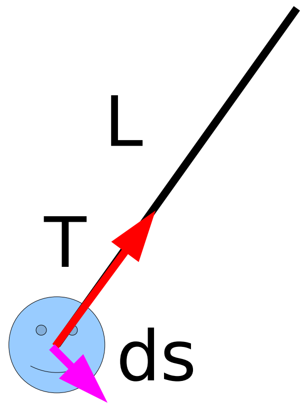
Review:

When an ideal (no friction) pendulum is released at angle ϕ out of its equilibrium position, how much work is done by the **tension** in the string during the first half period?

h: Initial Height of the ball above its equilibrium level

T: Period, m: mass, L: length of the pendulum



0!

General result (Chapter 7):

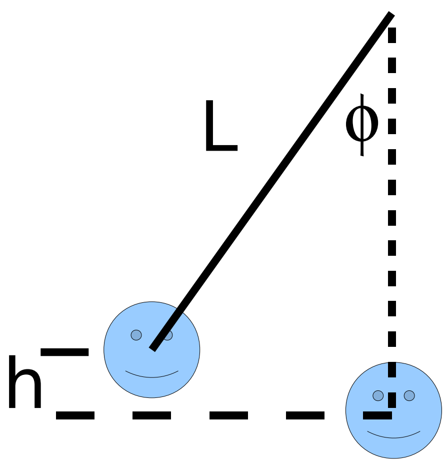
If Force **perpendicular** to displacement

=>

Work = 0

Review:

When an ideal (no friction) pendulum is released at angle ϕ out of its equilibrium position, how much work is done by **gravity** during the first half period?



What is the difference in potential energy after one half period?
0, pendulum reaches the same height!
Work is again 0.

After a quarter period: $\Delta U=-mgh=-W$
Next quarter: $\Delta U=mgh=-W$

Review: Potential Energy

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{s}$$

Work done by **gravitational** or **elastic** **(spring)** force acting on the particle

$$\Delta U_{ab} = -W_{ab}$$

This work changes potential energy

$$\Delta U_{ab} = -\int_a^b \vec{F} \cdot d\vec{s}$$

Equation to determine potential energy. F: **gravitational** or **elastic** force

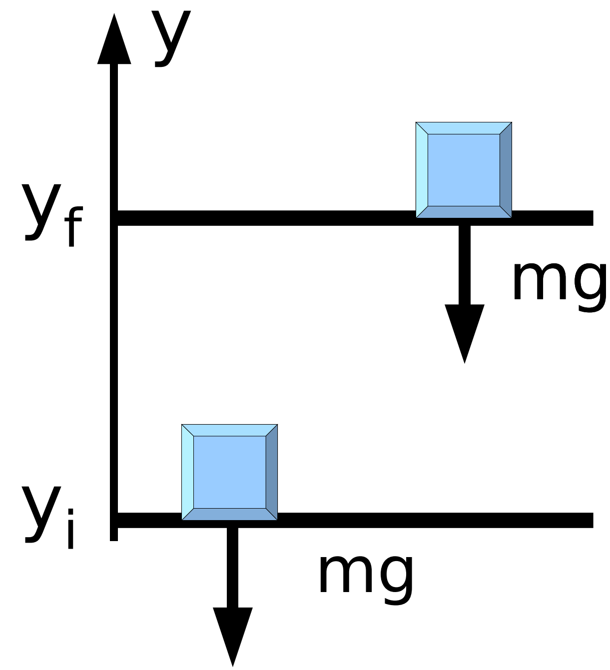
$$\Delta U = -\int_{x_i}^{x_f} F\,dx$$

In 1-dimension, good enough for most HW (not all!)

Gravitational Potential Energy

Textbook explanation:

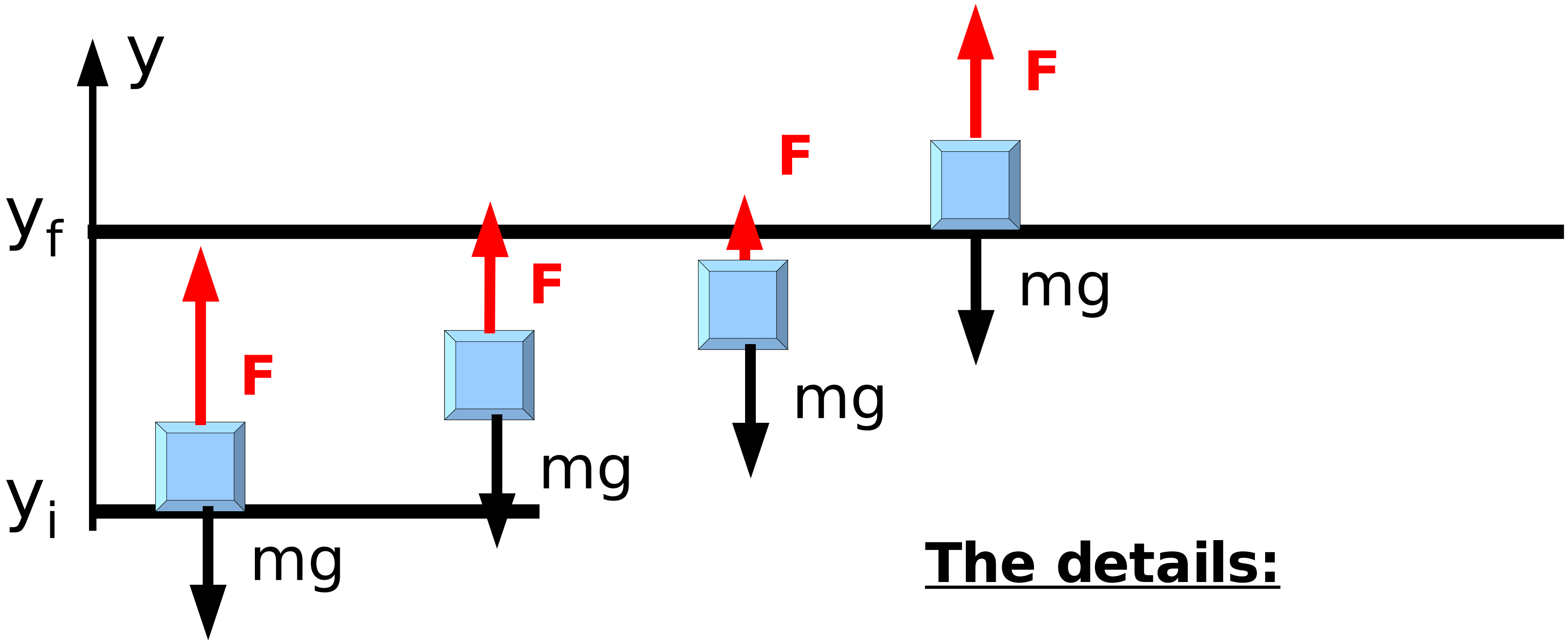
$$\Delta U_{ab} = -\int_{y_i}^{y_f} F\,dy$$



$$\Delta U_{ab} = -\int_{y_i}^{y_f} (-mg)\,dy = mg\,(y_f - y_i) = mgh$$

The gravitational potential energy associated with a particle – Earth system depends only on the vertical position y (or height h) of the particle relative to the reference position y=0. It does not depend on the horizontal position.

Gravitational Potential Energy



<u>The details:</u>

First: We have to apply a force **F** which is at least as large as mg to lift it up. The difference between F and mg (**the net force**) will accelerate the mass and increase the kinetic energy K.

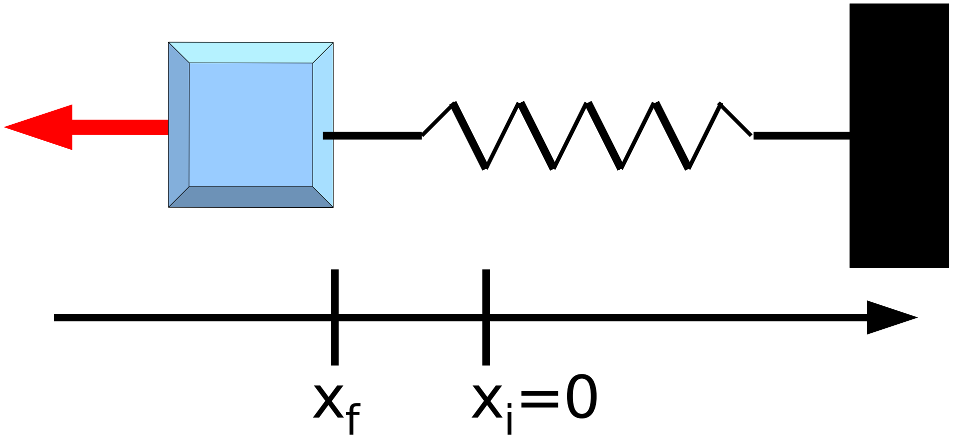
Second: We have to reduce the force F to decelerate the mass again before we reach the final height. During this time the net force is pointing against the displacement and reduces K.

Third: We have to match mg to keep the height steady.

The increase in U only cares about the work against mg.

Elastic Potential Energy

$$\Delta U_{ab} = -\int_{x_i}^{x_f} F dx$$



$$\Delta U_{ab} = -\int_{x_i}^{x_f} (-kx)\, dx = \frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2}kx^2$$

The elastic potential energy associated with a particle – spring system depends on the extension or compression of the spring relative to the **relaxed** length ($x=0$) of the spring.

Work and kinetic Energy

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{s}$$

Net Work done by a **net** force acting on a particle

Recall Chapter 7:

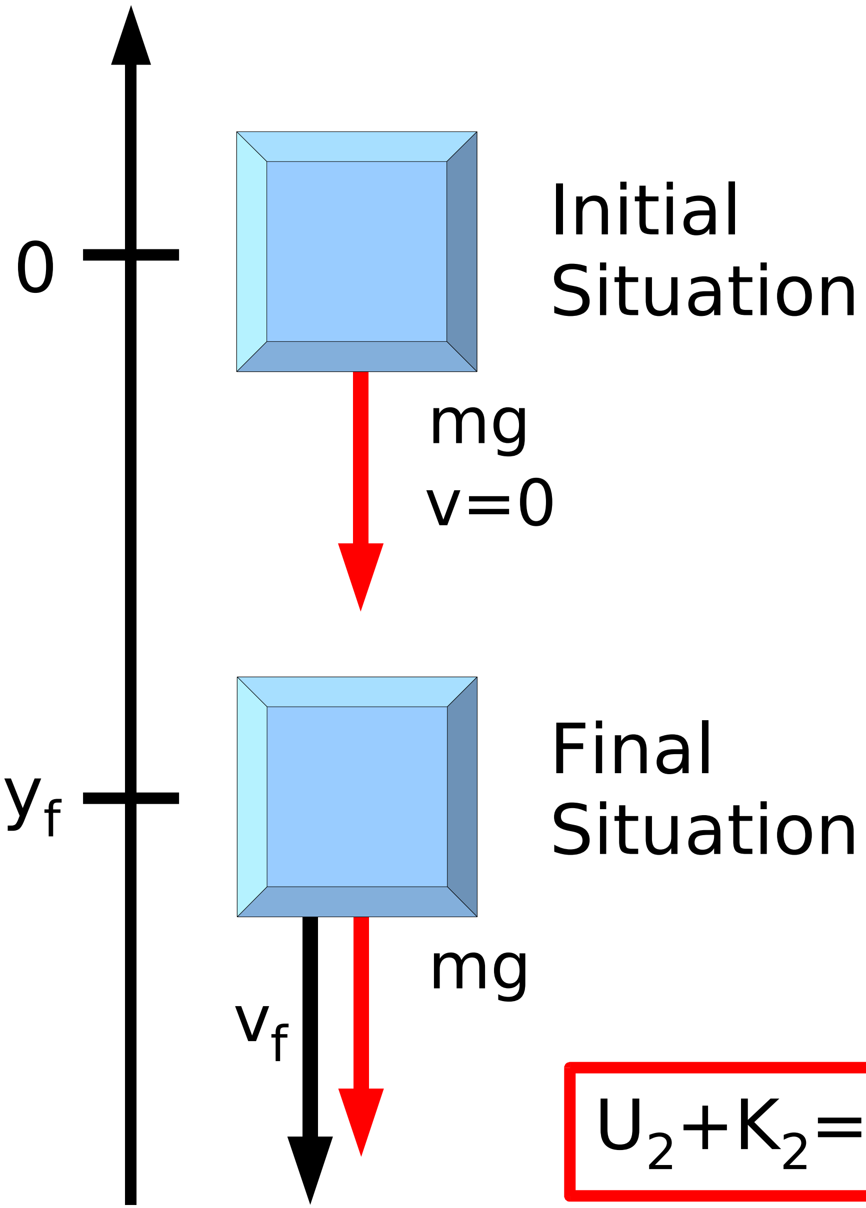
- A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. Zero work if it is perpendicular.

- Change in kinetic energy of a particle = **net** work done

$$\Delta K = W$$

- Net work: Includes **all** forces including **friction**

Example:



Only Force is mg.

$U_1=0$

$K_1=0$

Equation of motion gives:

$v_f=-gt$ and $y_f=-\frac{1}{2}gt^2$

$U_2=mgy_f=-\frac{1}{2}mg^2t^2$

$K_2=\frac{1}{2}mv_f^2=\frac{1}{2}mg^2t^2$

$$U_2+K_2=-\frac{1}{2}mg^2t^2+\frac{1}{2}mg^2t^2=0=U_1+K_1$$

Conservation of Energy

Positive Work done by force on particle

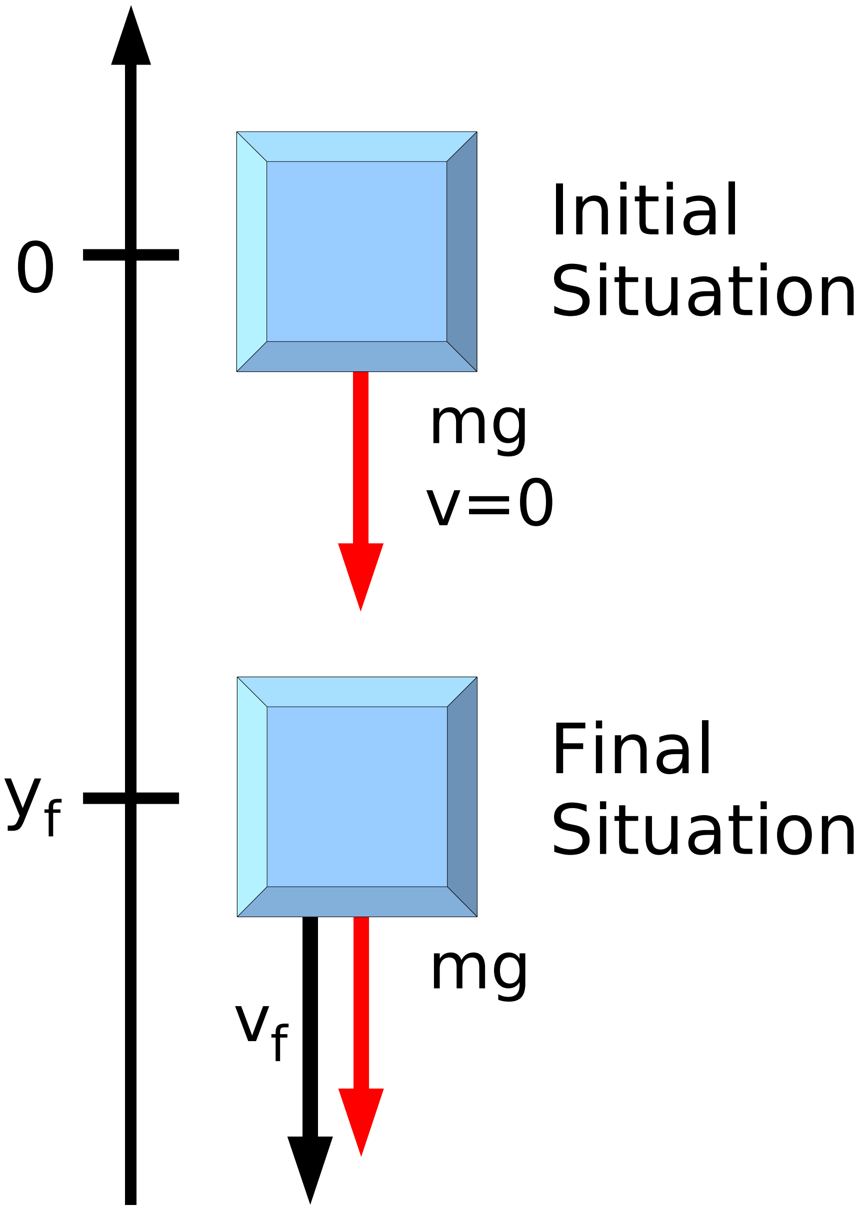
- Increases kinetic energy — $\Delta K = W$
- Decreases potential energy — $\Delta U = -W$

$$\left.\begin{array}{l}\Delta K = W \\ \Delta U = -W\end{array}\right\} \Delta K = -\Delta U$$

$$K_2 - K_1 = -(U_2 - U_1) \rightarrow K_2 + U_2 = K_1 + U_1 = E_{mec}$$

In an isolated system where only conservative forces cause energy changes, the kinetic and potential energies can change, but their sum, the mechanical energy of the system, cannot change.

Conservation of Energy



When the mechanical energy of a system is conserved, we can relate the sum of kinetic and potential energy at one instant to that at another instant w/o considering the intermediate motion (or the equation of motion)

$$U_2+K_2=U_1+K_1$$

$$\Delta U_{ab} = -\int_a^b \vec{F}_{G,e} \cdot d\vec{s}$$

Equation to determine change in potential energy. F: **gravitational** or **elastic** force

$$\Delta K_{ab} = -\int_a^b \vec{F}_{Net} \cdot d\vec{s}$$

Equation to determine change in kinetic energy. F: **net** force

Conservative system when net force is a combination of conservative forces (gravitational and/or elastic). No friction or drag or external forces

Potential Energy Curves

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx$$

allows to calculate the change in potential energy between two points if we know the force F(x)

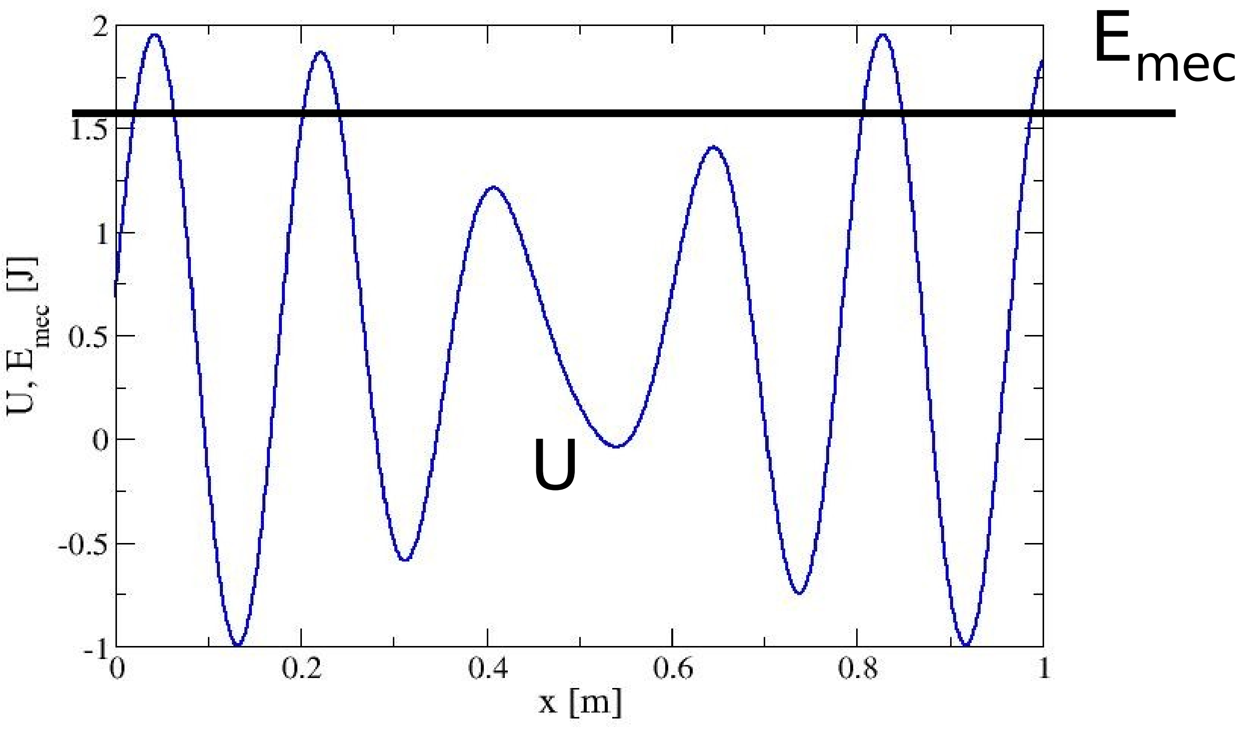
$$F(x) = -\frac{dU(x)}{dx}$$

allows to calculate the force from the change in potential energy

No surprise: This is just the mathematical relation between integrals and derivatives

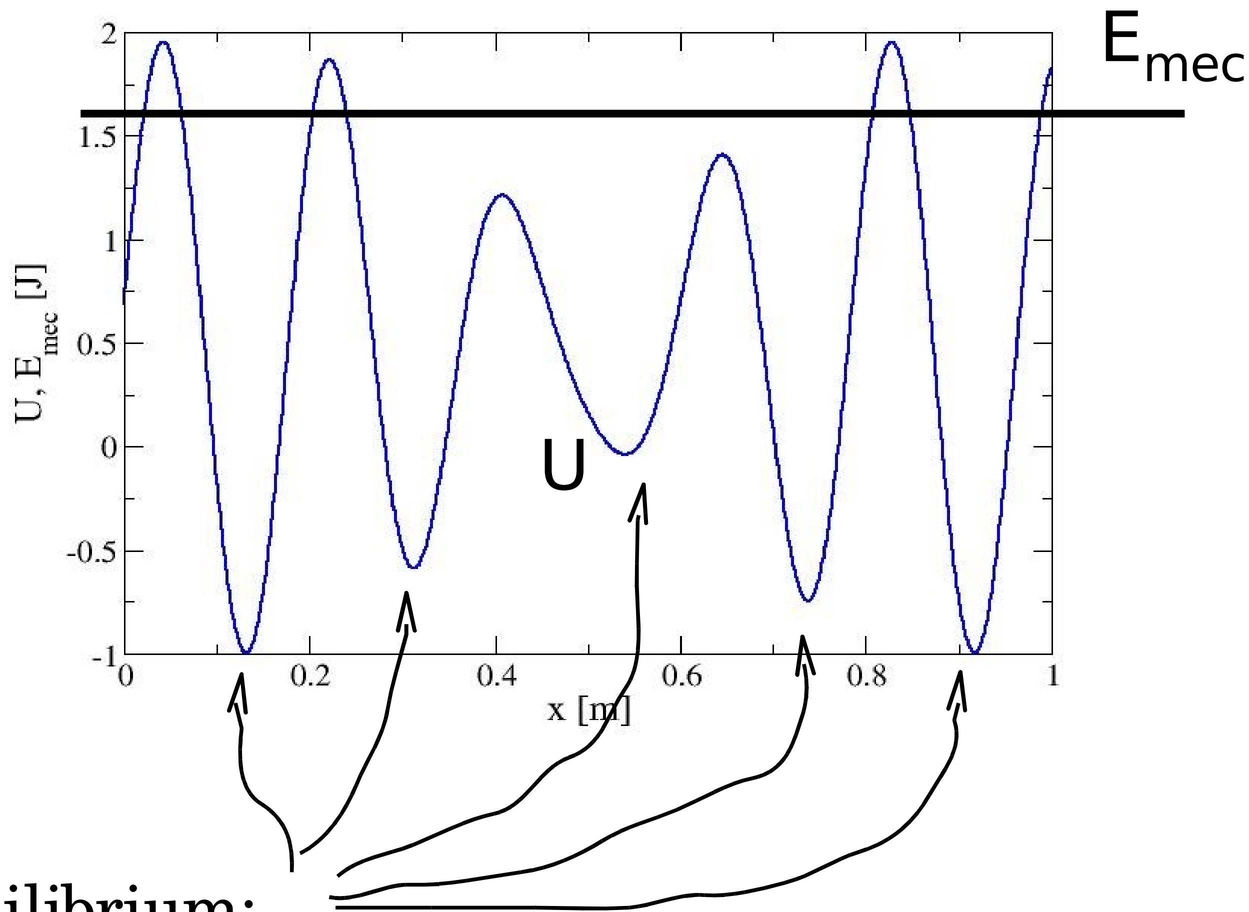
Still very useful to understand the behavior of a system

Potential Energy Curves



Imagine this is a (frictionless) roller coaster and you release a marble at one location!

Potential Energy Curves



Stable equilibrium:

A marble placed at the minima of U(x) would not move

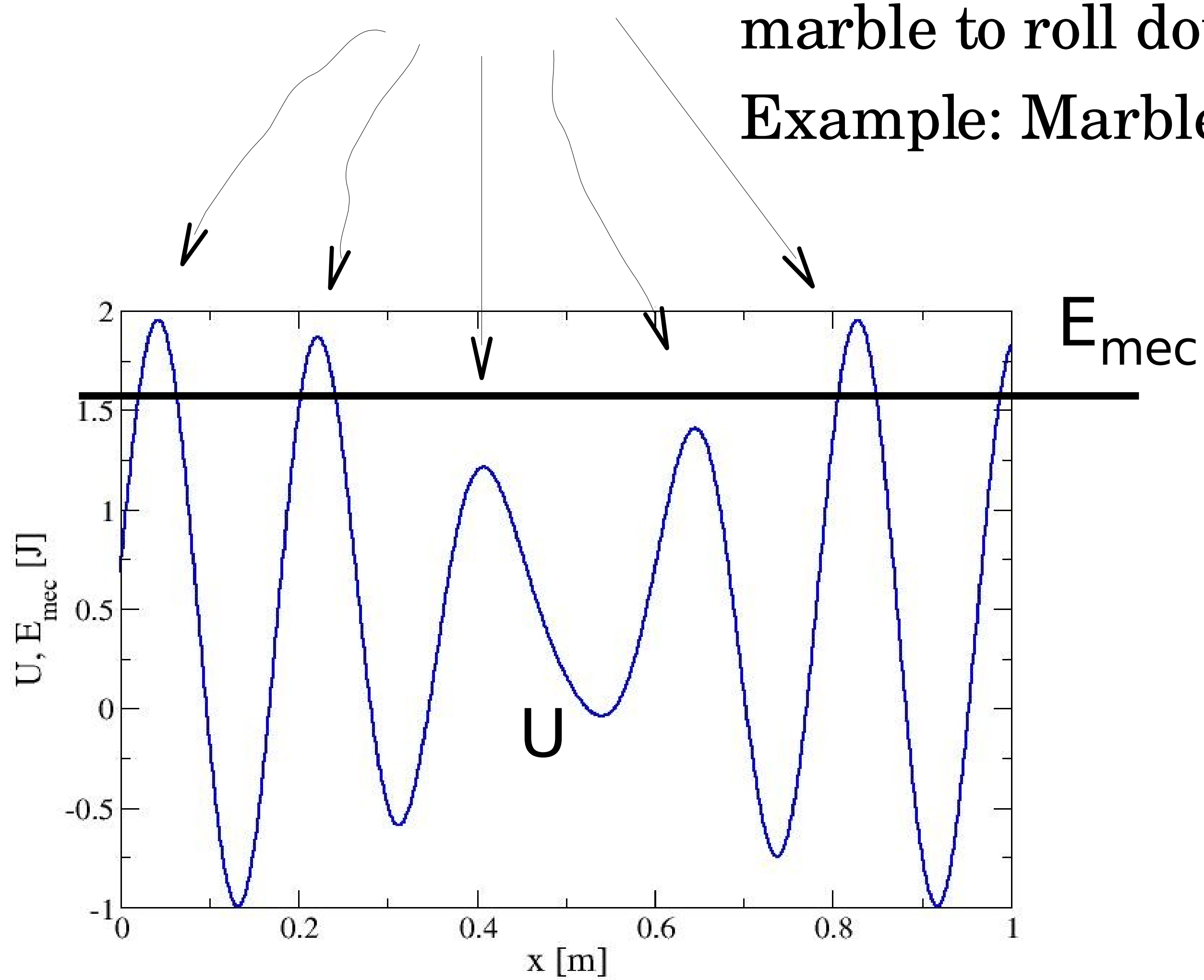
Example: Marble on the bottom of a dish

Potential Energy Curves

Unstable equilibrium:

Any distortion would cause the marble to roll down

Example: Marble on a billiard ball

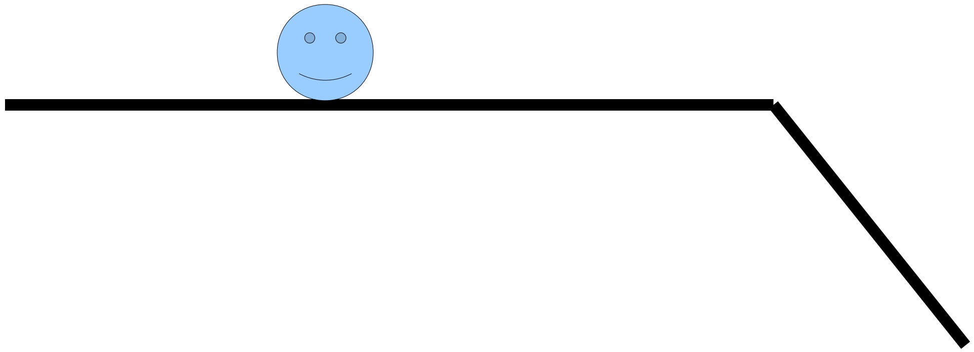


Potential Energy Curves

Neutral equilibrium:

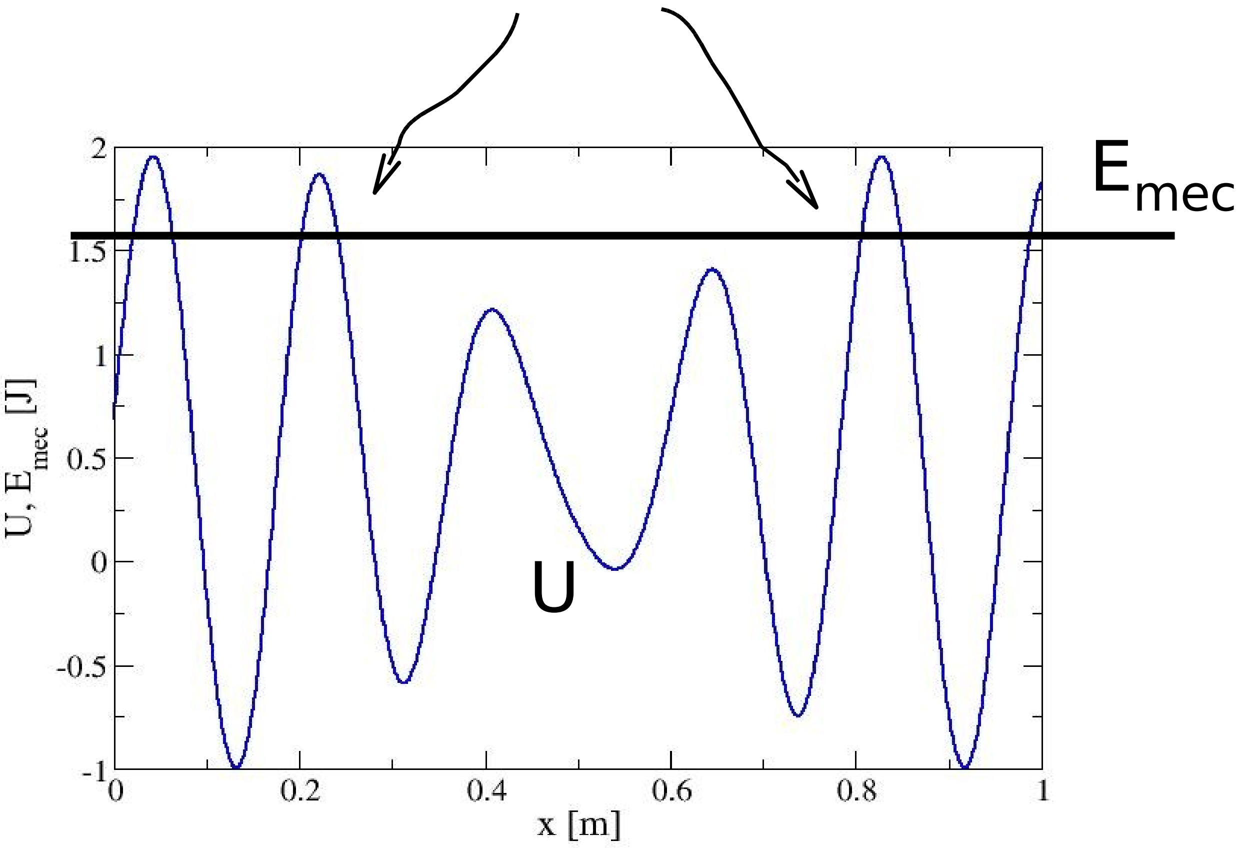
Potential energy does not change as a function of the position locally. No force acts on it even if we move it by a small amount.

Example: A marble on a flat table



Potential Energy Curves

Turning Points:



Points where the kinetic energy is zero (v=0) and the marble turns around.

Note that always

$K \geqslant 0$ and $U \leqslant E_{mec}$