Complete Inelastic Collisions in 1-D

Additional information:
Both masses stick together after inelastic collision

$$\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$v_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{(m_1 + m_2)} = \vec{v}_{com}$$

The familiar center of mass velocity

In a coordinate system in which $\vec{v}_{com} = 0$, the entire kinetic energy is transferred into thermal energy

Elastic Collisions in 1-D

Elastic collision:
- Kinetic energy is conserved
- Momentum is conserved

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

2 equations and usually 2 unknowns.
Solving this can be a little cumbersome.

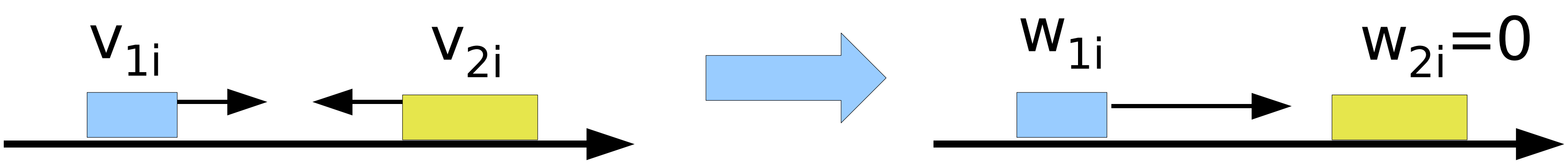
One trick: Go into a coordinate system where one of the masses is initially at rest.

Elastic Collisions in 1-D

One trick: Go into a coordinate system where one of the masses is initially at rest

Change all velocities by one of the initial velocities

Say we make: $w_{2i} = v_{2i} - v_{2i} = 0$ $\qquad w_{1i} = v_{1i} - v_{2i}$

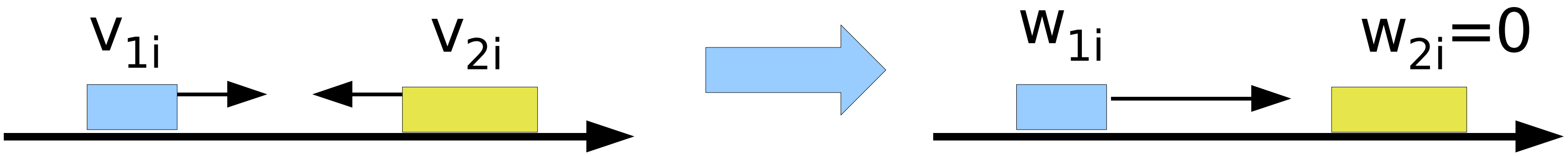


$$\frac{1}{2} m_1 w_{1i}^2 = \frac{1}{2} m_1 w_{1f}^2 + \frac{1}{2} m_2 w_{2f}^2$$

$$m_1 w_{1i} = m_1 w_{1f} + m_2 w_{2f}$$

Solve for w_{1f} and w_{2f} and then transform back: $v_{1f} = w_{1f} + v_{2i}$ and $v_{2f} = w_{2f} + v_{2i}$

Elastic Collisions in 1-D



$$\frac{1}{2}m_1 w_{1i}^2 = \frac{1}{2}m_1 w_{1f}^2 + \frac{1}{2}m_2 w_{2f}^2$$

$$m_1 w_{1i} = m_1 w_{1f} + m_2 w_{2f}$$

Solutions: $w_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} w_{1i}$

$$w_{2f} = \frac{2m_1}{m_1 + m_2} w_{1i}$$

Elastic Collisions in 1-D

Solution: $w_{1f} = \frac{m_1 - m_2}{m_1 + m_2} w_{1i}$ $\quad w_{2f} = \frac{2m_1}{m_1 + m_2} w_{1i}$

Special cases (recall: $w_{2i} = 0$):

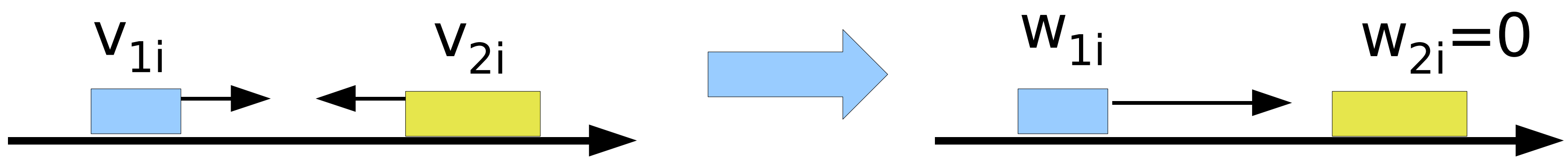
$m_1 = m_2$ first mass stands still
(Billiard balls w/o rotation)

$m_1 << m_2$ first mass bounces off with same speed. (Super ball on wall.)

$m_1 >> m_2$ first mass continues w/o change
Smaller mass advances twice as fast

Do you see the symmetry between the last two cases?

Elastic Collisions in 1-D



Back transform: $v_{1f}=w_{1f}+v_{2i}$ and $v_{2f}=w_{2f}+v_{2i}$

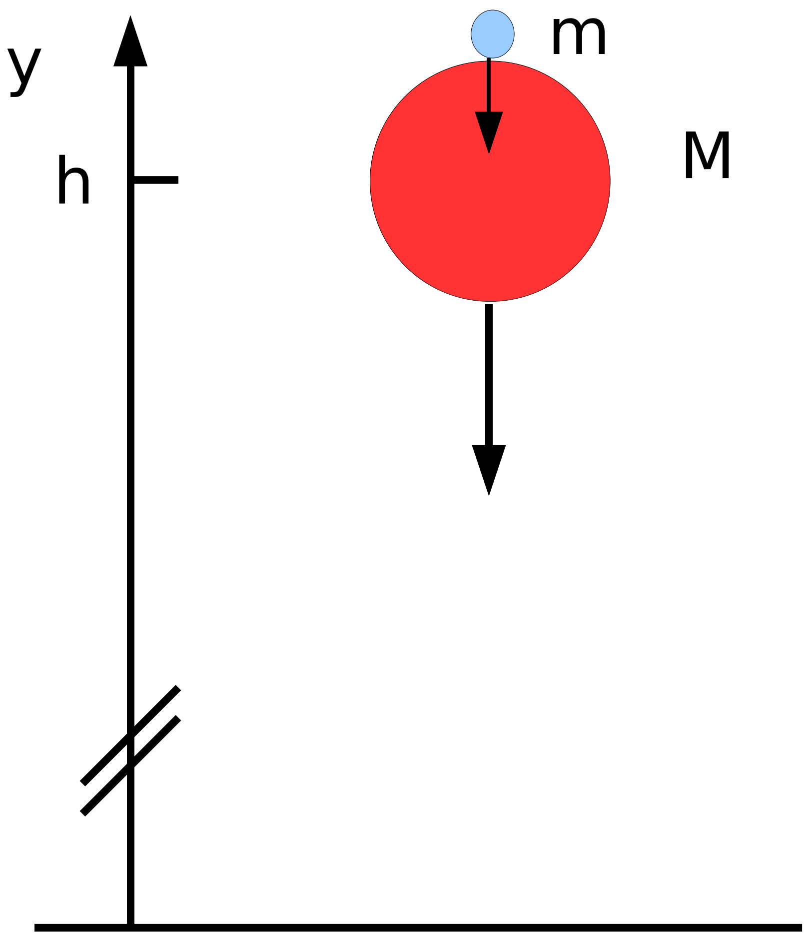
Solutions: $v_{1f}=\dfrac{m_1-m_2}{m_1+m_2}(v_{1i}-v_{2i})+v_{2i}$

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_{1i}+\frac{2m_2}{m_1+m_2}v_{2i}$$

Because of Symmetry (Change the indices):

$$v_{2f}=\frac{m_2-m_1}{m_1+m_2}v_{2i}+\frac{2m_1}{m_1+m_2}v_{1i}$$

Examples



h >> Diameter of balls

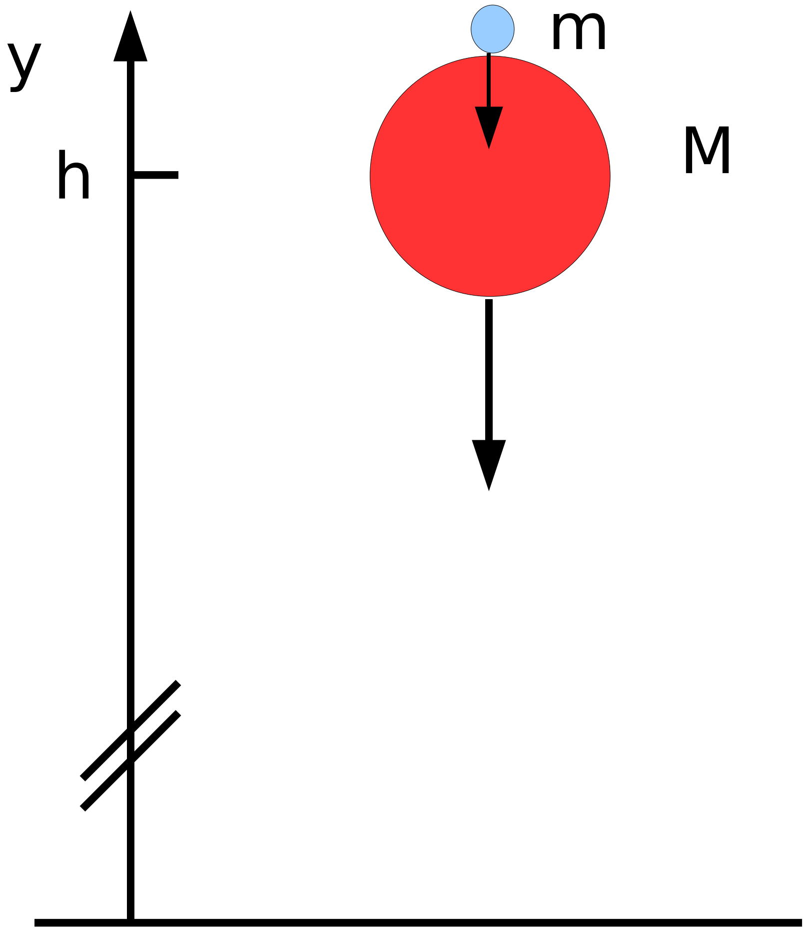
Both dropped at same time from height h (have a small gap). Large ball rebounds elastically from floor, then the small ball rebounds elastically from large ball.

a)What m would cause the large ball to stop when the two collide?

b) What height does the small ball then reach?

Parameters: M=0.63kg, h=1.8m (Problem 9.69)

Examples

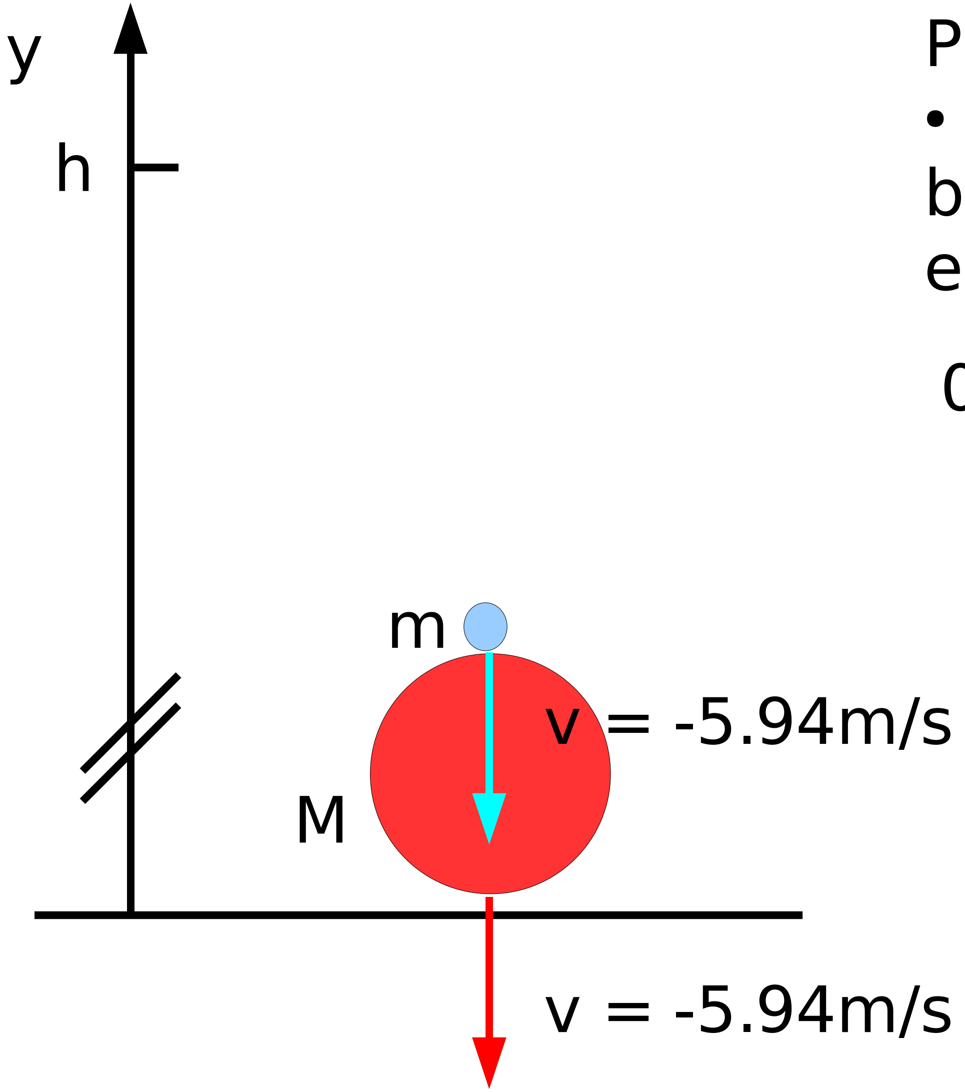


Parameters: M=0.63kg,
h=1.8m (Problem 9.69)

Plan:

- Step 1: Get velocities of both balls when they hit from energy conservation $K_f=P_i$
- Step 2: Elastic collision betw. large ball and ground. Use energy conservation to get upward momentum of large ball
- Step 3: Elastic collision betw. large and small ball. Momentum and energy conservation should allow to solve this.
- Step 4: Kinetic energy of small ball turns into potential energy at max height.

Examples



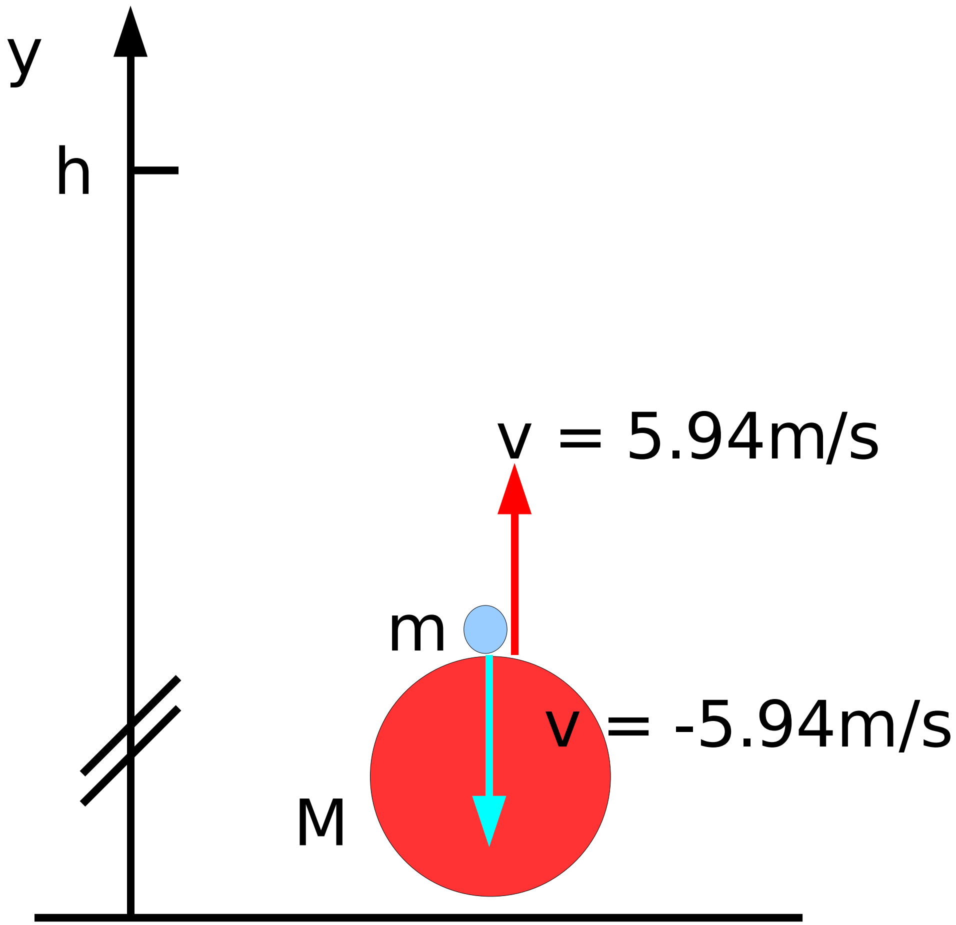
Plan:

- Step 1: Get velocities of both balls when they hit from energy conservation $K_f = P_i$

$0.5Mv^2 = Mgh$ → $v = (2gh)^{1/2}$

independent of mass:
v = -5.94m/s
(- because it goes down)

Examples

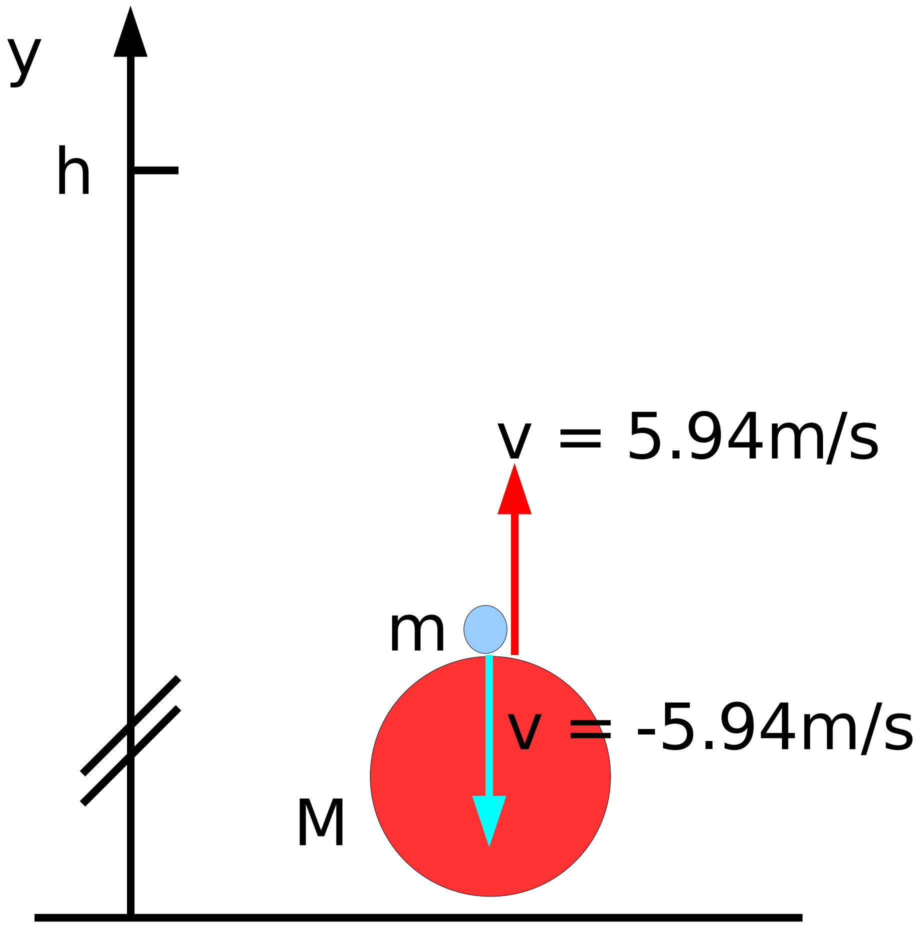


Plan:

- Step 1: v = -5.94m/s
- Step 2: Elastic collision betw. large ball and ground. Use energy conservation to get upward momentum of large ball

Collision of large ball with larger object (Earth): Ball turns around, same speed

Examples



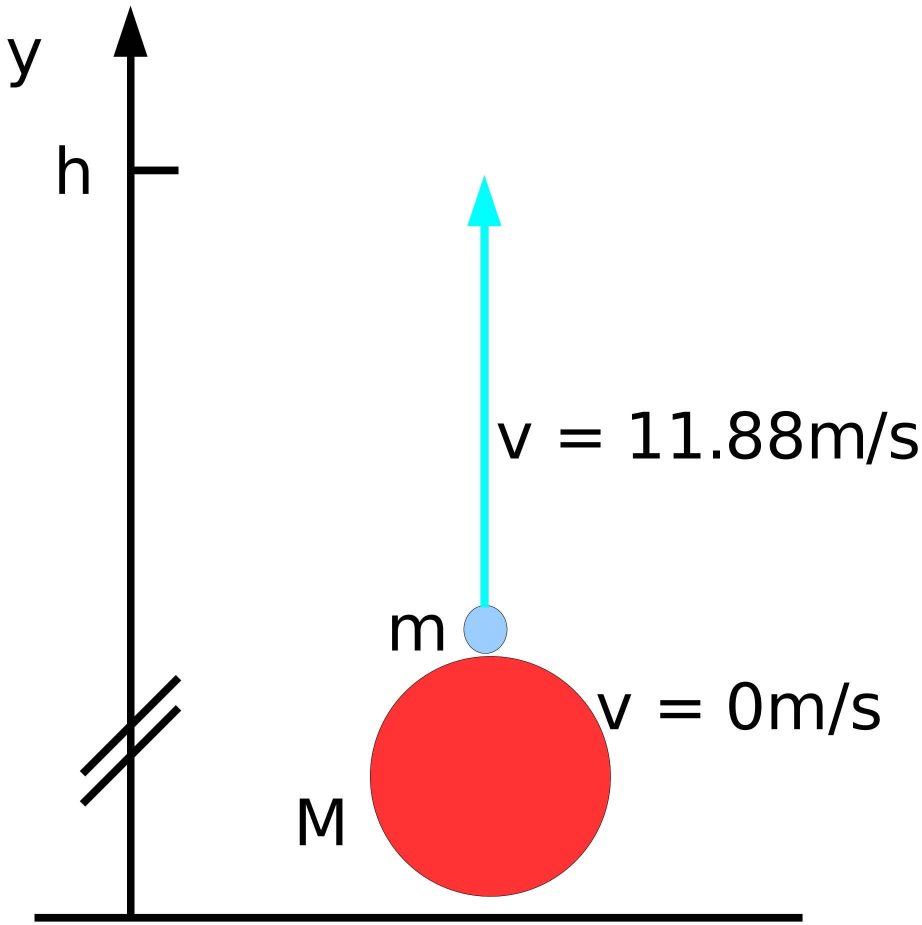
Plan:

- Step 1: $v = -5.94 m/s$
- Step 2: $v_{Mi} = 5.94 m/s$
  $v_{mi} = -5.94 m/s$
- Step 3: Elastic collision betw. large and small ball. Use:

$$v_{Mf} = \frac{M-m}{M+m} v_{Mi} + \frac{2m}{M+m} v_{mi}$$

Solve: $v_{Mf} = 0$ with: $v_{Mi} = -v_{mi}$ leads to $m = M/3 = 0.21 kg$

Examples



Plan:

- Step 2: $v_M = 5.94$m/s

  $v_m = -5.94$m/s

- Step 3: m=M/3 = 0.21kg

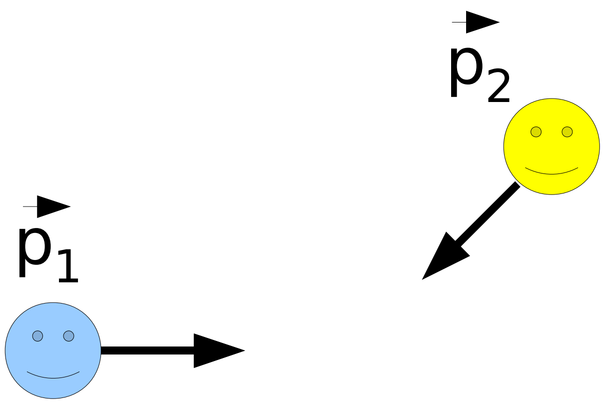
  What height does the small ball then reach?
  Need kinetic energy first:

$$v_{mf} = \frac{m-M}{M+m} v_{mi} + \frac{2M}{M+m} v_{Mi}$$

Use $v_{Mi} = -v_{mi}$ and M=3m leads to $v_{mf} = 2v_{Mi} = 11.88$m/s

$K = 0.5mv^2_{mf} = mgH = U$ $\Rightarrow$ $H = v^2_{mf}/2g = 4h = 7.2$m

Elastic Collisions in 2-D



Conserved momentum in both dimensions:

x-comp.: $p_{1x,i}+p_{2x,i}=p_{1x,f}+p_{2x,f}$

y-comp.: $p_{1y,i}+p_{2y,i}=p_{1y,f}+p_{2y,f}$

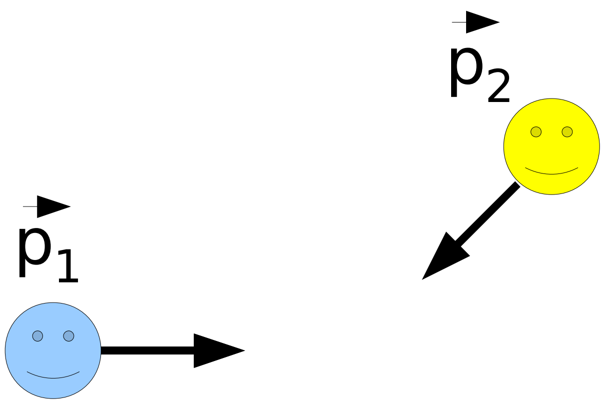
Additional equation: $K_{1i}+K_{2i}=K_{1f}+K_{2f}$

One question you should ask yourself when you study:
- Which information would I need to solve this problem?
- In other words: Is this problem completely defined

⇨ that would mean, you should in principle be able to solve it!

In this case: Assume we have the masses and initial velocities, is that enough?

Elastic Collisions in 2-D



Conserved momentum in both dimensions:

x-comp.: $p_{1x,i}+p_{2x,i}=p_{1x,f}+p_{2x,f}$

y-comp.: $p_{1y,i}+p_{2y,i}=p_{1y,f}+p_{2y,f}$

Additional equation: $K_{1i}+K_{2i}=K_{1f}+K_{2f}$

⇨ **3 equations**

Usually known: $m_1, m_2, \vec{v}_{1i}, \vec{v}_{2i}$ ⟶ $\vec{p}_1, \vec{p}_2$ and K_{1i}, K_{2i}

Unknowns: $\vec{v}_{1f}, \vec{v}_{2f}$ 2x2 components = **4 unknowns**

Need one more parameter: For example angle betw. $\vec{v}_{1f}, \vec{v}_{2f}$

Elastic Collisions in 2-D

Why is this the case?
Why is the problem not completely defined?

This will lead to different results.
It actually depends on the way the spheres hit each other.

Generally:

Variable: $\vec{v}_{1i},\vec{v}_{2i},\vec{v}_{1f},\vec{v}_{2f},m_1,m_2$

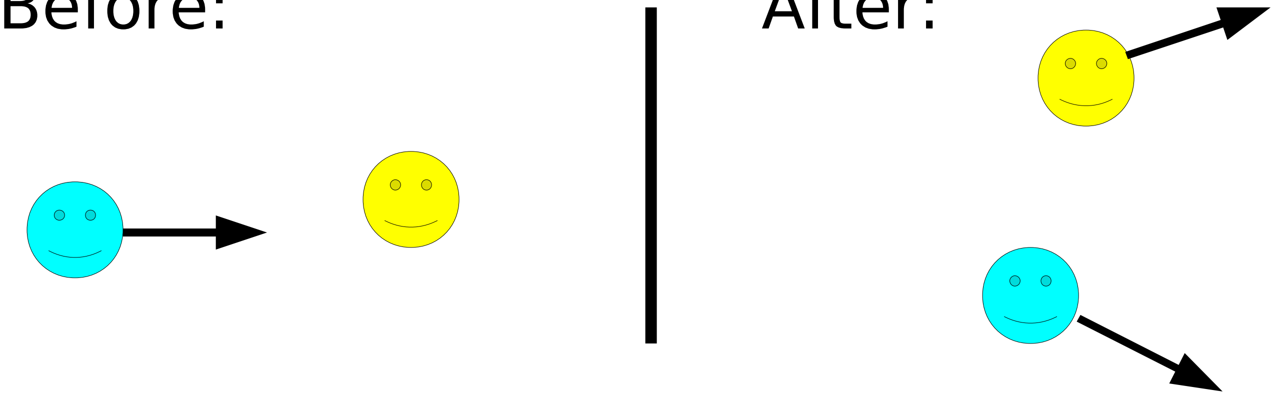
4x2 components + 2 = 10 variables, only 3 equations

Elastic Collisions in 2-D

4x2 components + 2 = 10 variables, only 3 equations

See Section 9.11, Fig 9:23:

Before: After:



Book: Equation 9-79 to 9-81 contain seven variables:
"*Two masses m_1, m_2; three speeds v_{1i},v_{1f},v_{2f}; and two angles θ_1,θ_2. If we know any four of these quantities, we can solve the three equations.*"

Why 7 and not 10? 3 variables in Fig 9-23 are set to 0 (known).

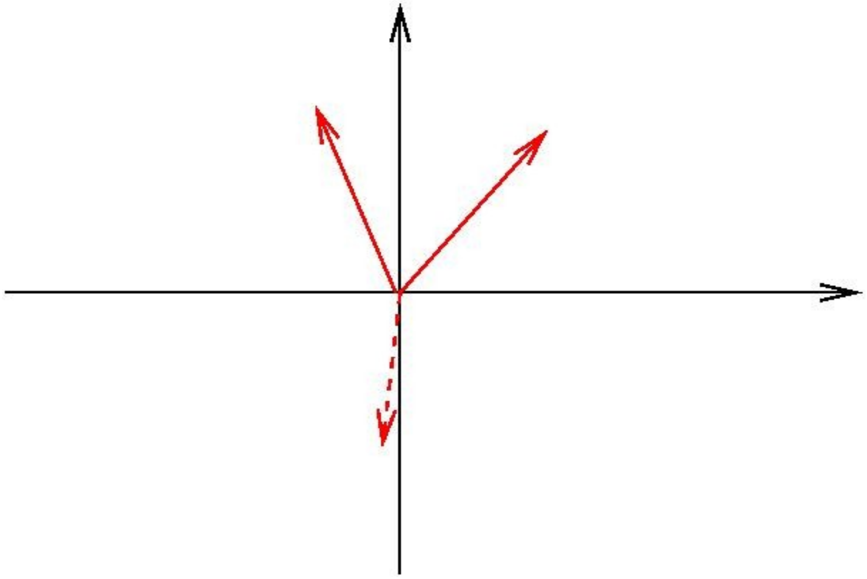
Conservation of momentum

Example 133: A 2.65kg stationary package explodes into three parts that then slide across the floor. Assume that the origin of our coordinate system is at the location of the package before the explosion.
In that case the initial velocities and masses of two of the pieces are:

Part 1: M_1=0.5kg $\vec{v}_1$=(10.0i +12.0j)m/s

Part 2: M_2=0.75kg $|v_2|$= 14m/s at an angle of 110deg

Q: What is $\vec{v}_3$?



Conservation of momentum

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Part 2: M_2=0.75kg $|v_2|$= 14m/s at an angle of 110deg

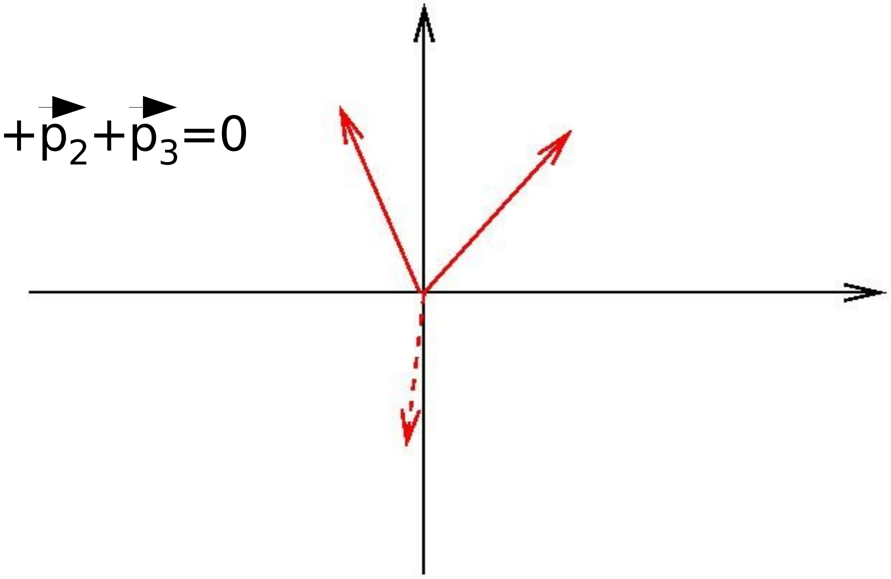
Q: What is $\vec{v}_3$?

Use conservation of momentum: $\vec{p}_1+\vec{p}_2+\vec{p}_3=0$

$\vec{p}_1$=0.5*(10i +12j) = 5.0i+6.0j

$\vec{p}_2$=0.75*14[cos(110)i +sin(110)j]
= -3.59i+9.87j

$\vec{v}_3=-(\vec{p}_1+\vec{p}_2)/M_3$ = [-1.1i -11.3j]

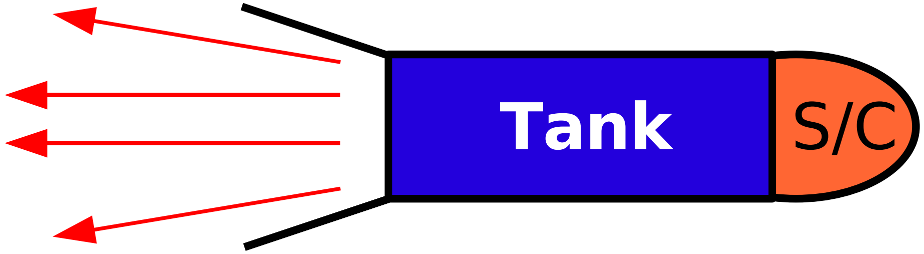


The Rocket Science

After this lecture, you can claim you are a

Rocket Scientist.

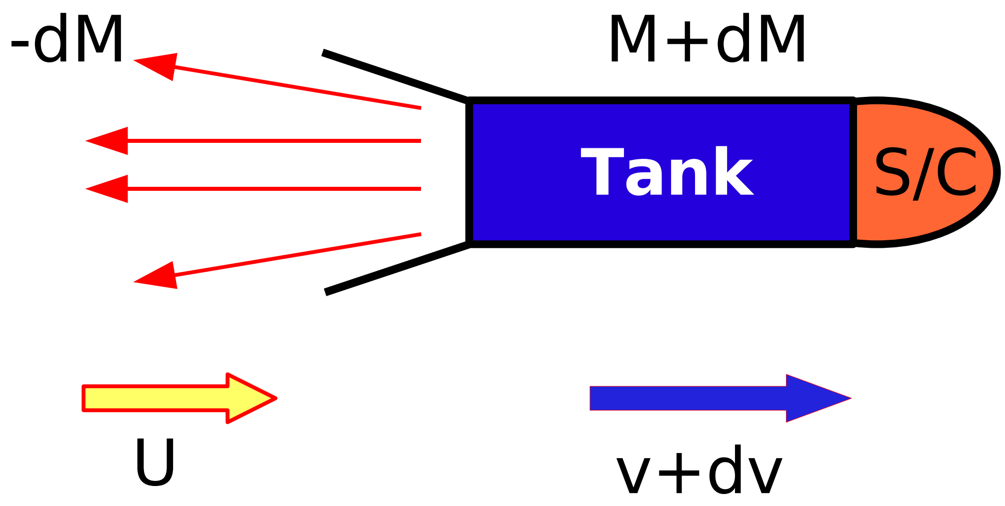
(Maybe not the best one out there, but who cares ...)



Most of the mass of a rocket is fuel (very inefficient...)

Momentum of system is conserved, but mass of rocket is reduced by mass of fuel which is send out at the back with high speed.

The Rocket Science



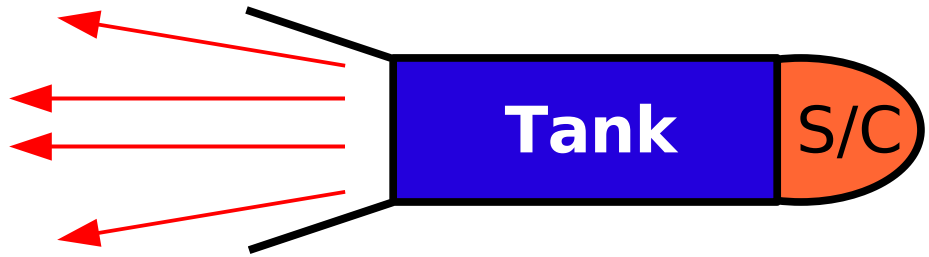
$v+dv=U+v_{rel}$

At time t: $P_i(t) = M(t)v(t)$

At time (t+dt): $P_f(t+dt) = (M(t)+dM)(v(t)+dv) - dM\,U$

M(t): Mass of rocket
dM : Mass change during time intervall dt (dM < 0)
dv : Velocity change during dt
U : Velocity of exhaust
v_{rel}: Relative velocity between exhaust and rocket

The Rocket Science



At time t: $P_i(t) = M(t)v(t)$

At time (t+dt): $P_f(t+dt) = (M(t)+dM)(v(t)+dv) - dM\,U$

Use: $P_i(t)=P_f(t+dt)$ and $v+dv=U+v_{rel}$

$M\,dv = -dM\,v_{rel}$ $\xrightarrow{1/dt}$ $Ma = Rv_{rel} = T$ (Thrust)

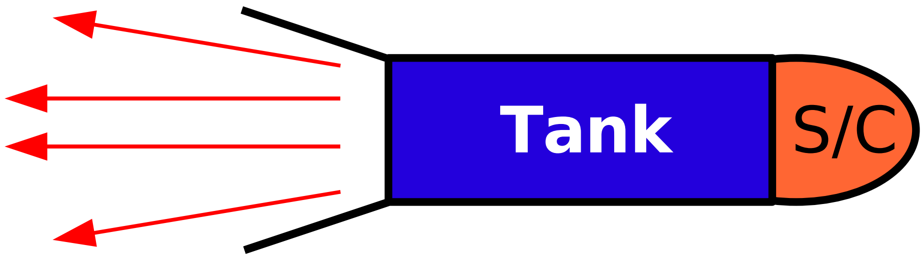
$R=-\dfrac{dM}{dt}$ positive mass loss rate

1st Rocket Equation

Note: Thrust depends on relative velocity

Acceleration increases in time as M goes down

The Rocket Science



At time t: $P_i(t) = M(t)v(t)$

At time (t+dt): $P_f(t+dt) = (M(t)+dM)(v(t)+dv) - dM\,U$

Use: $P_i(t)=P_f(t+dt)$ and $v+dv=U+v_{rel}$

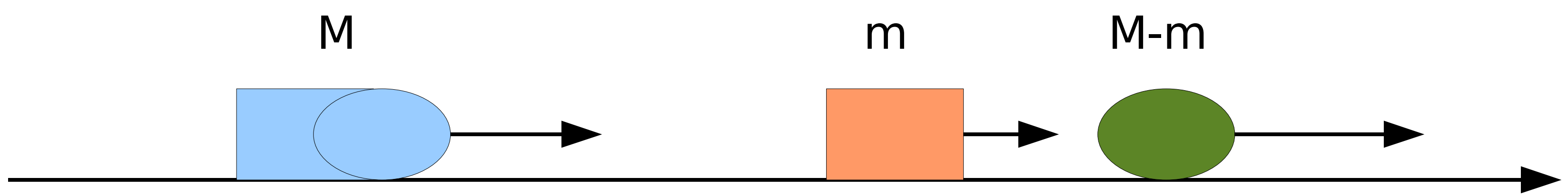
$M\,dv = -dM\,v_{rel}$ ⇒ $dv = -v_{rel}\dfrac{dM}{M}$

$$\int_{v_i}^{v_f} dv = -v_{rel}\int_{M_i}^{M_f}\frac{dM}{M}$$ ⇒ $$v_f - v_i = v_{rel}\ln\frac{M_i}{M_f}$$

2nd Rocket Equation

Examples

132: A rocket of mass M moves along an x-axis at the constant speed of v_i=40m/s. A small explosion separates the rocket into a rear section of mass m1 and a front section. Both sections move along the x-axis. The relative speed between the rear and the front sections is 20m/s.
a) What is the minimum speed of the front section and how large is m in this case ?
b) What is the maximum speed of the front section and how large is m in this case?



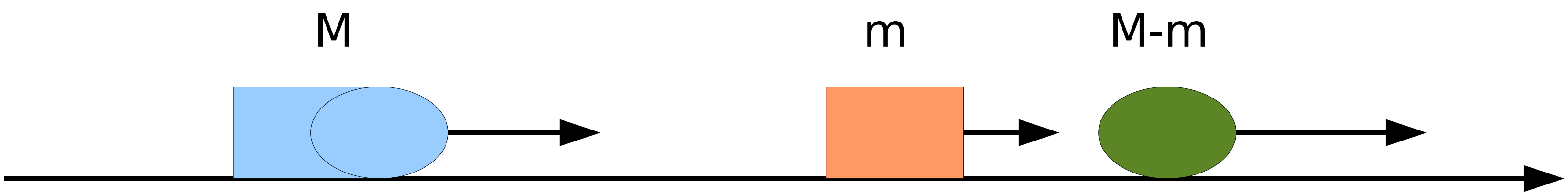
Idea: Conservation of momentum: $Mv_i = m(v_f-20) + (M-m)v_f$

→ $v_f=v_i+20(m/M)$

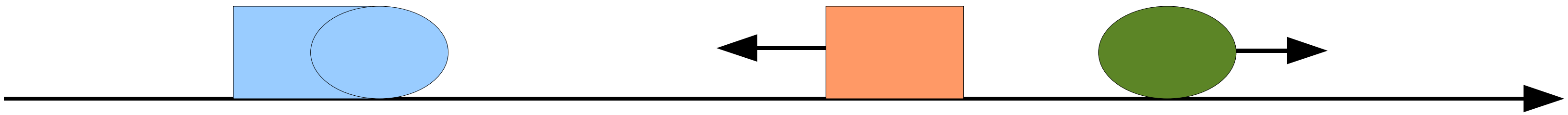
to a) $v_f=v_i$ = 40m/s when m=0

to b) v_f=60m/s when m=M

Examples



Different way to solve this: Transform into a coordinate system where w_i=0m/s. Then it is just an explosion between two objects.



Still use conservation of momentum:

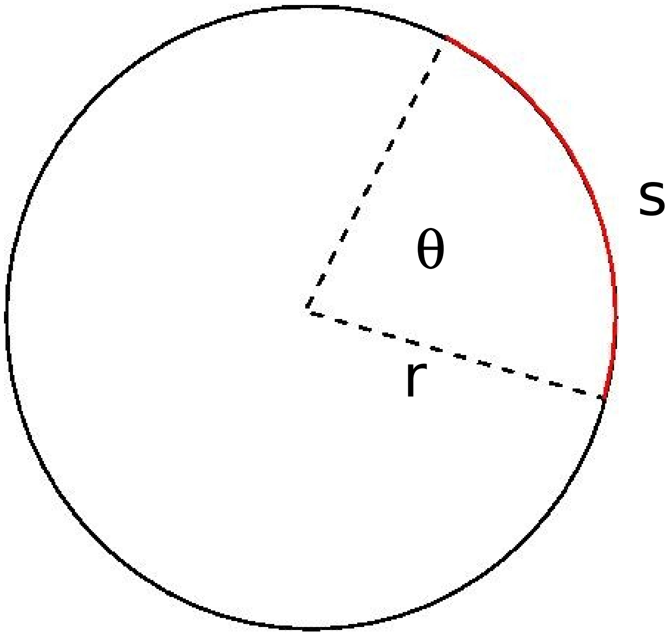
$$mw_m = m(w_f - 20) = (M-m)w_f \Rightarrow w_f = 20m/(2m-M)$$

to a) $w_f = 0$ when m=0 $\Rightarrow$ $v_f = 40$m/s

to b) $w_f = 20$m/s when m=M $\Rightarrow$ $v_f = 60$m/s

Chapter 10: Rotation

Rotational variable: $\theta = s/r$



Angle measured in radian:
Length s of the line segment
if the radius of the circle is r=1

Compare with degrees:
1 rev. = 360deg = 2π rad
1 rad = 57.3deg = 0.159 rev.

Angular displacement: $\Delta\theta = \theta_2 - \theta_1$

An angular displacement in the
- counterclockwise direction is positive
- clockwise direction is negative

Chapter 10: Rotation

Rotational variable: $\theta = s/r$

Angular displacement: $\Delta\theta = \theta_2 - \theta_1$

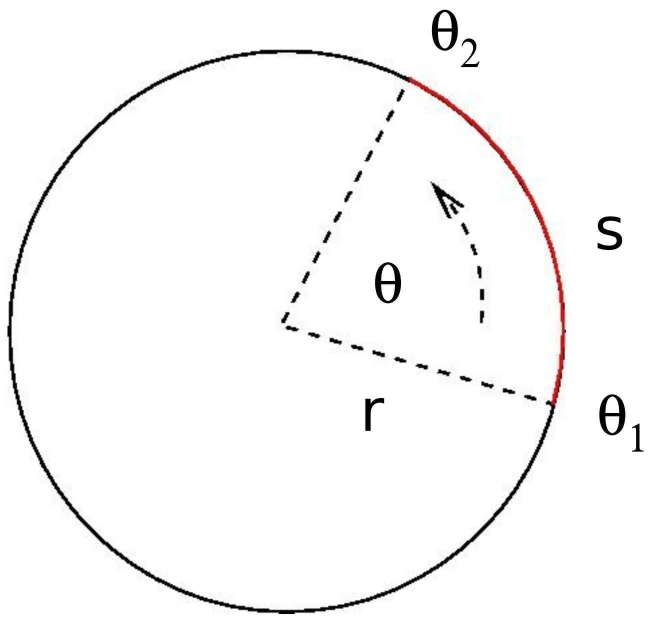
If the angular displacement happens during time Δt:

average angular velocity:

$$\omega_{avg} = \Delta\theta/\Delta t$$

instantaneous angular velocity:

$$\omega = \lim_{\Delta t \to 0} \Delta\theta/\Delta t = \frac{d\theta}{dt}$$



If the angular velocity changes over time:

$$\alpha = \lim_{\Delta t \to 0} \Delta\omega/\Delta t = \frac{d\omega}{dt}$$

angular acceleration

Chapter 10: Rotation

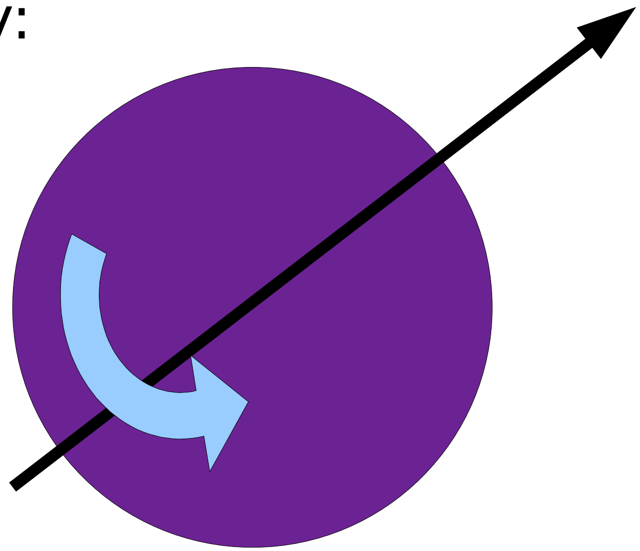
Comparison
linear motion vs angular motion
for constant acceleration

Displacements:	$x - x_0$	$\theta - \theta_0$
Velocities:	v	ω
Accelerations:	a	α
Velocities:	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
Displacements:	$x - x_0 = v_0 t + 0.5at^2$	$\theta - \theta_0 = \omega_0 t + 0.5\alpha t^2$
Velocities:	$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha\,(\theta - \theta_0)$

Chapter 10: Rotation

Angular variables are vectors:

Consider a solid body:



It can rotate around any axis.
This axis and the right hand rule defines the direction of the vector.
The length is equal to the magnitude of the rotation.