PHY3054 - Exam 1 - Solutions - Fall 1994 - Selman Hershfield

1. The direction of the force is determined using opposite charges attract and like charges repel. The magnitude of the force is determined using Coulomb's law.

$$
\begin{aligned}
\text{F on } Q_3 \text{ by } Q_1 &= \frac{k|+2\times10^{-9}C||+8\times10^{-9}C|}{(2cm)^2} \text{ in +x direction} = +3.6\times10^{-4}N \\
\text{F on } Q_3 \text{ by } Q_2 &= \frac{k|+2\times10^{-9}C||-1\times10^{-9}C|}{(1cm)^2} \text{ in –x direction} = -1.8\times10^{-4}N \\
\Rightarrow \text{F net on } Q_3 &= \mathbf{+1.8\times10^{-4}N}
\end{aligned}
$$

2. The electron is attracted to the top $1C$ charge (force at 90 degrees), attracted to the right $1C$ charge (force at 330 degrees), and repelled from the left $-1C$ charge (force at 30 degrees). The magnitudes of each of these forces is the same because the separation, r, and magnitudes of the charges are the same. Drawing a vector diagram shows that the net force on the electron is away from the $-1C$ charge (**30 degrees**).

3. To do this problem, we add the forces as vectors using Coulomb's law to determine the magnitude. The directions are determined by opposite charges attract and like charges repel. The distance of each charge to the center is determined by

$$\cos(30) = \frac{0.5m}{r} \Rightarrow r = 0.577m.$$

Thus, the magnitude of each of the three forces is

$$|F_1| = \frac{k|e||1C|}{r^2} = 4.3\times10^{-9}N,$$

and the net force on the electron is

Force	x component	y component
F on e by top charge	0	$+\lvert F_1\rvert$
F on e by left charge	$+\lvert F_1\rvert\cos(30)$	$+\lvert F_1\rvert\sin(30)$
F on e by right charge	$+\lvert F_1\rvert\cos(30)$	$-\lvert F_1\rvert\sin(30)$
net F on e	$+2\cos(30)\lvert F_1\rvert$	$+\lvert F_1\rvert$

The magnitude of the net force is

$$|F| = \sqrt{F_x^2+F_y^2} = 2|F_1| = \mathbf{8.6\times10^{-9}N}.$$

4. The force on the electron is

$$F = \frac{mv^2}{r} = \frac{ke^2}{r^2} \Rightarrow v = \sqrt{\frac{ke^2}{mr}} = \mathbf{2.2\times10^6 m/s}.$$

5. So long as nothing touches the electroscope it will remain neutral. The negative rod repels negative charges, leaving the ball at the top of the electroscope positively charged and the foil negatively charged. **The overall charge on the electroscope is zero, and the foil is negatively charged.**

6. This problem relies on the basic definition of the electric field, $E = F/q$. Since the force on the second charge is in the opposite direction to the first charge, the two charges have opposite sign. $\Rightarrow$ The second charge is negative. Since the magnitude of the force is half as much, the second charge is half as big. $\Rightarrow$ **Q = -0.5 micro-C**

7. Electric field lines come from positive charges and end at negative charges. They do not go in circles as in Fig. 3.1. **Fig. 3.1 is incorrect.**

8. This is a problem involving constant acceleration:

$$a = \frac{F}{m_e} = \frac{eE}{m_e} = 7.0 \times 10^{14} m/s^2.$$

Since the electron starts from rest, the distance it travels is given by

$$d = \frac{1}{2}at^2 = \frac{1}{2}(7.0 \times 10^{14} m/s^2)(2 \times 10^-8 s)^2 = 0.14m = \mathbf{14cm}.$$

9. The electric field can only be zero where the fields due to Q_1 and Q_2 point in opposite directions. This occurs both for $x < 0\,m$ and for $x > 1.6m$; however, since Q_2 is greater in magnitude, one must be further away from it in order to have the magnitudes of the fields due to Q_1 and Q_2 be equal. This means the field is zero in the region $x < 0\,m$. Setting the magnitudes equal,

$$\begin{aligned} \frac{k|Q_1|}{(-x)^2} &= \frac{k|Q_2|}{(-x+1.6m)^2} \\ \Rightarrow (-x+1.6m)^2 &= 2x^2 \\ \Rightarrow -x+1.6m &= \pm\sqrt{2}x \\ \Rightarrow x &= -3.86m \text{ or } +0.66m, \end{aligned}$$

gives two possible solutions. Only the first one has $x < 0$: **x = -4m**.

10. Charges reside only on the surface of a conductor so the total charge on the middle shell is the sum of the charges on its inside and outside surfaces.

    If we draw a Gaussian surface inside the middle shell, then the net charge enclosed is zero because the electric field is zero inside a conductor. The charge enclosed is the charge on the sphere ($+1C$) and the charge on the inner surface of the middle shell (unknown). $\Rightarrow$ The charge on the inner surface of the middle shell is $-1C$.

    Similarly, if we draw a Gaussian surface inside the outer shell, the net charge enclosed is zero because the electric field is zero inside the conductor. The enclosed charges are the sphere ($+1C$), the inside surface of the middle shell ($-1C$), the outside surface of the middle shell (unknown), and the inside surface of the outer shell ($-1C$). Consequently, the charge enclosed on the outside surface of the middle shell is $+1C$.

The sum of the charges on the inside and outside surfaces of the middle shell is $-1C + 1C = \mathbf{0C}$.

11. This is a conservation of energy problem. The initial kinetic energy of the electron is zero. The initial and final potential energy of the electron are:

$$\begin{aligned} U_i &= \frac{kQ_eQ_1}{2m} + \frac{kQ_eQ_2}{1m} = 0J \\ U_f &= \frac{kQ_eQ_1}{3m} + \frac{kQ_eQ_2}{2m} = -2.88 \times 10^{-20} J. \end{aligned}$$

Consequently, the final kinetic energy is

$$\frac{1}{2} m_e v_f^2 = K_f = K_i + U_i - U_f = +2.88 \times 10^{20} J,$$

and the final velocity of the electron is $\mathbf{2.5 \times 10^5 m/s}$.

12. The force on the charge is $qE = 0.5N$ upwards. The work done by the field is the force times the distance traveled parallel to the force, which is $1m$ from the picture. Thus, the work done by the field is $0.5N \times 1m = \mathbf{0.5J}$.

13. The potential energy of the charge configuration is the sum of the potential energies due to each pair of point charges. In this problem there are three pairs:

$$U = \frac{kQ_1Q_2}{1m} + \frac{kQ_1Q_3}{2m} + \frac{kQ_2Q_3}{1m} = \mathbf{0.36J}.$$

14. The electric field goes away from the positive charges and towards the negative charges. Drawing a vector diagram for the electric field shows that the fields due to the diagonal point charges cancel except for the left most and right most charges. These charges produce a field going to the right. The electrical potential (voltage) is the sum of the potentials, $V = kq/r$, for each point charge. Since $|q|$ and r are the same for each point charge and because there are equal numbers of positive and negative point charges, the net electrical potential is zero. **V is zero, and E is not zero.**

15. The electrons are attracted to the positive plates and repelled from the negative plates. Thus, in order to focus the beam on the upper right hand corner of the screen, **positively charge the right horizontal deflection plate, and positively charge the upper vertical deflection plate.**

16. Since the charge on the $2\mu F$ capacitor is $2\mu C$, the voltage across it is $(2\mu C)/(2\mu F) = 1V$. Since capacitors in parallel have the same voltage, this is also the voltage across the $4\mu F$ capacitor and across the combined $1\mu F$ and $3\mu F$ capacitors. The effective capacitance of the $1\mu F$ and $3\mu F$ capacitors in series is $0.75\mu F$. Because capacitors in series have the same charge, the charge on the $1\mu F$ capacitor is the same as the charge on the $0.75\mu F$ effective capacitor, which is $(0.75\mu F)(1V) = \mathbf{0.75V}$.

17. Because capacitors in parallel have the same voltage drop, the $2\mu F$ capacitor has the same voltage as the $4\mu F$ capacitor: **1V**.

18. The energy stored in a capacitor is $U = 0.5\,Q^2/C$. Since the charge remains unchanged, the energy stored is inversely proportional to the capacitance, C. When the dielectric constant goes from $K = 1$ (air) to $K = 2$, the capacitance goes up by a factor of 2, and consequently, **the energy will go down by a factor of 2.**

19. The effective capacitance of the top and bottom branches are

$$\begin{aligned} \text{C of top branch} &= \left(\frac{1}{6\mu F} + \frac{1}{2\mu F}\right)^{-1} = 1.5\mu F \\ \text{C of bottom branch} &= \left(\frac{1}{4\mu F} + \frac{1}{2\mu F}\right)^{-1} = 1.33\mu F \end{aligned}$$

Adding the two branches in parallel gives $2.83\mu F \approx \mathbf{2.8\mu F}$.

20. All three circuits look the same except for the placement of the bottom $1F$ capacitor. In Figs. 10.1 and 10.2 this $1F$ capacitor is in parallel with the $4F$ capacitor; however, in Fig. 10.3 the $1F$ capacitor is in parallel with the $3F$ capacitor. **Figs. 10.1 and 10.2 are the same, and Fig. 10.3 is different.**