Unusual metal-insulator transition in disordered ferromagnetic films

K.A. Muttalib[a*], P. Wölfle[b†], R. Misra[c‡], and A.F. Hebard[a§]
[a] *Department of Physics, University of Florida, Gainesville FL 32611-8440.*
[b] *Institute for Condensed Matter Theory and Institute for Nanotechnology, DFG-Center for Functional Nanostructures, Karlsruhe Institute of Technology, D-76128 Karlsruhe, Germany and*
[c] *Department of Physics, Penn State University, University Park, PA 16802-6300*

We present a theoretical interpretation of recent data on the conductance near and farther away from the metal-insulator transition in thin ferromagnetic Gd films of thickness $b \approx 2$ - 10 nm. For increasing sheet resistances a dimensional crossover takes place from $d = 2$ to $d = 3$ dimensions, since the large phase relaxation rate caused by scattering of quasiparticles off spin wave excitations renders the dephasing length $L_\phi \lesssim b$ at strong disorder. The conductivity data in the various regimes obey fractional power-law or logarithmic temperature dependence. One observes weak localization and interaction induced corrections at weaker disorder. At strong disorder, near the metal-insulator transition, the data show scaling and collapse onto two scaling curves for the metallic and insulating regimes. We interpret this unusual behavior as proof of two distinctly different correlation length exponents on both sides of the transition.

Keywords: Metal-Insulator transition, disordered systems, ferromagnetic films

I. INTRODUCTION

The effect of disorder on the conductivity of metals has been recognized as a fundamental theme of theoretical condensed matter physics ever since P. W. Anderson introduced the concept of electron localization by a random potential[1] in 1958. A new phase of research on this problem began after the scaling theory of Anderson localization had been proposed[2] in 1979. While the quantum corrections to the conductivity on the metallic side have been observed and interpreted in numerous systems (for a review see [3]), it has been more difficult to access the disorder induced metal-insulator transition (MIT). One of the best-known examples for the latter is the MIT in the impurity band of doped semiconductors, e.g. Si doped with P . Unfortunately, it turned out to be difficult to control the systems so well that the desired close approach to the MIT critical point could be achieved. As a consequence the experimental results on e.g. the critical exponents turned out to scatter widely. The experimental data we would like to interpret here have been collected on thin films of ferromagnetic Gd . This raises the immediate question why to study films and not bulk materials, considering that the MIT depends sensitively on dimension, and it is unclear whether a MIT exists in a two-dimensional (interacting) system at high electron density (for a review see [4]). The answer is that (1) the strength of disorder may be much better controlled in a film geometry, and (2) the quasi-two-dimensional systems may be effectively three-dimensional. The decisive criterion for three-dimensionality is whether the phase relaxation length is less than the film thickness $L_\phi < b$. On both counts, ferromagnetic Gd films have an advantage over $3d$-materials: (i) these films may be prepared with sheet resistances R_0 (at a reference temperature of $T = 5K$) varying over a very large range, from 400Ω up to $50k\Omega$, enclosing the range around the resistance quantum $R_Q = h/e^2 \approx 25k\Omega$ where one expects the MIT to happen; and (ii) in ferromagnetic films the scattering of electrons off spin waves offers a powerful mechanism of phase breaking, helping to satisfy the criterion of three-dimensionality in the temperature range above a few K.

The theory of disorder driven MITs is part of a larger topic, concerning the critical behavior near quantum phase transitions in general. Powerful theoretical methods have been developed to describe localization by disorder in the case that electron-electron interactions may be neglected[5]. The existence of a quantum phase transition showing critical properties has been established in dimensions $d > 2$. The critical exponents of the correlation length, ν, of the conductivity in the metallic phase, s, and of the dynamical scaling, z, have been determined for $3d$-systems as $z = 3$ (analytically[6]) and in numerical simulations $\nu = s = 1.58$ (case of orthogonal symmetry[7]). Including interactions[8], the problem is much more difficult. The perturbative renormalization group method has been developed to a high level of sophistication, but in many cases of interest the RG-flow moves the system into the strong coupling regime, which is not accessible by presently available methods[4,9].

In this paper we review and extend the quantum transport theory of disordered ferromagnetic films as developed and applied to experiments in [10–13]. We first recall our results on the phase relaxation rate and length induced by scattering off spin wave excitations in these systems, which dominates the usual Coulomb interaction induced (Altshuler-Aronov) contributions[14]. A first ap-

*Corresponding author:
Electronic address: muttalib@phys.ufl.edu;
Tel: 352-392-6699; Fax: 352-846-0295.
†Electronic address: peter.woelfle@kit.edu
‡Electronic address: rxm74@psu.edu
§Electronic address: afh@phys.ufl.edu

plication of this is the weak localization correction to the conductivity in both effectively two-dimensional and three-dimensional weakly disordered systems. In general, increasing the disorder strength at fixed temperature will induce a transition from effectively two to three dimensions. In the effectively $3d$ regime a metal-insulator transition takes place. We discuss the scaling regime near the metal-insulator transition in some detail. A comparison with experimental data allows to extract the critical exponents. We find, as published elsewhere[12], that the correlation length exponent is substantially different on both sides of the transition, while the dynamical exponent is the same. It is interesting to note that in the scaling regime the condition of three-dimensionality is met at any temperature $T > 20mK$, provided the film thickness exceeds a certain threshold value.

II. PHASE RELAXATION TIME AND LENGTH

We start by reviewing the theory of the phase relaxation rate in ferromagnetic disordered metals. The quantum phase of an electron may be changed by several effects. First, the interaction with external or internal magnetic fields (in a ferromagnet) will limit the phase coherence of an electron traversing a closed loop in opposite directions (the weak localization correction ultimately responsible for localization occurs only when the phase difference accumulated is sufficiently small). For ferromagnetic Fe and Gd films we have estimated the corresponding energy shift $\omega_H = 4(\epsilon_F \tau_{tr})(eB_{in}/m^*c)$ as small, $\omega_H < 1K$ (in units of temerature, see [10]), in the regime of temperature and disorder studied in experiments [10–12]. Here ϵ_F, τ_{tr} denote the Fermi energy and the transport relaxation time, m^* is the effective mass and B_{in} is the internal magnetic field (which is equal to the externally applied field normal to the film). Likewise, the effect of spin-orbit scattering on the phase shift may be estimated to be less than the effect of inelastic scattering in the temperature range $5K < T < 50K$ considered here. As shown in [13], the dominant inelastic phase breaking processes in ferromagnetic films are given by scattering of electrons off spin wave excitations. It may be shown that the phase relaxation rate $1/\tau_\phi$ in the limit of spin wave gap $\Delta << 1/\tau_\phi$ for Gd[15] in effectively $d = 3$ dimensions is given by

$$\left(\frac{1}{\tau_\phi}\right)_{3d} = \left[\frac{nJ^2}{\pi^2}\frac{D}{(D^2+A^2)\sqrt{A}}\right]^{2/3} T^{2/3}\,, \qquad (2.1)$$

where D is the diffusion coefficient, $n = k_F^3/3\pi^2$ is the electron density where k_F is the Fermi wave vector, J is the exchange coupling and A is the spin wave stiffness (spin wave dispersion $\omega_q = \Delta + Aq^2$). This is in stark contrast to the contribution to dephasing by the e-e-Coulomb-interaction[14], which is $(1/\tau_\phi)|_{3d,C} \propto T^{3/2}$. We may relate D to the sheet resistance R by $D = 1/(Rbe^2\frac{\partial n}{\partial \mu})$, where b is the film thickness, $(\partial n/\partial \mu)$ is the compressibility and we are using units with $\hbar = 1$. For Gd we estimate[15,16] the exchange energy $Jn \approx 80\ meV$, $k_F \approx 1\times 10^8\ cm^{-1}$, $(\partial n/\partial \mu) = (m^* k_F/\pi^2)$ with $m/m^* \approx 1/3$ and $A = nJk_F^{-2} \approx 1.1 \times 10^{-29} erg \cdot cm^2$. The diffusion coefficient is then given in terms of R, b, and the resistance quantum $R_Q \equiv (h/e^2) = 25.81k\Omega$ as

$$D = D_0 \frac{R_Q}{R}\frac{1}{bk_F}, \qquad (2.2)$$

where $D_0 \equiv k_F/(h\frac{\partial n}{\partial \mu}) \approx 0.6\ cm^2/s$. We thus find an estimate of the phase relaxation rate (in units of Kelvin) for a three-dimensional film of

$$\left(\frac{1}{\tau_\phi}\right)_{3d} \approx 1.3\, T_0 \left[\frac{R}{R_Q}\frac{bk_F}{1+A^2/D^2}\right]^{2/3}\left(\frac{T}{T_0}\right)^{2/3} \qquad (2.3)$$

where $T_0 = 1K$.

In effectively $d = 2$ dimensions the temperature power law changes to $T^{1/2}$,

$$\left(\frac{1}{\tau_\phi}\right)_{2d} = \left[\frac{nJ^2}{\pi^2}\frac{D}{(D^2+A^2)}\right]^{1/2} T^{1/2}. \qquad (2.4)$$

Again the temperature dependence of the phase relaxation rate caused by Coulomb interaction[14] is quite different, $(1/\tau_\phi)|_{2d,C} \propto T$.

The phase relaxation length (in cm) is obtained for an effectively three dimensional system as

$$\begin{aligned} L_\phi &= \sqrt{D\tau_\phi} \approx 1.8\times 10^{-6}\left[1+\left(\frac{A}{D}\right)^2\right] \\ &\times \left(\frac{R_Q}{R}\frac{1}{bk_F}\right)^{5/6}\left(\frac{T_0}{T}\right)^{1/3}. \end{aligned} \qquad (2.5)$$

It is seen that L_ϕ decreases with increasing resistivity $\varrho = Rb$ (disorder) and in general increases as the temperature is lowered. We will see later, however, that the temperature dependence is partially or completely compensated by the temperature dependence of the sheet resistance $R(T)$. The effective dimensionality of the film is determined from the condition $L_\phi = b$: for $L_\phi < b$ the system is effectively three-dimensional, otherwise it is two-dimensional. The boundary curve in the $T - R$ plane (at fixed thickness b) is given by

$$\frac{T}{T_0} = \left(\frac{17}{bk_F}\right)^{11/2}\left(\frac{R_Q}{R}\right)^{5/2}. \qquad (2.6)$$

Note that this is an implicit equation since $R = R(T)$. In Figure 1 we schematically show the boundary between these two regimes, for the case of Gd as studied in [11,12].

III. WEAK LOCALIZATION

In the quasi two-dimensional regime, i.e. when $L_\phi > b$, the weak localization correction to the conductivity takes

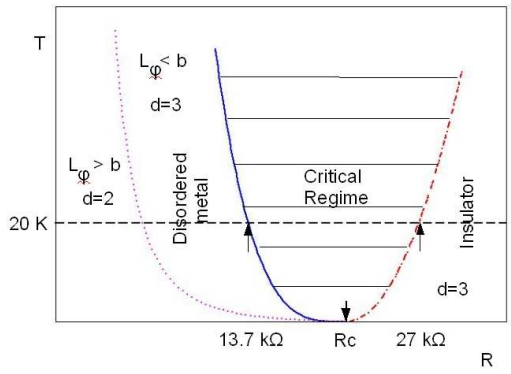

FIG. 1: (Color online) The critical regime and the boundary between two and three dimensionality of the system as given by the phase-breaking length scale $L_\phi = b$ shown by the dotted (magenta) line. The solid (blue) boundary line between the critical regime and the disordered metal is given by $T = (1/\tau)[(R_c - R)/R_c]^{\nu_- z}$, while the dash-dotted (red) line separating the critical regime and the insulator is given by $T = (1/\tau)[(R - R_c)/R_c]^{\nu_+ z}$. Note that as discussed after Eq. (4.5), L_ϕ eventually diverges near R_c for $T < T_x \approx 20mK$.

the form ($L_{00} \equiv (\pi R_Q)^{-1}$)

$$\begin{aligned} \delta\sigma_{2d} &= -L_{00} \ln \frac{\tau_\phi}{\tau_{tr}} \\ &= -\alpha L_{00} \ln \frac{T_0}{T} + const, \end{aligned} \tag{3.1}$$

where $\alpha = 1/2$ for contributions from spin-waves in 2d[13]. The experimental conductivity data in the regime $400\Omega < R(T_1) < 3k\Omega$ show a contribution $\delta\sigma = -\alpha L_{00} \ln \frac{T_0}{T}$, where $\alpha \approx 0.75$[11]. The difference between the theoretical and experimental coefficients may be due to a positive contribution to α from Coulomb interaction contribution[14]. Note that in the usual case of dephasing due to Coulomb interaction, $\alpha = 1$. As shown in [11], there is a further contribution to the conductivity linear in T, interpreted as a quantum correction caused by the effective e-e-interaction mediated by spin wave excitations.

In the quasi three-dimensional regime, at $L_\varphi < b$ but still outside the scaling regime, we expect a weak localization correction of the form

$$\begin{aligned} \delta\sigma &= c_3' L_{00} \left(\frac{\tau_{tr}}{\tau_\phi}\right)^{1/2} + const \\ &= c_3 \left(\frac{T}{T_0}\right)^{1/3} + const. \end{aligned} \tag{3.2}$$

IV. SCALING REGIME NEAR THE METAL-INSULATOR TRANSITION

Near the metal-insulator transition, the observables obey power laws in the disorder parameter or in frequency/temperature. We will use the sheet resistance $R_0 = R(T_0)$ at a reference temperature T_0 as the disorder parameter. Then, the dc conductivity $\sigma(R_0)$ follows a power law $\sigma \sim t^s$; here $t = (1 - R_0/R_c)$ where R_c is the resistivity at the critical point and s is the conductivity exponent. The dynamical conductivity at the critical point, on the other hand, is characterized by the dynamical exponent z as $\sigma(\omega; R_c) \sim \omega^{1/z}$. The zero temperature correlation length on the metallic side ($R < R_c$) diverges at the critical point as $\xi \sim t^{-\nu_-}$ and the localization length ($R > R_c$) diverges as $\xi \sim |t|^{-\nu_+}$. In $d = 3$ dimensions the relation $s = \nu_-$ holds. The exponents in $d = 3$ have not been calculated in a reliable way up to now. Note that while the critical exponents ν_- and ν_+ may in principle be different, all theoretical and experimental works so far have either assumed or observed $\nu_- = \nu_+$. In contrast, we have found that for ferromagnetic thin films, the two exponents are distinctly different, describing a very unusual asymmetric transition[12]. At finite temperature, the correlation length acquires the T-dependence $\xi^{-1}(R_0, T) = a|R_0 - R_c|^{\nu_\pm} + bT^{1/z}$.

The critical exponents can not be measured experimentally at the $T = 0$ critical point as a function of the control parameter t, but must be inferred from finite temperature measurements. Here the property of quantum phase transitions helps, to exhibit an extensive critical scaling regime, which reaches up in temperature into the experimentally accessible domain. In the scaling regime the only remaining length scale is the correlation length ξ. The conductivity, in units of the quantum of conductivity R_Q^{-1}, has dimension $(length)^{-1}$ and is therefore proportional to ξ^{-1}. Using that frequency ω and wavevector q scale as $\omega \propto q^z$, we find that temperature $T \propto \xi^{-z}$. The temperature dependent conductivity therefore obeys the scaling law

$$\sigma(T; R_0) = \xi^{-1} G(\pm 1, \xi T^{1/z}), \quad t \gtrless 0. \tag{4.1}$$

At the critical point, when $\xi \to \infty$, it follows that $G(\pm 1, \xi T^{1/z}) \sim T^{1/z}$. Away from the critical point, using $\xi(T = 0) \propto |R_0 - R_c|^{-\nu_\pm}$ where $\nu_\pm$ is the correlation length exponent at $R_0 \gtrless R_c$, the conductivity should obey the scaling,

$$\sigma(T; R_0) = |R_0 - R_c|^{\nu_\pm} G_1(\pm 1, |R_0 - R_c|^{-\nu_\pm} T^{1/z}). \tag{4.2}$$

By plotting the conductivity data in that way the data were found to collapse onto two curves for $R_0 \gtrless R_c$. The values of the critical exponents found are $z = 2.56$, $s = \nu_- = 1.38$ and $\nu_+ = 0.77$. In the regime considered, the scaling functions $G_1(\pm 1, |R_0 - R_c|^{-\nu_\pm} T^{1/z})$ turned out to be straight lines, $G_1(\pm 1, y) \approx G_1(\pm 1, 0) + By + \widetilde{G}_1(y)$, with $\widetilde{G}_1(y)$ negligible for $0 \leqslant y \lesssim y_h$ where y_h is the maximum value obtained in the experiment. The constant $G_1(\pm 1, 0)$ is positive (negative) at $R_0 < R_c$ ($R_0 > R_c$). A negative constant is not in conflict with the requirement of positivity of σ, as the critical regime is restricted to the domain $T > \xi^{-z}$ (in appropriate units of elastic relaxation rate and mean free path). In Figure 2 we show

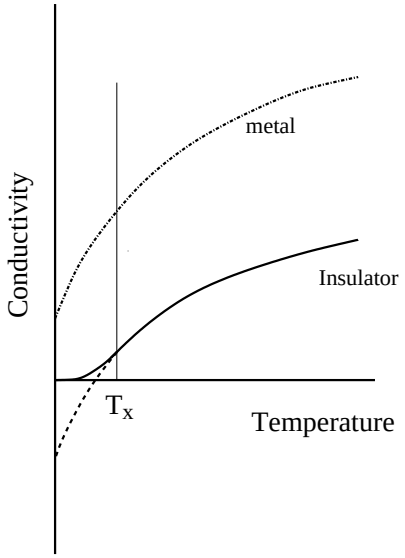

FIG. 2: Schematic figure showing the temperature T dependence of conductivity near the metal-insulator transition. Note that a power law fit within the scaling regime ($T \gg T_X$) with an apparent negative intercept actually denotes an insulating phase with a non-power-law behavior outside the scaling regime at very low temperatures ($T \ll T_X$).

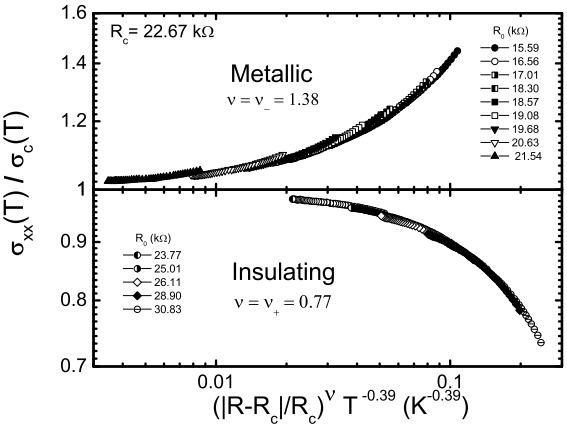

FIG. 3: Data from Ref. 12 re-plotted according to Eq. (4.3).

schematically how the T-dependence of σ is different below and above the transition. In the limit $R_0 \to R_c$ at finite T, the conductivity is expected to be a differentiable function of R_0, i.e. the derivatives on both sides of the transition have to be equal. The constraints this requirement puts on the form of the scaling function in this limit are more clearly seen using the equivalent form of the scaling relation

$$\sigma(T; R_0) = T^{1/z} G_2((R_0 - R_c)T^{-1/z\nu_\pm}). \qquad (4.3)$$

In order that all derivatives of σ be continuous at R_c there is only one possibility: all derivatives must vanish, $\lim_{R_0 \to R_c}(\partial^n \sigma / \partial R_0^n) = 0$. In other words, the scaling function $G_2(x)$ must have an essential singularity at $x = 0$. In Figure 3 we show a scaling plot of the data of [12] in the latter form. It turns out that in this way of plotting, which weighs the deviations differently, the fit might be improved on the metallic side by taking a slightly smaller value of ν_-, between 1.2 and 1.28. The best fit for ν_+ on the insulating side remains unchanged.

It is interesting to note that taking the value of $z \approx 2.5$ obtained from the data, and the corresponding temperature dependence of the resistivity at the critical point, $R_c(T)/R_Q = [R_c(T_0)/R_Q](T/T_0)^{-p} \approx 1.7(T/T_0)^{-0.4}$ (where we used $R_c(5K) \approx 22.67k\Omega$ from experiment and calculated $R_c(T_0 = 1K) = R_c(5K) \times (1/5)^{-p} \approx 43.16k\Omega$), one finds that the temperature dependence drops out of the expression for the phase relaxation length given in (2.5):

$$L_\phi \times k_F \approx 1.17 \times 10^2 \times \left(\frac{1}{bk_F}\right)^{5/6} \qquad (4.4)$$

as long as $A/D \ll 1$ and can be neglected in (2.5). The condition $L_\phi < b$ guaranteeing effective three-dimensionality reduces then to $bk_F \gtrsim 13.4$, or $b > b_3 \approx 1.3\ nm$, which is independent of temperature (see Figure 1). However, at very low temperatures, when $D \to 0$, the factor containing A/D in (2.5) diverges, leading to a divergent phase-breaking length

$$\begin{aligned} L_\phi \times k_F &\approx 1.8 \times 10^2 \times \left(\frac{A}{D}\right)^{2/3} \left(\frac{D}{D_0}\right)^{5/6} \left(\frac{T_0}{T}\right)^{1/3} \\ &\approx 11.7 \times \left(\frac{1}{bk_F}\right)^{1/6} \left(\frac{T_0}{T}\right)^{4/15}. \end{aligned} \qquad (4.5)$$

Then the condition for crossover to two-dimensionality, or $L_\phi > b$ for a fixed b, becomes $T/T_0 < T_x = 20mK$. This implies that once the condition $b > b_3$ is satisfied, the $2d - 3d$ crossover line falls down to T_x, which is very low but still finite.

V. CONCLUSION

In this paper we extended the theoretical interpretation of the experimental data on the conductivity of disordered ferromagnetic films of Gadolinium reported in [12]. We presented a phase diagram in the temperature-disorder plane (Fig.1) showing the crossover from two-dimensional to three-dimensional behavior for increasing disorder at given temperature and the boundary of the critical regime near the metal-insulator transition. We reviewed the results on the phase relaxation rate induced by scattering of electrons off spin wave excitations derived in [13]. A first consequence of these results is a

revised interpretation of the weak localization contribution to the conductivity in the effectively $2d$ - regime discussed in [11]. We predict a new power law dependence of the weak localization correction in the effectively $3d$ - regime, which has not been observed yet. Closer to the metal-insulator transition, in the critical regime, we review the scaling behavior of the conductivity as observed in experiment. The highly unusual finding of two different critical exponents of the correlation length on both sides of the transition would have dramatic consequences for a single parameter (ξ) scaling theory. It would require the conductivity as a function of R_0 to have an essential singularity at $R_0 = R_c$ for nonzero temperature.

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It is with great pleasure that we dedicate this paper to Costas Soukoulis on the occasion of his 60th birthday, whose work significantly advanced our understanding of electron transport in disordered systems and of wave propagation in photonic crystals and metamaterials. We wish him many more years of activity in physics.

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