Organizational Stuff

- How to get to the lectures
- Check them out:
  They might incl. hints for HW problems
- 1st HW is due: Tomorrow, Thursday 09/04 11pm

Chapter 3: Vectors

Vectors:

- Used to describe quantities which have
  - Magnitude
  - Direction

Scalars:

- Used to describe quantities which have only
  - Magnitude

Chapter 3: Vectors

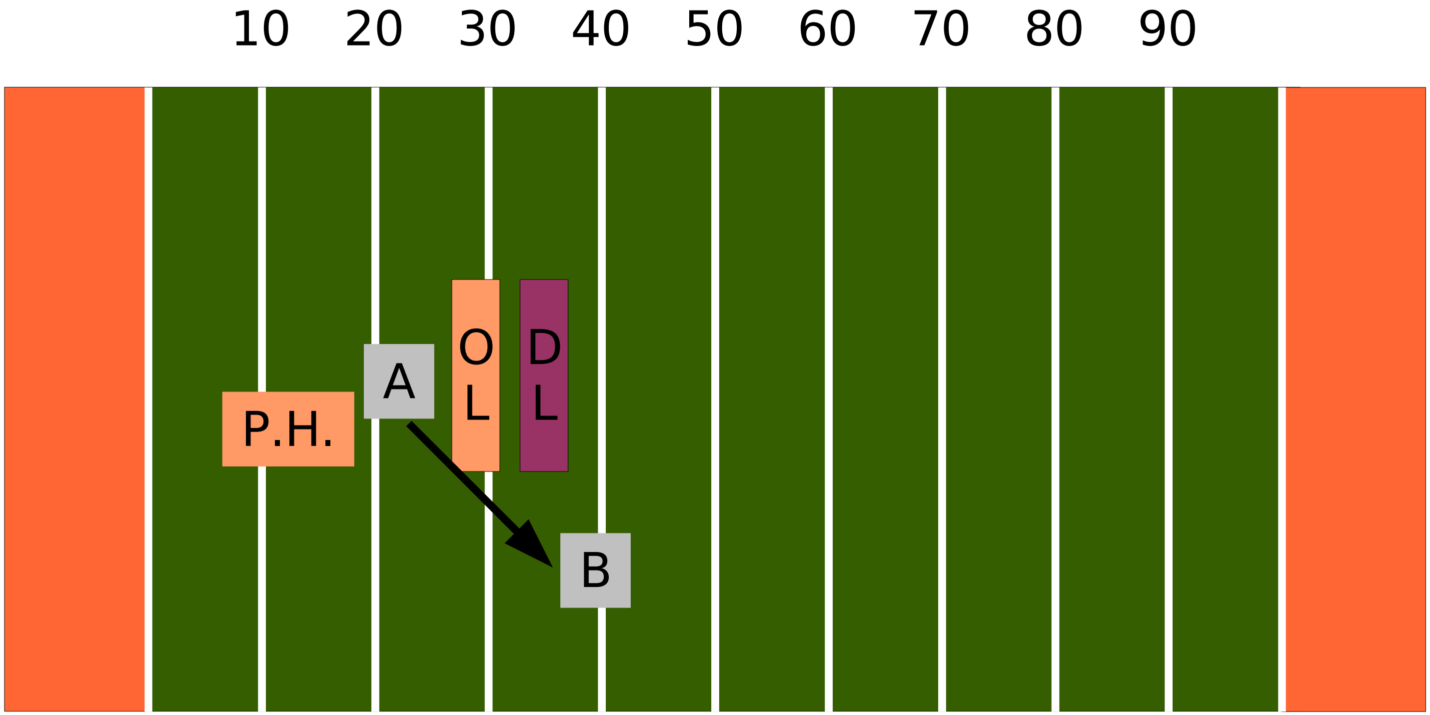
Today:

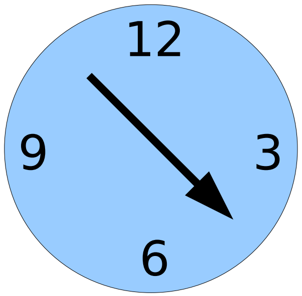
1. Understanding vectors
2. Adding vectors
3. Vectors and coordinate systems
4. Adding vectors using coordinates

Friday:

1. Multiplying vectors with a scalar
2. Multiplying vectors
   a) Scalar (Dot) Product
   b) Vector (Cross) Product

Example: Displacement Vector





P.H. running from A to B.
Magnitude: 15yd, Direction: 4:30 (One way to define it)
Gain of 10yd (Here only the 3 o'Clock component counts).
Obviously: Direction is important!

Vectors

Vectors (Examples):

- Displacement
- Velocity
- Acceleration
- Force
- ...

Scalar (Examples):

- Energy
- Temperature
- Pressure
- Mass
- ...

This chapter teaches you how to do calculations with vectors

Vectors

- The quantity (magnitude and direction) described by the vector exists independently of the way you describe it.

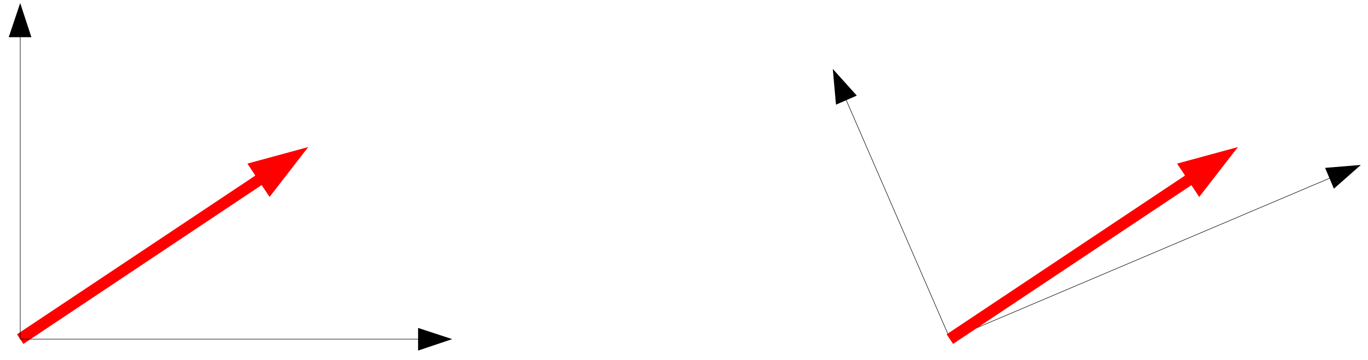
Example for the same displacement:

- Take Newberry Road and drive 60 miles west
- Let's go to Cedar Key
- Shortest route to the Gulf coast

all describe the same 'displacement'

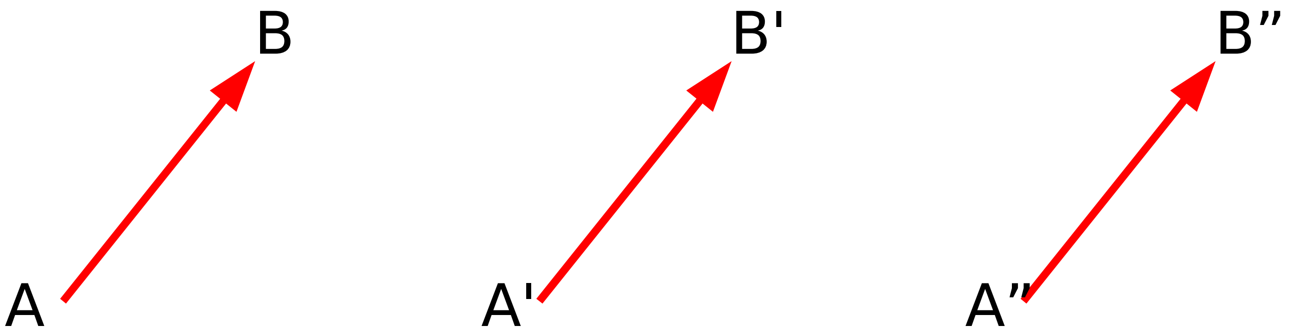
Difficulties with Vectors

- In Physics we use coordinate systems to describe vectors. But the physics (what happens) has to be independent from the coordinate system (So you are free to pick a good one).



Difficulties with Vectors

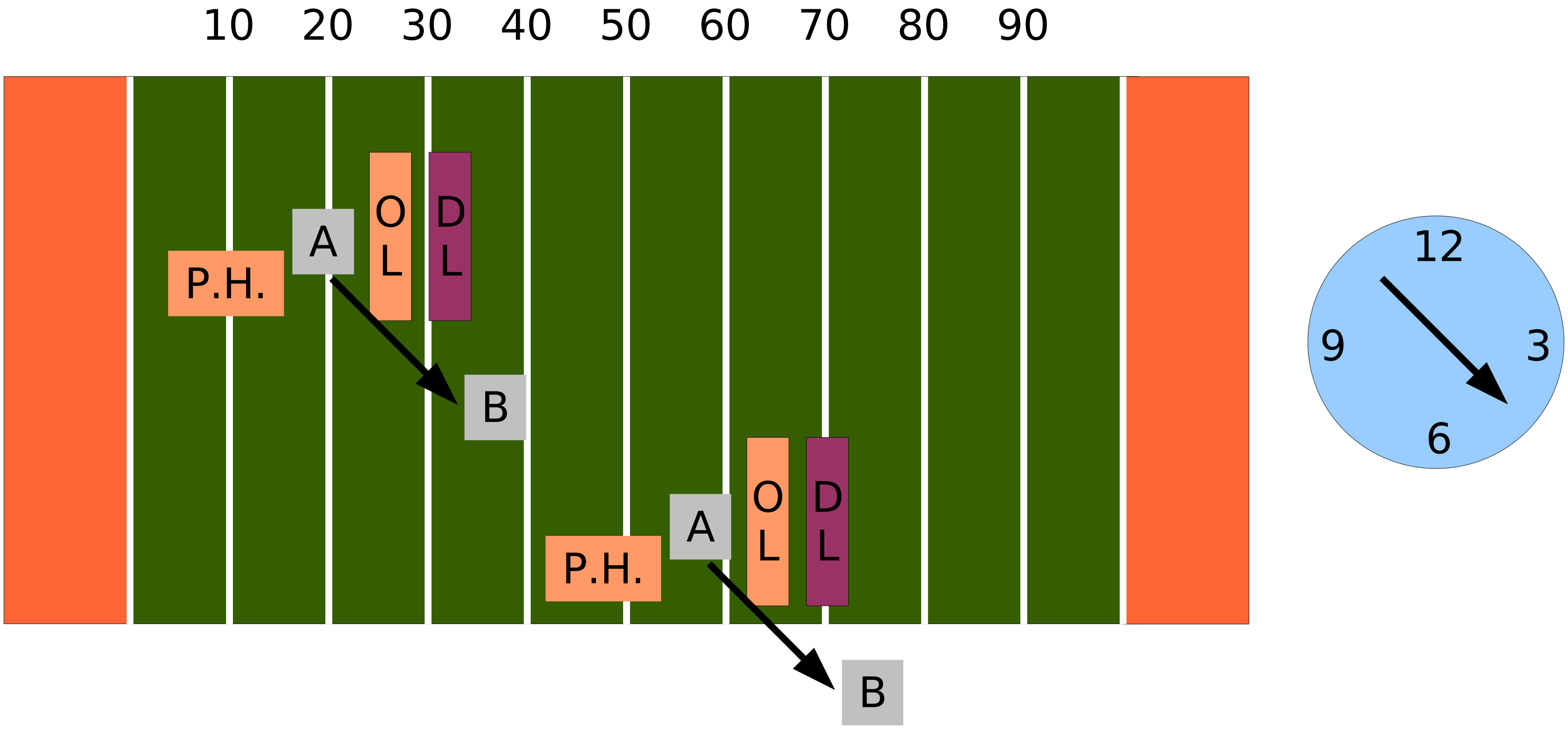
- Vectors can be shifted w/o changing their value



All vectors $\overrightarrow{AB}$, $\overrightarrow{A'B'}$, $\overrightarrow{A''B''}$ describe the same displacement

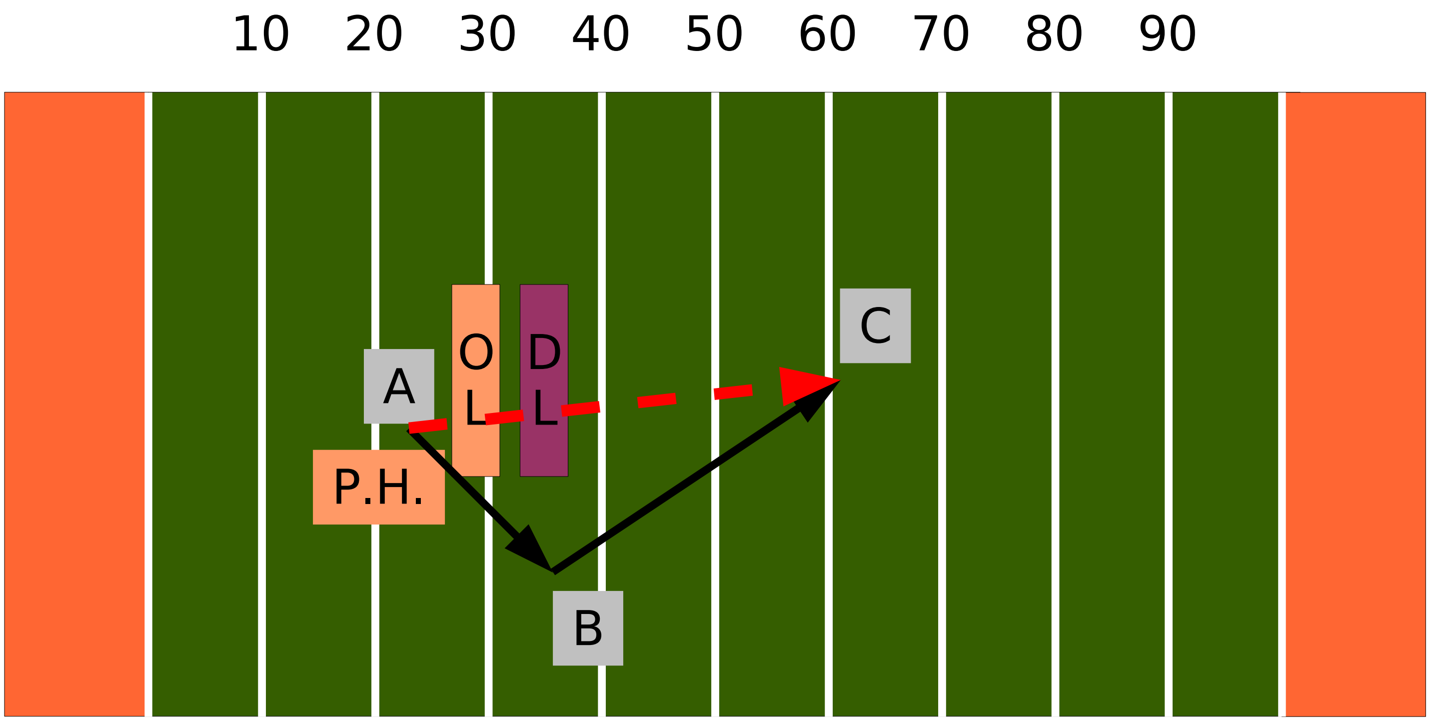
This 'shifting' sometimes causes conceptual problems!

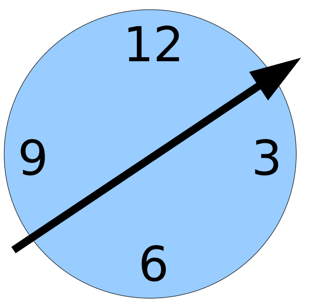
Example: Displacement Vector



If you only shift parts of the problem, you might get different results (such as getting out of bounds)
Use common sense (once you do it, Physics is easy!)

Adding Vectors



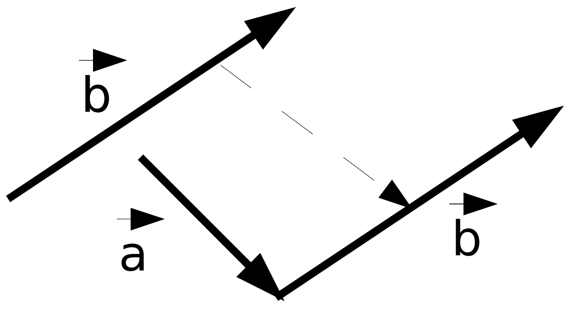


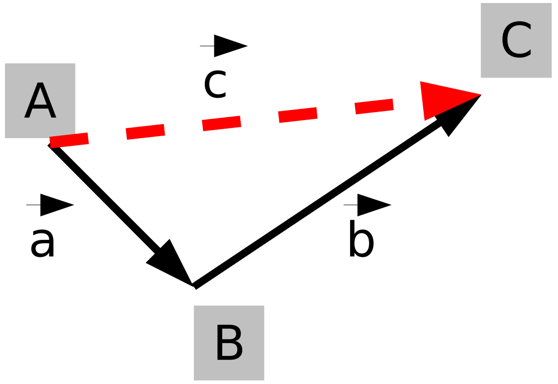
P.H. running from A to B then turns the corner to C.
From A to B: Magnitude: 15yd, Direction: 4:30
From B to C: Magnitude: 25yd, Direction: 2:00

How can we calculate the displacement from A to C?

Adding Vectors

Geometrically: $\vec{a} + \vec{b} = \vec{c}$

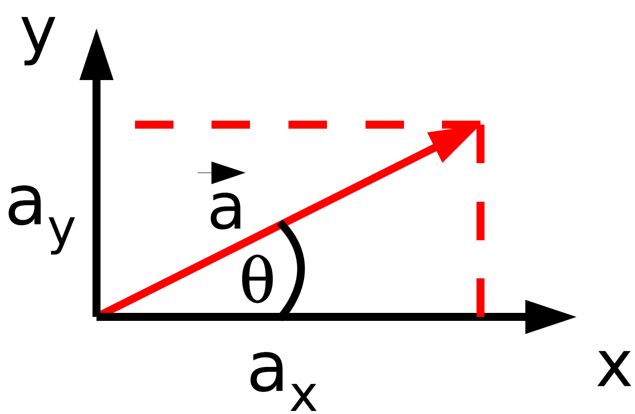




Move the beginning of the 2nd arrow (or vector) to the tip of the 1st and then draw an arrow from the beginning of the 1st vector to the tip of the 2nd.

- This does not even require a coordinate system although getting real numbers such as the 'gain' (progress down field) requires now geometry.

Vector Components



$$a_x = a\cos\theta$$

$$a_y = a\sin\theta$$

<u>Recall:</u>

$$\frac{a_y}{a_x} = \frac{a\sin\theta}{a\cos\theta} = \tan\theta$$

$$a_x^2 + a_y^2 = a^2$$

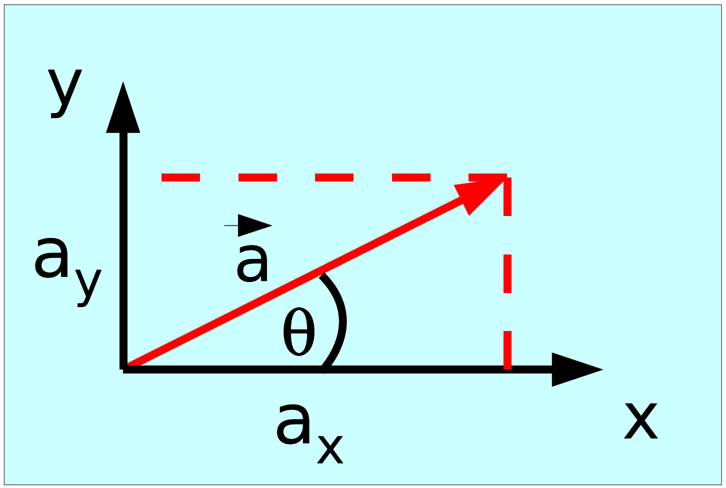
Algebraic addition:

- Requires coordinate system to define vector components

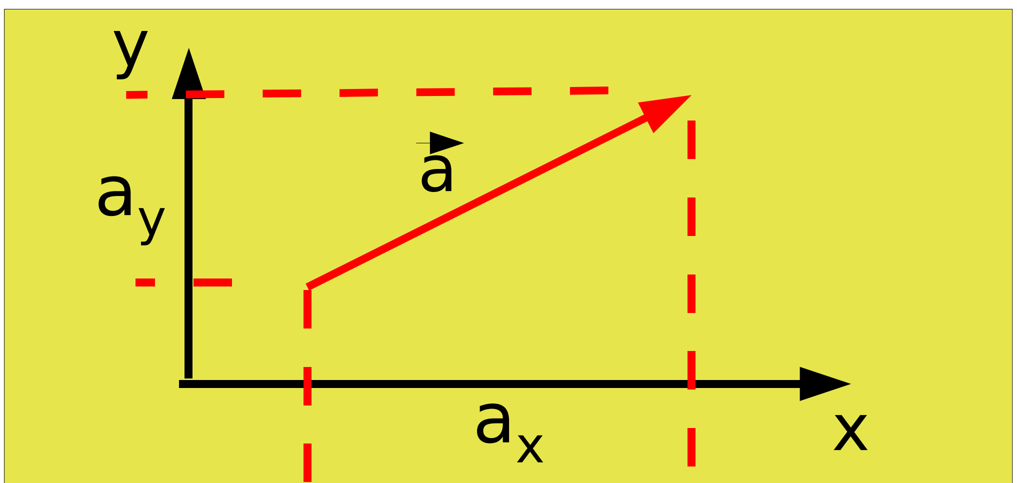
We use two different ways to describe a vector in 2-D (with respect to a specific coordinate system)

- Magnitude a and angle θ with respect to x-axis
- Components a_x and a_y

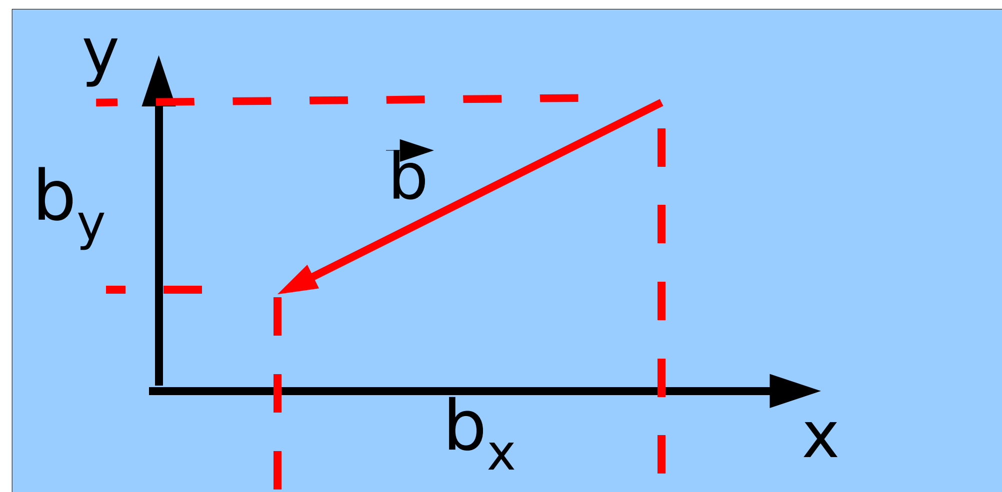
Vector Components



A vector component is the (signed) projection of the vector on a particular axis of the coordinate system. Positive and negative components are possible.

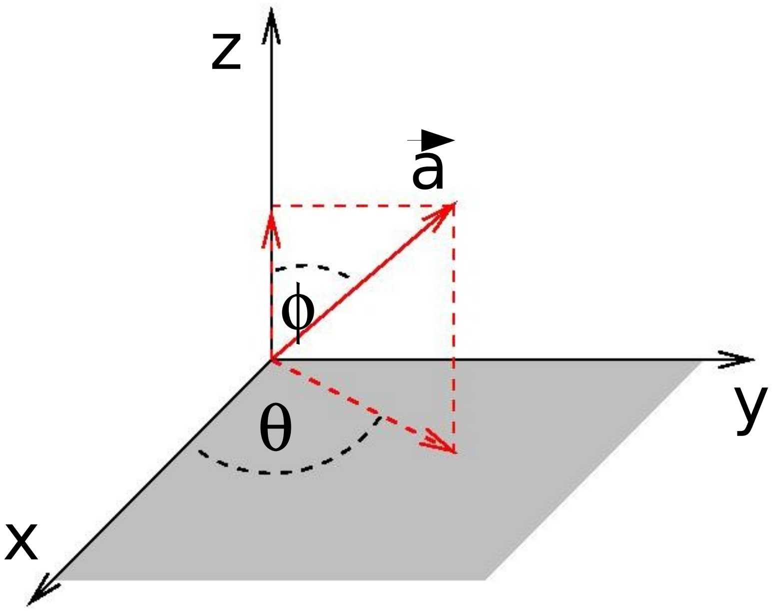


Projection: From the start to the tip of the vector (NOT from 0).



$b_x = -a_x < 0$ $\qquad$ $b_y = -a_y < 0$

Vector Components in 3-D



$$a_x = r\cos\theta\sin\phi$$

$$a_y = r\sin\theta\sin\phi$$

$$a_z = r\cos\phi$$

$$\cos\phi = \frac{a_z}{r} \qquad \tan\theta = \frac{a_y}{a_x}$$

$$a_x^2 + a_y^2 + a_z^2 = r^2$$

Two often used ways to describe a vector in 3-D:

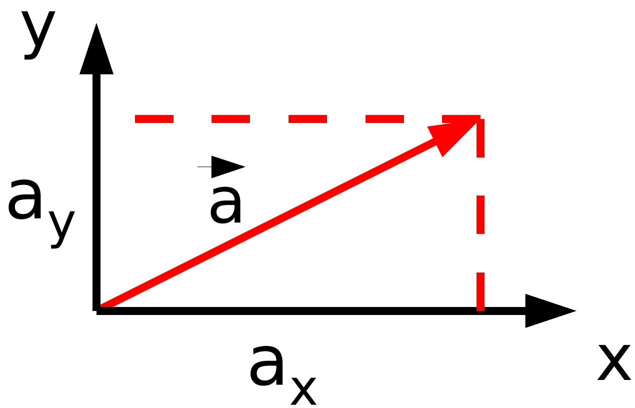
- Spherical coordinates: (r, ϕ, θ)
  - r: magnitude
  - ϕ : angle with the z-axis
  - θ : angle between the projection onto the x-y plane and the x-axis
- Cartesian Coordinates: (x,y,z)

Vectors

Different ways to write vectors in cartesian coordinates:

$\vec{a}$: Is the symbol (or name) for the vector
Is independent from the coordinate system

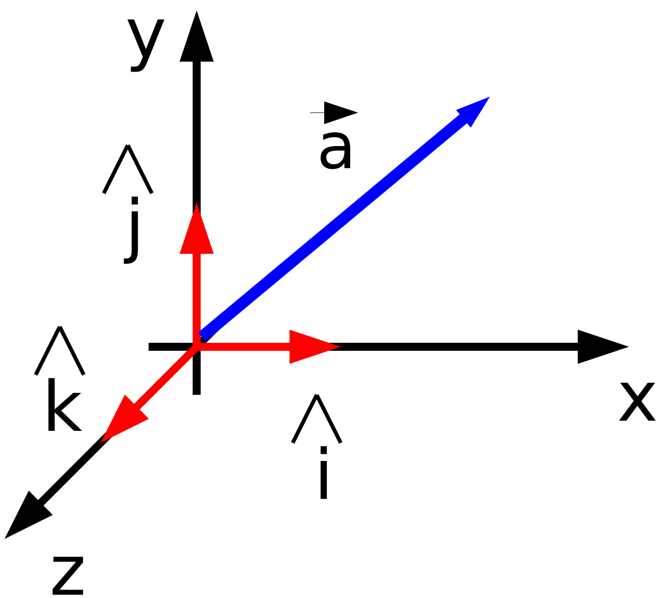
Once we have a coordinate system:



$$\vec{a} = (a_x, a_y, a_z) = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

both are used in other text books (and probably occasionally by me or the TA's).
The book uses Unit Vector-notation (next slide).

Unit Vectors



Unit vectors ($\hat{i}$, $\hat{j}$, $\hat{k}$) have a magnitude of 1 unit:

- Displacement: 1 m

(or feet or any other length unit)

- Velocities: 1 m/s

(or any other velocity unit)

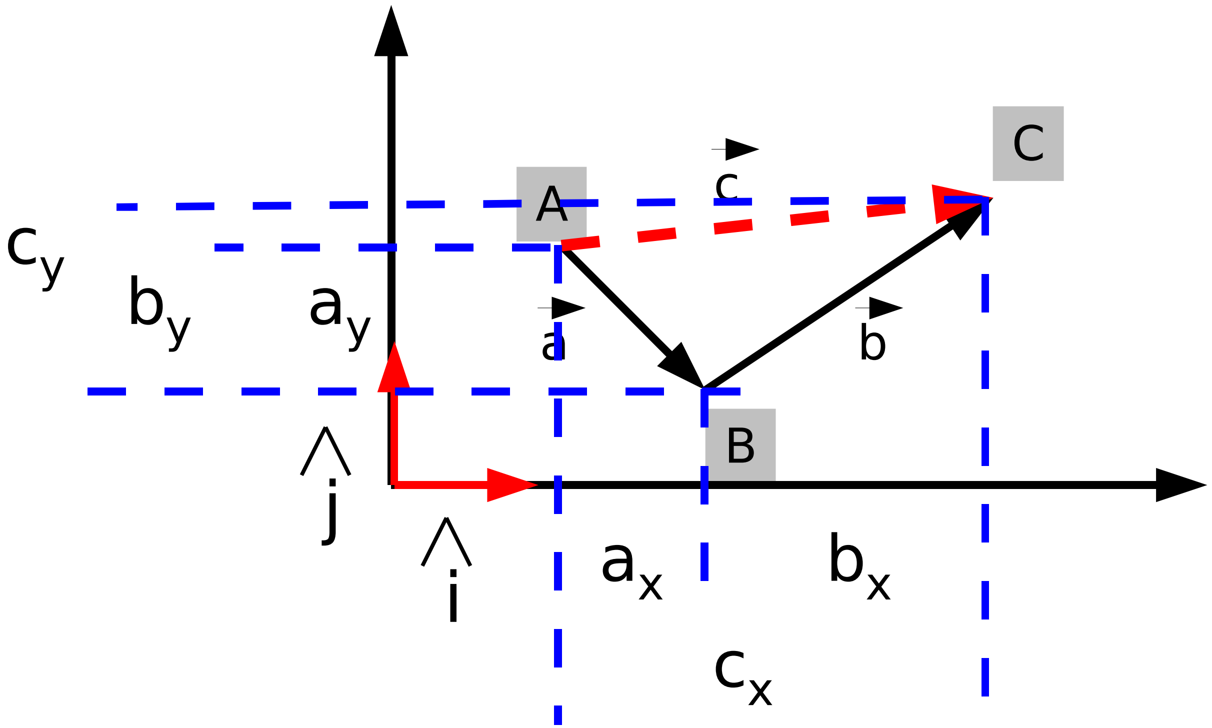
- ...

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

and are parallel to your coordinate axes

a_x, a_y, a_z: Are the Vector components

Adding Vectors



$\vec{a} + \vec{b} = \vec{c}$

$a_x + b_x = c_x$

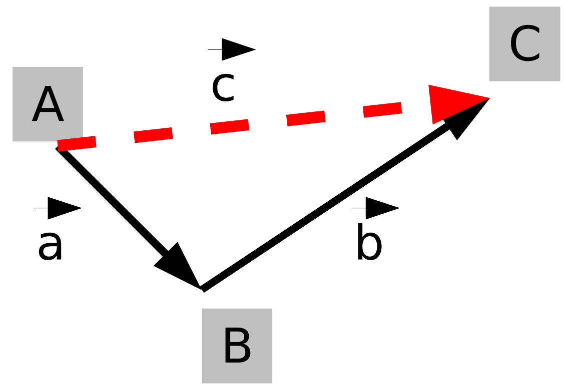
$a_y + b_y = c_y$ Note that $a_y < 0$ here

$a_z + b_z = c_z$ if we have a 3rd component

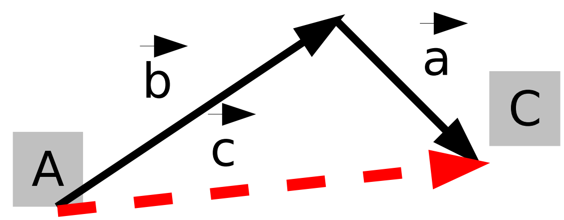
Adding Vectors

Useful laws for adding vectors:

- Commutative
  (Doesn't matter with which displacement you start)



$\vec{a} + \vec{b} = \vec{c}$



$\vec{b} + \vec{a} = \vec{c} = \vec{a} + \vec{b}$

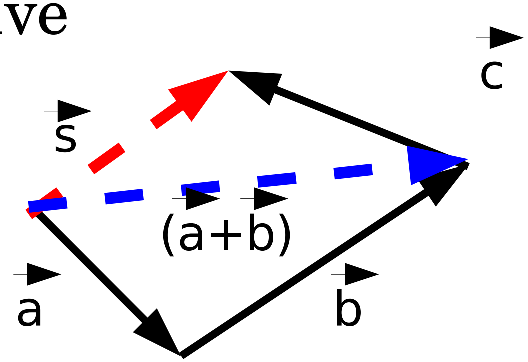
Obviously as
$a_x+b_x=b_x+a_x$
same for y and z

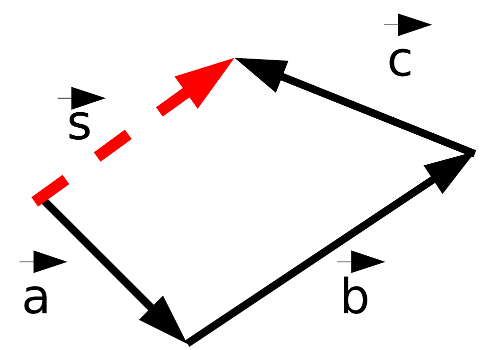
Adding Vectors

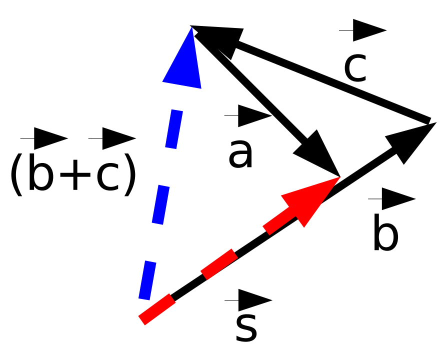
Useful laws for adding vectors:

- Associative

$(\vec{a}+\vec{b})+\vec{c}=\vec{s}=\vec{a}+(\vec{b}+\vec{c})=\vec{a}+\vec{b}+\vec{c}$







Obviously as

$a_x+(b_x+c_x)=(a_x+b_x)+c_x$

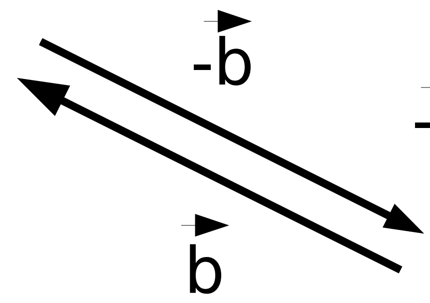
same for y and z

Adding Vectors

Useful laws for adding vectors:

The 'negative': $-\vec{b}$

- same magnitude
- opposite direction

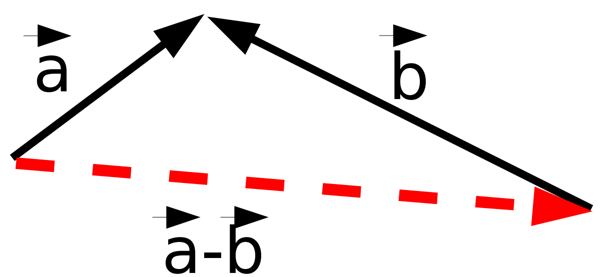


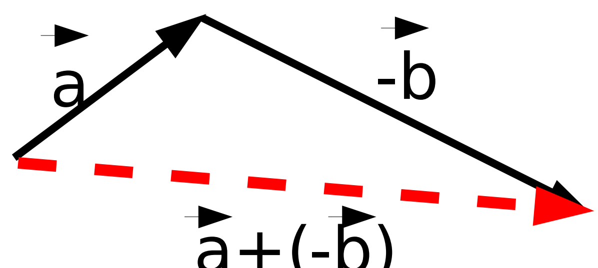
$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$-\vec{b} = -b_x\hat{i} - b_y\hat{j} - b_z\hat{k}$$

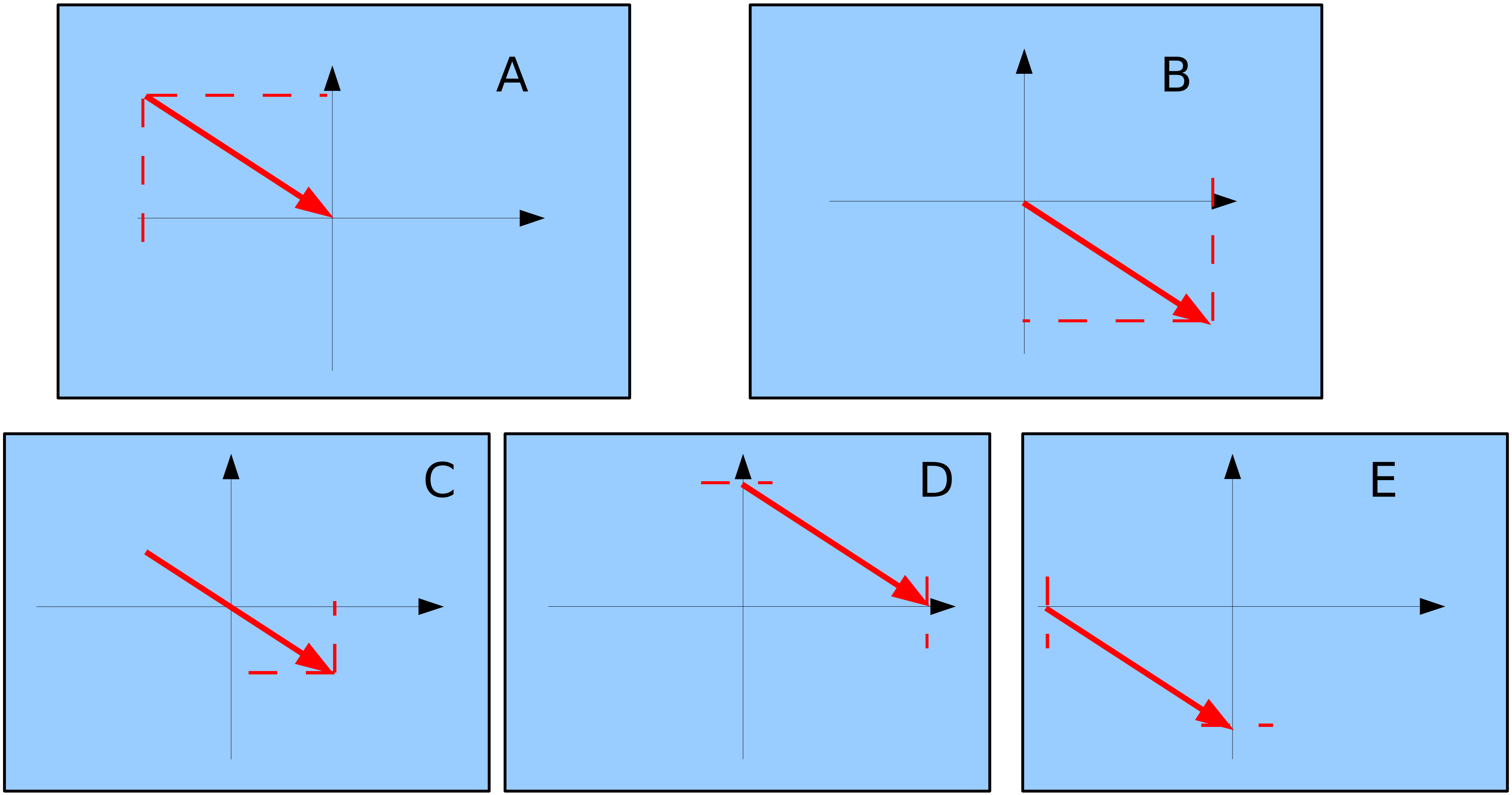
Subtraction: $\vec{a}-\vec{b} = \vec{a}+(-\vec{b})$

- Add the negative





Which of the following figures shows how to find the components of vector $\vec{a}$?



Multiplying Vectors

3 different types of multiplications using vectors:

1. Multiplying a Vector by a scalar
2. Scalar product between two vectors
3. Vector product between two vectors

Multiplying Vectors

1. Multiplying a Vector $\vec{a}$ by a scalar s:

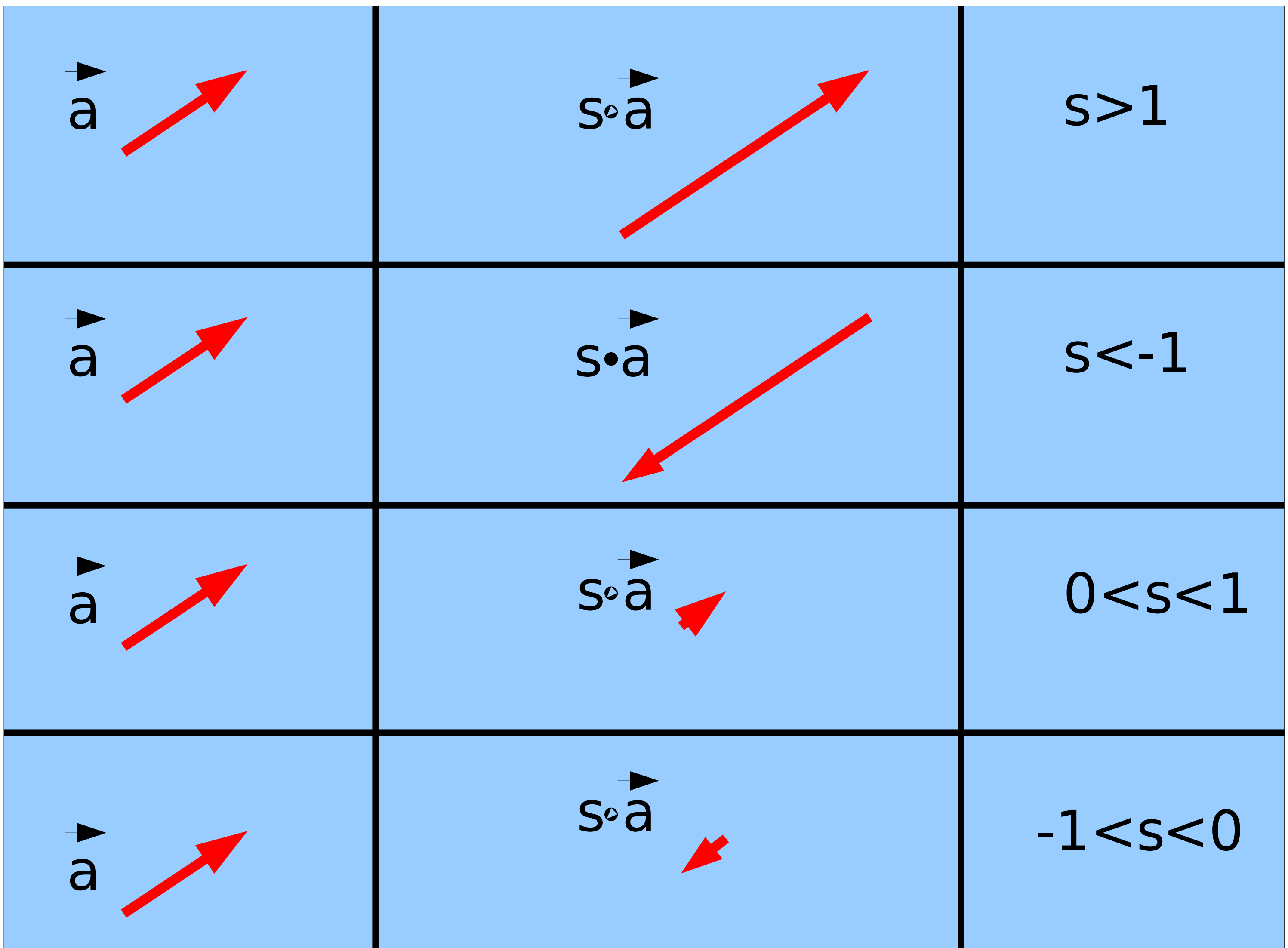
$$s \bullet \vec{a} = (sa_x)\hat{i} + (sa_y)\hat{j} + (sa_z)\hat{k}$$

- streches the vector by a factor s if s > 1
- shortens the vector if 0 < s < 1
- turns the vector around and stretches it if s < -1
- turns the vector around and shortens it if -1 < s < 0

- Division by s is equal to multiplication by 1/s

Multiplying Vectors

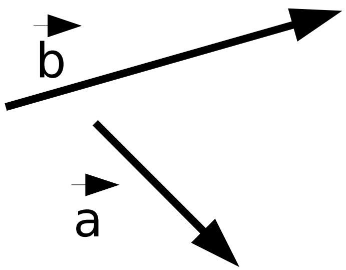
$$s\cdot\vec{a} = (sa_x)\hat{i} + (sa_y)\hat{j} + (sa_z)\hat{k}$$

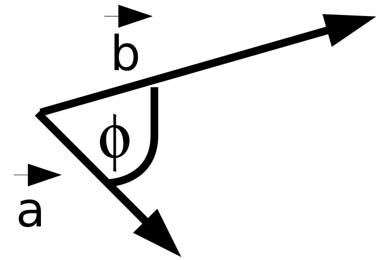


Scalar Product

2. Scalar (dot) product between two vectors

$$\vec{a} \cdot \vec{b} = ab\cos(\phi)$$





$$\vec{a} \cdot \vec{b} = ab\cos(\phi) = a_x b_x + a_y b_y + a_z b_z$$

Measures the projection of one vector (say $\vec{a}$) onto the other vector (say $\vec{b}$) times the amplitude of the second vector (b).

Scalar Product

2. Scalar (dot) product between two vectors

$$\vec{a}\cdot\vec{b} = ab\cos(\phi) = a_x b_x + a_y b_y + a_z b_z$$

Can be used to calculate ϕ:

$$\cos(\phi) = \frac{a_x b_x + a_y b_y + a_z b_z}{a\,b}$$

Recall:

$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} = \vec{a}\;\vec{a}$$

$$b = \sqrt{b_x{}^2 + b_y{}^2 + b_z{}^2} = \vec{b}\cdot\vec{b}$$

and will be used for physics problems starting chapter 7

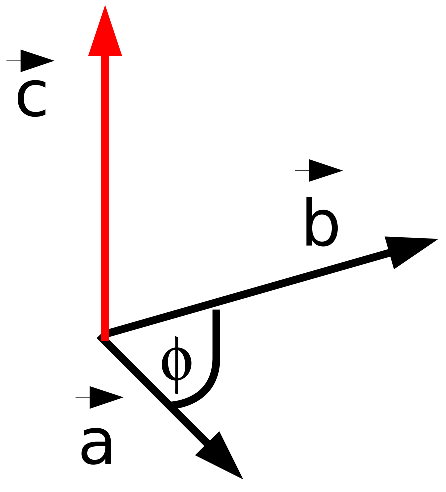
Vector Product

3. Vector product between two vectors

generates 3rd vector:

$$\vec{a} \times \vec{b} = \vec{c} \quad \text{with} \quad c = ab\sin(\phi)$$

$\vec{c}$: orthogonal to $\vec{a}$ and $\vec{b}$

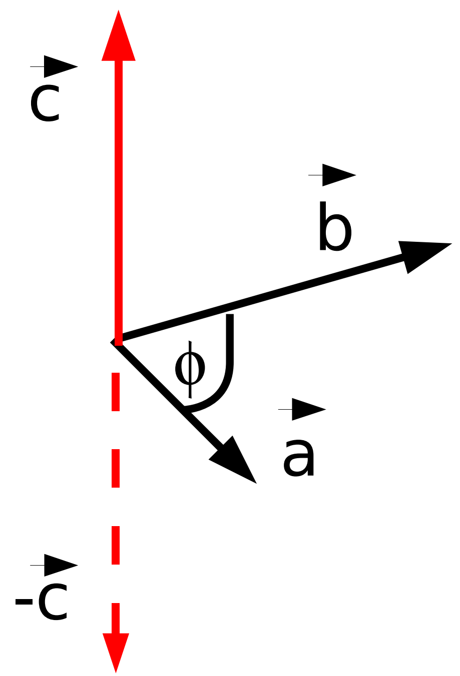


Direction:
Right hand rule

Vector Product

3. Vector product between two vectors

$$\vec{a} \times \vec{b} = \vec{c} = (a_y b_z - b_y a_z)\,\hat{i}$$
$$+ (a_z b_x - b_z a_x)\,\hat{j}$$
$$+ (a_x b_y - b_x a_y)\,\hat{k}$$



Note that:

$$\vec{a} \times \vec{b} = \vec{c} = -(\vec{b} \times \vec{a}) = -(-\vec{c})$$

Consistent with

$c = ab\sin(\phi)$ with

$\sin(-\phi) = -\sin(\phi)$

HITT 1

Calculate the scalar product between:

$\vec{a} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

A: 10

B: 8

C: 14

D: -12

E: 26

HITT 2

Calculate the scalar product between:

$\vec{a} = 4\hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

A: 10

B: 8

C: 14

D: -12

E: 26