Chapter 9

Collision and Impulse (The fun stuff)
Only internal forces

Elastic Collisions

<u>Conservation of momentum:</u>

$\vec{P}_i = (2m)v = \vec{P}_f$

Potential solutions for

$\vec{P}_f = m(2\vec{v})$ 1 mass, double v

$\vec{P}_f = (2m)\vec{v}$ 2 masses, same v

...

<u>Conservation of energy:</u>

$E_i = 0.5(2m)v^2 = E_f$

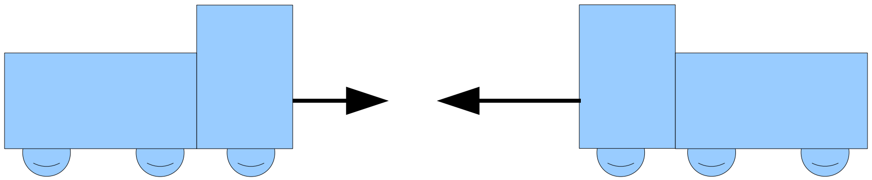
Potential solutions for

$E_f = 0.5m\,(2^{1/2}v)^2$ 1 mass, sqrt(2)v

$E_f = 0.5(2m)v^2$ 2 masses, same v

Only one solution fulfills both conservation laws

Inelastic Collisions



'Inelastic' when the mechanical energy is NOT conserved.

Example Car crash:
Mechanical Energy deforms metal, deformation is non-elastic.
Metal heats up.

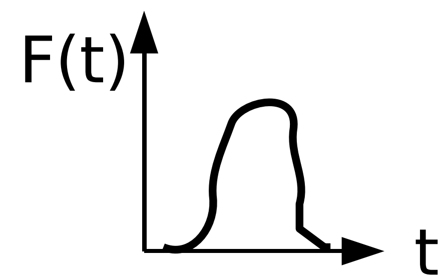
But this is only caused by internal forces!
Linear momentum P is conserved!
(Friction of non-rotating tires stops motion)

Collision and Impulse

How does this all work?

Single collision:

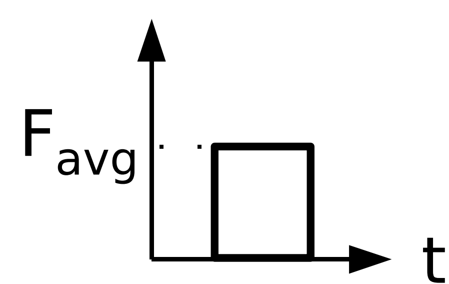
- A varying force $\vec{F}(t)$ is acting over a short period of time between the colliding objects.
- This causes a change in the momentum of each colliding object:



$$d\vec{p} = \vec{F}(t)dt$$

- Net change in momentum of <u>each</u> colliding object:

$$\Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt = \vec{F}_{avg}\Delta t = \vec{J}$$

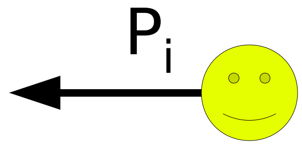


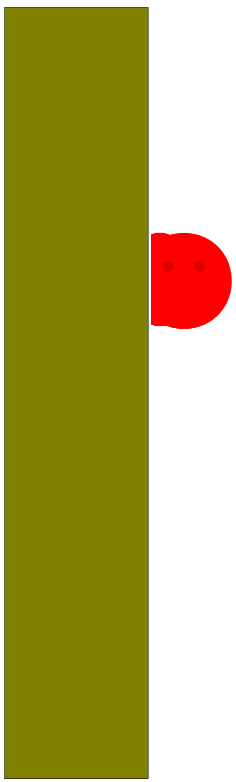
Change in momentum

Impuls of the collision

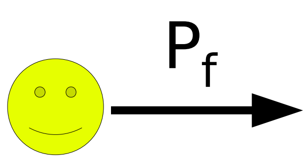
Collision and Impulse

Example: Tennis ball hitting a wall





Tennis ball deforms.
Acts as a compressed spring
which pushes the ball back.
Also heats up ball (friction in the ball)
which reduces mechanical energy
(good ball: small friction, small loss)



$$\vec{J} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i \lesssim 2\vec{P}_i$$

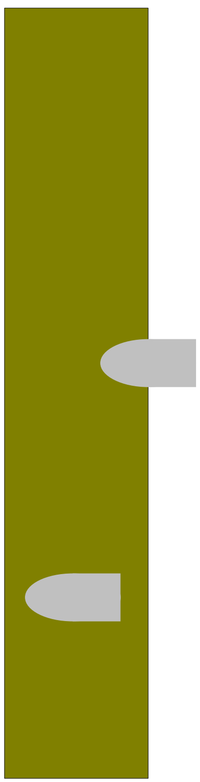
Completely elastic collision (no losses) = $2\vec{P}_i$ <-> $\vec{P}_f = \vec{P}_i$

Wall absorbs also a momentum of $\Delta\vec{P}$,
but mass ~ infinity (compared to tennis ball), so v~0

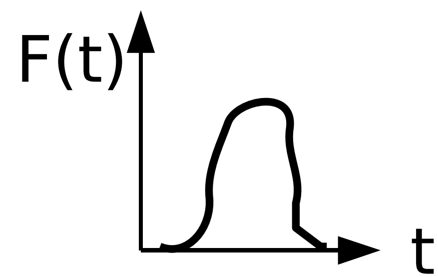
Collision and Impulse

Example: Bullet hitting a wall

J=Area
under F-t graph



$\vec{P}_i$



Bullet penetrates into
the wall. All kinetic energy
dissipates into thermal energy.

$$\vec{J} = \Delta\vec{P} = -\vec{P}_i$$

Momentum is 'swallowed' by wall
(Meaning: Wall has now momentum
but transfers it to the ground,
huge mass -> no velocity)

Collision and Impulse

If you drop an egg and it hits the ground floor in the kitchen it shatters.
If you drop an egg in your bed (assume same height) and it hits your mattress it survives (maybe).

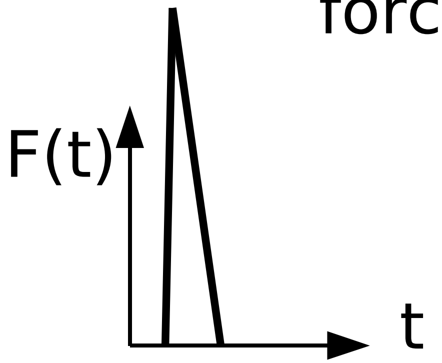
What is different in the two cases:
a) Momentum
b) Impuls
c) Force stopping the fall

Collision and Impulse

If you drop an egg and it hits the ground floor in the kitchen it shatters.
If you drop an egg in your bed (assume same height) and it hits your mattress it survives.

What is different in the two cases:
a) Momentum
b) Impuls
c) Force stopping the fall

Case 1: High peak force



Same Areas under F(t)

Case 2: Low peak force

Example

Example (9.27)

A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25m/s. It rebounds with an initial speed of 10m/s.

a) What impulse acts on the ball?

b) If the ball is in contact with the floor for 0.02s, what is the magnitude of the average force on the floor from the ball?

c) How much mechanical energy is lost (How?)

All in 1-dimensions, no vectors needed.

to a)

$J = P_f - P_i = m\ (10m/s - (-25m/s)) = -1.2kg\ 35m/s = 42kg\ m/s$

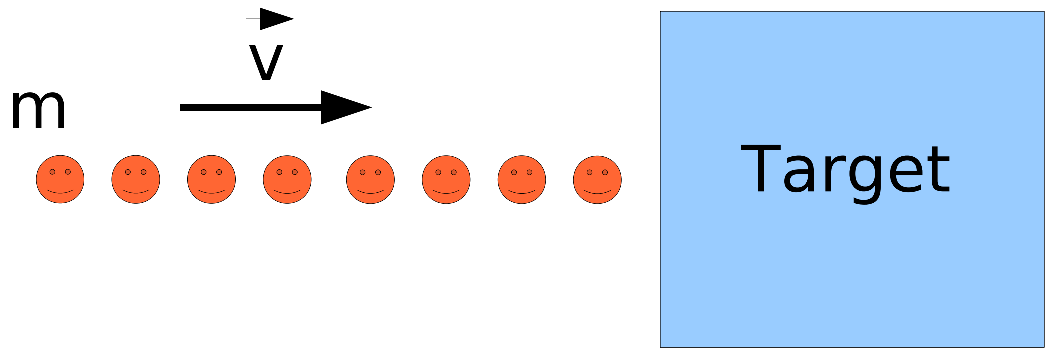
to b)

$J = F_{avg}\ \Delta t$ --> $F_{avg} = J/\Delta t = 42/0.02\ N = 2100N$

to c) $\Delta K = 0.5m\ (v_f^2 - v_i^2) = -315J$

Series of Collisions

Another possibility of having an average force:



Assumption:
Target has infinite mass (bolted to the ground)

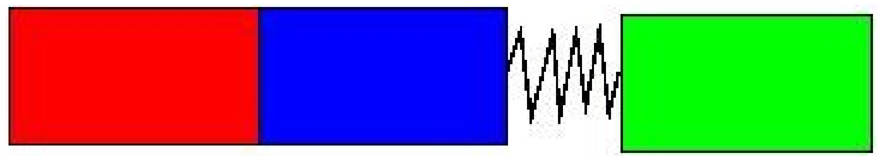
During each collision the momentum of the colliding projectile changes by Δp [with $mv \lesssim \Delta p \lesssim 2mv$ (Why?)]

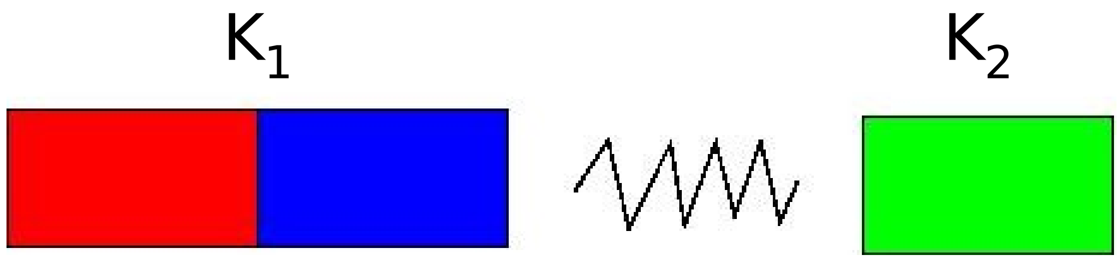
If n projectiles hit the target in a time interval Δt the impulse on the target during Δt is: $J = -n\,\Delta p$

and the average force is $F_{avg} = J/\Delta t = -n\Delta p/\Delta t$

Soda Cans

Soda cans and a spring. The 3 cans have equal mass.





Just after the spring is released (=explosion betw. cans):
Momentum is conserved:

$$(2m)v = p_1 = p_2 = m(2v)$$

Only way to conserve the momentum.
What about kinetic energies on both sides:

$$\text{General: } K = 0.5\ m\ v^2$$

Momentum and Kinetic Energy in Collisions

Assume a closed and isolated system
- No external forces
- Total Energy is conserved
- Linear Momentum is conserved

Just before collision:
- Energy: all kinetic K_i
- Momentum p_i

Just after elastic collision:
- $K_i = K_f$
- $p_i = p_f$ (If $M_1 >> M_2$, then $p_{2f} = -p_{2i}$
  like a super ball bouncing of the wall)

Just after inelastic collision:
- $K_i = K_f + E_{th}$ (caused by material deformation, friction)
- $p_f = p_i$ (Momentum is still conserved)

Momentum and Kinetic Energy in Collisions

Remarks:

- All real collisions are inelastic.
- Some can be approximated to be elastic (Newtons balls)
- If the two bodies stick together, the collision is **completely inelastic**.
  Largest possible loss of kinetic energy.

Think about a coordinate system in which the bodies rest after the collision... .

Inelastic Collisions

Recall: External forces will accelerate center of mass (and might rotate things around (torsion))

Collision only affected by internal forces → Momentum is conserved

Two masses:

$$\vec{p}_{1i}+\vec{p}_{2i}=\vec{p}_{1f}+\vec{p}_{2f}$$

$$m_1\vec{v}_{1i}+m_2\vec{v}_{2i}=m_1\vec{v}_{1f}+m_2\vec{v}_{2f}$$

But Energy not conserved.

Even if you know the individual masses m_1, m_2 and initial velocities v_{1i} and v_{2i}, you still can't calculate the final velocities (One equation, two unknowns).
Need either one final velocity or lost energy or ... .

Complete Inelastic Collisions in 1-D

Additional information:
Both masses stick together after inelastic collision

$$\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$v_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{(m_1 + m_2)} = \vec{v}_{com}$$

The familiar center of mass velocity

In a coordinate system in which $\vec{v}_{com} = 0$, the entire kinetic energy is transferred into thermal energy

Elastic Collisions in 1-D

Elastic collision:
- Kinetic energy is conserved
- Momentum is conserved

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

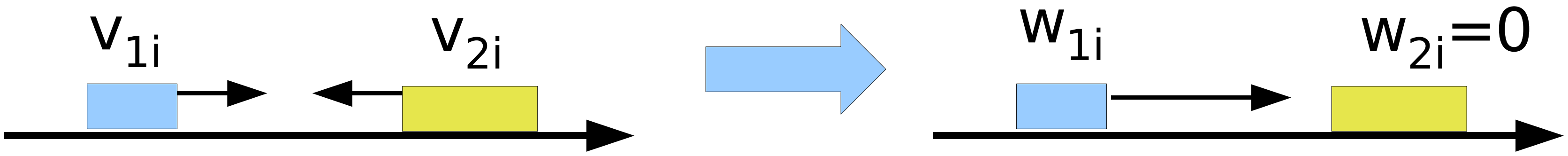
2 equations and usually 2 unknowns.
Solving this can be a little cumbersome.

One trick: Go into a coordinate system where one of the masses is initially at rest.

Elastic Collisions in 1-D

One trick: Go into a coordinate system where one of the masses is initially at rest.

Say we make: $w_{2i} = v_{2i} - v_{2i} = 0$ $\quad w_{1i} = v_{1i} - v_{2i}$

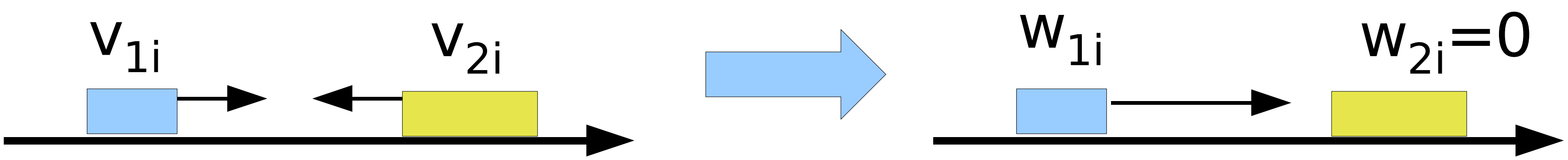


$$\frac{1}{2} m_1 w_{1i}^2 = \frac{1}{2} m_1 w_{1f}^2 + \frac{1}{2} m_2 w_{2f}^2$$

$$m_1 w_{1i} = m_1 w_{1f} + m_2 w_{2f}$$

Solve for w_{1f} and w_{2f} and then transform back: $v_{1f} = w_{1f} + v_{2i}$ and $v_{2f} = w_{2f} + v_{2i}$

Elastic Collisions in 1-D



$$\frac{1}{2} m_1 w_{1i}^2 = \frac{1}{2} m_1 w_{1f}^2 + \frac{1}{2} m_2 w_{2f}^2$$

$$m_1 w_{1i} = m_1 w_{1f} + m_2 w_{2f}$$

Solution: $w_{1f} = \frac{m_1 - m_2}{m_1 + m_2} w_{1i}$

$$w_{2f} = \frac{2m_1}{m_1 + m_2} w_{1i}$$

Elastic Collisions in 1-D

Solution: $w_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} w_{1i}$ $\qquad w_{2f} = \dfrac{2m_1}{m_1 + m_2} w_{1i}$

Special cases:

$m_1 = m_2$ — first mass stands still (Billiard balls w/o rotation)

$m_1 << m_2$ — first masses bounces off with same speed. (Super ball on wall.)

$m_1 >> m_2$ — first masses continues w/o change
Smaller mass advances twice as fast