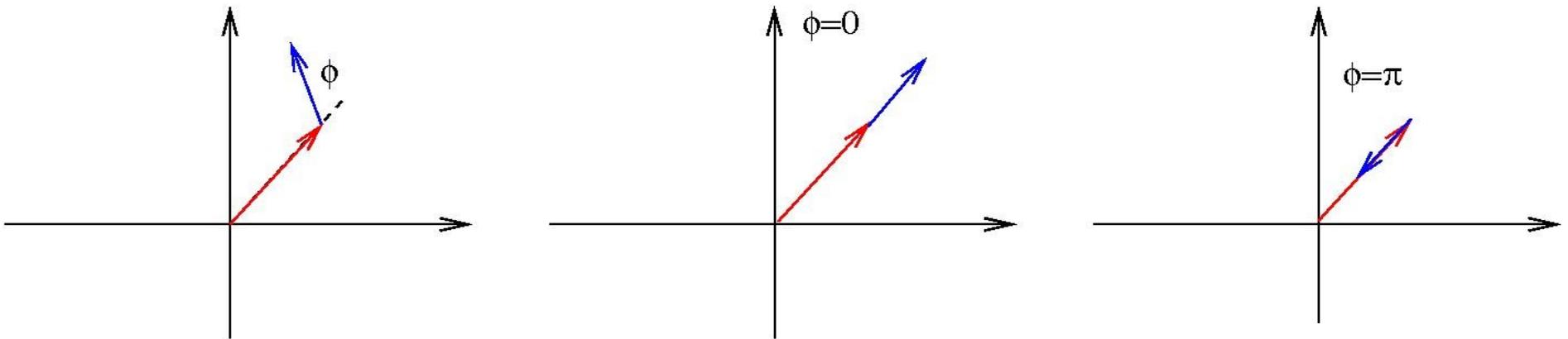
$$S_1 = S_m \sin(kL_1 - \omega t) \quad S_2 = S_m \sin(kL_2 - \omega t) = S_m \sin(kL_1 - \omega t + k\Delta L)$$

$$S_{\Sigma} = S_1 + S_2 = 2S_m \cos\left(\frac{k\Delta L}{2}\right) \sin\left(kL_1 - \omega t + \frac{k\Delta L}{2}\right)$$



Chapter 17: Waves II

Waves with identical frequencies but different amplitudes and initial phases:

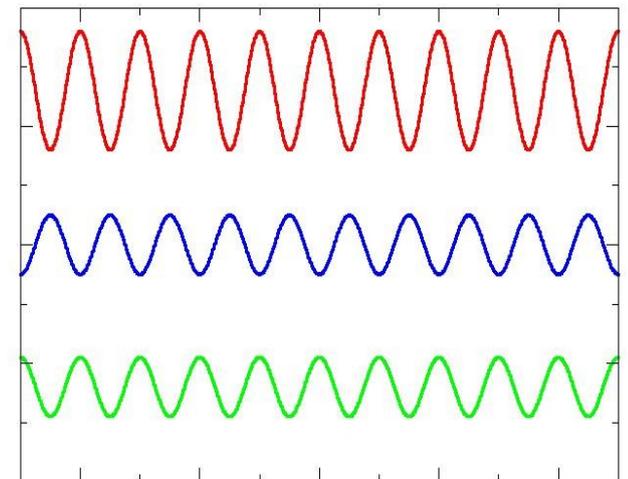


$$S_1 = S_{10} \sin(kx - \omega t + \phi_1) \quad S_2 = S_{20} \sin(kx - \omega t + \phi_2)$$

$$S_{\Sigma} = S_1 + S_2 = S_0 \sin(kL_1 - \omega t + \phi_0)$$

$$S_0 = \sqrt{S_{10}^2 + S_{20}^2 + 2S_{10}S_{20} \cos(\phi_1 - \phi_2)}$$

$$\tan \phi_0 = \frac{S_{10} \sin \phi_1 + S_{20} \sin \phi_2}{S_{10} \cos \phi_1 + S_{20} \cos \phi_2}$$



Chapter 17: Waves II

Intensity = Average Power/Area
= Average Energy transferred
per unit time through a unit area

Average: Averaged over several wavelength $\langle \cos^2 \rangle = 0.5 = \langle \sin^2 \rangle$

Recall the energy transported by a string:
We looked at dK/dt and dU/dt

The kinetic energy is obviously related to the longitudinal speed w of the gas molecules.

The potential energy must then be related to the pressure changes Δp in the pressure wave (the potential to accelerate gas molecules in a particular direction)

Chapter 17: Waves II

Derived this earlier:

Harmonic waves:	$p = \Delta p_0 \sin(kx - \omega t) \Rightarrow w = \frac{v\Delta p_0}{B} \sin(kx - \omega t) = \frac{ds}{dt}$
Displacement:	$s = \frac{1}{\omega} \frac{v\Delta p_0}{B} \cos(kx - \omega t) = s_m \cos(kx - \omega t)$
	$\Rightarrow s_m = \frac{1}{\omega} \frac{v\Delta p_0}{v^2\rho} \Rightarrow \Delta p_0 = (\omega v\rho) s_m$



$$dK = \frac{1}{2} dm w^2 \quad \text{The book uses: } v_s \text{ instead of } w$$

$$dK = \frac{1}{2} \underbrace{\rho A dx}_{=dm} w_m^2 \sin^2(kx - \omega t) \quad \left\langle \frac{dK}{dt} \right\rangle = \frac{1}{4} \rho A v \omega^2 s_m^2$$

without proof: $\left\langle \frac{dU}{dt} \right\rangle = \left\langle \frac{dK}{dt} \right\rangle \Rightarrow I = \frac{2 \left\langle \frac{dK}{dt} \right\rangle}{A} = \frac{1}{2} \rho v \omega^2 s_m^2$

Chapter 17: Waves II

$$I = \frac{2 \langle \frac{dK}{dt} \rangle}{A} = \frac{1}{2} \rho v \omega^2 s_m^2 = \frac{1}{2} \rho v w_m^2$$

Intensity is proportional to:

- Density
 - Velocity of sound
 - Angular frequency (or frequency) squared
 - Amplitude of displacement squared
- } or amplitude of transversal velocity squared

Chapter 17: Waves II

Variations with Distance:

We know from daily experience that

- You can't hear me in the back row when I don't use the microphone.
- some rooms have a very good acoustic:
 - concert halls etc. direct the sound to the audience

The changes in intensity depends on the environment (e.g. walls reflect the sound waves) and the source type (e.g. loud speakers which direct the sound waves). Can be complex.

But for an idealized point source of Power P
in otherwise empty space:

$$I = \frac{P}{4\pi r^2}$$

Intensity falls off as distance²

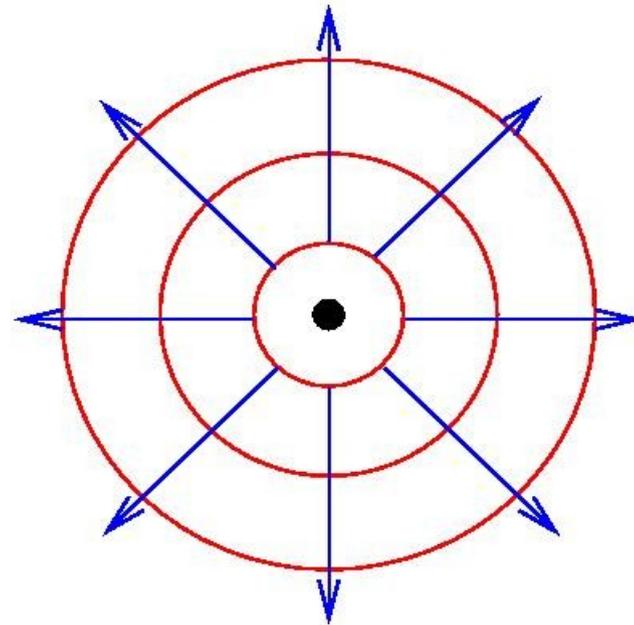
Chapter 17: Waves II

$$I = \frac{P}{4\pi r^2}$$

$A=4\pi r^2$ Area of a sphere with radius r

Why? Energy is conserved.

The amount of energy going through each sphere is the same (assuming no absorption or damping)



Recall that the intensity is proportional to the amplitudes squared \longrightarrow the amplitudes fall off with $1/r$!
General result for all waves:

$$\Delta p \sim 1/r \quad \text{and} \quad w \sim 1/r \quad \text{and} \quad I \sim 1/r^2$$

Chapter 17: Waves II

Interference and Intensity:

Remember that the amplitudes Δp and/or w (or Δs) interfere:

$$w_{10}(r,t) \sin(kr - \omega t + \phi_1) + w_{20}(r,t) \sin(kr - \omega t + \phi_2) = w_{net,0}(r,t) \sin(kr - \omega t + \phi_0)$$

with:
$$w_{net,0}(r,t) = \sqrt{w_{10}^2(r,t) + w_{20}^2(r,t) + 2w_{10}(r,t)w_{20}(r,t) \cos(\phi_1 - \phi_2)}$$

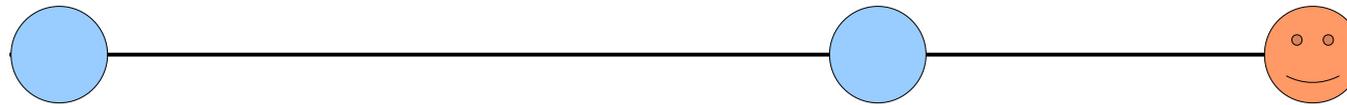
The amplitude only depends on the phase difference

$$I_{net}(r,t) = \frac{1}{2} \rho v w_{net,0}^2(r,t) = I_{10}(r,t) + I_{20}(r,t) + 2\sqrt{I_{10}(r,t)I_{20}(r,t)} \cos(\phi_1 - \phi_2)$$

This helps because in most cases we measure only intensities and not amplitudes.

Chapter 17: Waves II

Assume we have one idealized point source at $x=0$ generating a spherical wave which has an intensity of $I=1\text{mW/m}^2$ at $x=1\text{m}$. Assume we add now an identical point source at $x=-2\text{m}$ generating a spherical wave of the same frequency and power and assume that both waves interfere constructively at $x=1\text{m}$. What is the intensity at $x=1\text{m}$ now?



Source 2

Source 1

Detector

$$I_{net}(r,t) = \frac{1}{2} \rho v w_{net,0}^2(r,t) = I_{10}(r,t) + I_{20}(r,t) + 2\sqrt{I_{10}(r,t)I_{20}(r,t)} \cos(\phi_1 - \phi_2)$$

Intensities fall off with distance squared. Source 2 is 3 times further away \rightarrow its intensity is 9 times lower.

The interference is constructive (phase difference=0) and the cos-term is 1

$$I_{net} = 1 + \frac{1}{9} + 2\sqrt{1 \cdot \frac{1}{9}} = 1.78 \frac{\text{mW}}{\text{m}^2} \quad \text{well above } 1 + 1/9 = 1.11$$

Energy is still conserved, there are other areas where the waves interfere destructively

Chapter 17: Waves II

$$s_m = \frac{\Delta p_0^{max}}{v\rho\omega} = \frac{28\text{Pa}}{343\frac{\text{m}}{\text{s}} \cdot 1.21\frac{\text{kg}}{\text{m}^3} \cdot 2\pi \cdot 1000\text{Hz}} = 1.1 \times 10^{-5}\text{m} = 11\mu\text{m}$$

$$s_m = \frac{\Delta p_0^{min}}{v\rho\omega} = \frac{28\mu\text{Pa}}{343\frac{\text{m}}{\text{s}} \cdot 1.21\frac{\text{kg}}{\text{m}^3} \cdot 2\pi \cdot 1000\text{Hz}} = 1.1 \times 10^{-11}\text{m} = 11\text{pm}$$

6 orders of magnitude in terms of amplitude
12 orders of magnitude in terms of intensity

For these large ranges, it is more convenient to use the logarithmic scale.

Recall: $10^y = x \iff y = \log(x)$

Usually talking about sound level and not intensity:

$$\beta = (10\text{dB}) \log \frac{I}{I_0} \quad I_0 = 10^{-12} \frac{\text{W}}{\text{m}^2} \text{ standard reference intensity}$$

Chapter 17: Waves II

Recall: $10^y = x \iff y = \log(x)$

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$$\beta = (10\text{dB}) \log \frac{I}{I_0} \quad I_0 = 10^{-12} \frac{\text{W}}{\text{m}^2} \text{ standard reference intensity}$$

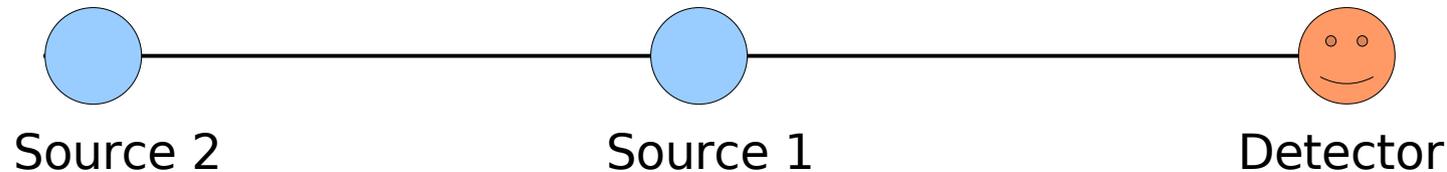
Typical examples:

Hearing threshold	0dB
leaves in the wind	10dB
Conversation	60dB
Rock Concert	110dB
Pain threshold	120dB
Jet engine	130dB

The log-scale is actually very good to describe sound on a human scale. Our perception of loudness (intensity) is also roughly logarithmic.

Chapter 17: Waves II

Assume we have one idealized point source at $x=0$ generating a spherical wave which has an intensity of $I=1\text{mW/m}^2$ at $x=1\text{m}$. Assume we add now an identical point source at $x=-2\text{m}$ generating a spherical wave of the same frequency and power and assume that both waves interfere **destructively** at $x=1\text{m}$. What is the intensity at $x=1\text{m}$ now?



A: 2mW/m^2

C: 1mW/m^2

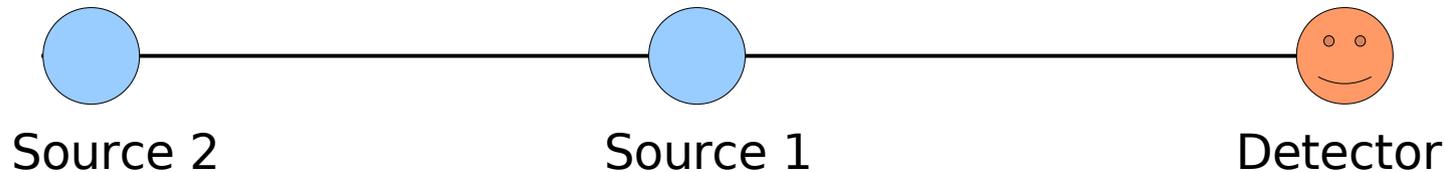
E: 0.5mW/m^2

B: 0.25mW/m^2

D: 0mW/m^2

Chapter 17: Waves II

Assume we have one idealized point source at $x=0$ generating a spherical wave which has an intensity of $I=1\text{mW/m}^2$ at $x=1\text{m}$. Assume we add now an identical point source at $x=-2\text{m}$ generating a spherical wave of the same frequency and power and assume that both waves interfere **destructively** at $x=1\text{m}$. What is the intensity at $x=1\text{m}$ now?



B: 0.25mW/m^2

$$I_{net} = 1 + \frac{1}{4} - 2\sqrt{1 \cdot \frac{1}{4}} = \frac{1}{4}$$

↑
destructive

Interference

Interference between two waves of different frequency, but same amplitude, and propagation direction:

$$y_1(x,t) = y_m \sin(k_1x - \omega_1t) \quad y_2(x,t) = y_m \sin(k_2x - \omega_2t)$$

use: $\Delta k = \frac{k_1 - k_2}{2}$ $\Delta \omega = \frac{\omega_1 - \omega_2}{2}$ $\bar{k} = \frac{k_1 + k_2}{2}$ $\bar{\omega} = \frac{\omega_1 + \omega_2}{2}$

$$y'(x,t) = y_1(x,t) + y_2(x,t) = 2y_m [\cos(\Delta kx - \Delta \omega t) \sin(\bar{k}x - \bar{\omega}t)]$$

