PHY3054 - Exam 2 - Solutions - Fall 1994 - Selman Hershfield

1. The resistance of a wire is $R = \rho l/A$, where ρ is the resistivity, l is the length of the wire, and $A = \pi r^2$ is the cross-sectional area. We are given: $5\Omega = \rho_{Cu} l_{Cu}/A_{Cu}$, $\rho_{Ag} = 0.85\rho_{Cu}$, $l_{Ag} = 3l_{Cu}$, and $d_{Ag} = 2d_{Cu}$, which also means $r_{Ag} = 2r_{Cu}$. Thus, the resistance of the silver wire is:

$$R = \frac{\rho_{Ag} l_{Ag}}{\pi r_{Ag}^2} = \frac{(0.85\rho_{Cu})(3l_{Cu})}{\pi(2r_{Cu})^2} = \frac{(0.6375)\rho_C u l_{Cu}}{\pi r_{Cu}^2} = (0.6375)(5\Omega) = \mathbf{3.2\Omega}.$$

2. The 12 V battery can be modeled by an ideal 12 V voltage source and a 5 Ω resistor connected in series. When the switch is open, no current flows through the circuit. Also, no current flows through an ideal voltmeter, which has infinite resistance. Since no current is flowing, there is no voltage drop across the resistors. The voltmeter measures the full 12V of the battery. When the switch is closed, current will flow. The current is

$$I = \frac{V}{R_{total}} = \frac{12V}{(5+15+16)\Omega} = \frac{1}{3}A.$$

The voltmeter measures the voltage drop across the 15 Ω resistor, which is

$$V = IR = \frac{1}{3}A(15\Omega) = 5V.$$

Thus, the difference between the voltmeter readings in the two cases is

$$12V - 5V \quad = \quad \mathbf{7V}.$$

3. The net voltage drop due to the batteries is $0V$ so no current flows. **0A**

4. The current through the 1Ω resistor in Fig. 3 is denoted by I. This is also the current through the 4Ω and 6Ω resistors combined in parallel, which have an effective resistance of 2.4Ω. Thus, the voltage drop across each of the resistors in parallel is $(I\,2.4\Omega)$. The current through the 4Ω resistor is $(I\,2.4\Omega)/4\Omega = 0.6I$. The current through the 6Ω resistor is $(I\,2.4\Omega)/6\Omega = 0.4I$. $\Rightarrow$ The power dissipated by each of the resistors is

$$\begin{aligned} \text{P by } 1\Omega &= (I)^2 1\Omega = I^2 \\ \text{P by } 4\Omega &= (0.6I)^2 4\Omega = 1.44\, I^2 \\ \text{P by } 6\Omega &= (0.4I)^2 6\Omega = 0.96\, I^2. \end{aligned}$$

The 4Ω resistor dissipates the most power, and the 6Ω resistor dissipates the least.

5. The first resistor has a resistance of

$$R_1 = \frac{V}{I} = \frac{3V}{1A} = 3\Omega.$$

The second resistor has a resistance of

$$R_2 = \frac{V}{I} = \frac{3V}{2A} = 1.5\Omega.$$

The series combination of the two resistors is

$$R_1 + R_2 = 4.5\Omega,$$

which when connected across a 3V battery will draw a current of

$$I = \frac{V}{R} = \frac{3V}{4.5\Omega} = \mathbf{0.67A}.$$

6. **Done correctly.**

7. The 5 Ω and 10 Ω resistors in series at the top add to become a 15 Ω resistor. Combining the resulting 15 Ω resistor in parallel with the 10 Ω resistor gives

$$\left(\frac{1}{15\Omega} + \frac{1}{10\Omega}\right)^{-1} = 6\Omega.$$

When this is combined in series with the 4 Ω resistor, we get the resistance of the top branch is 10 Ω.

The equivalent resistance of the 10 Ω and 15 Ω resistors at the bottom is

$$\left(\frac{1}{10\Omega} + \frac{1}{10\Omega}\right)^{-1} = 5\Omega.$$

Adding this effective resistor in series with the 5 Ω resistor yields 10 Ω for the net resistance of the bottom branch.

The top and bottom branches combined in parallel give

$$\left(\frac{1}{10\Omega} + \frac{1}{10\Omega}\right)^{-1} = \mathbf{5\Omega}.$$

8. Label the current in the left loop as I_1 going clockwise, and the current in the right loop as I_2 going counterclockwise. Because of the point rule, their sum is: $I = I_1 + I_2$. Applying the loop rule to the left loop going clockwise yields

$$\begin{aligned} 0 &= 2V - (2\Omega)I_1 - (2\Omega)(I_1 + I_2) - (2\Omega)I_1 \\ &= 2V - (6\Omega)I_1 - (2\Omega)I_2. \end{aligned}$$

Applying the loop rule to the right loop going counterclockwise yields

$$\begin{aligned} 0 &= 2V - (1\Omega)I_2 - (2\Omega)(I_1 + I_2) - (1\Omega)I_2 \\ &= 2V - (2\Omega)I_1 - (4\Omega)I_2. \end{aligned}$$

There are a number of ways to solve these two equations. One shortcut in this case is to subtract the two equations:

$$\begin{aligned} 0 &= -(4\Omega)I_1 + (2\Omega)I_2 \\ I_2 &= 2I_1. \end{aligned}$$

Substituting back into either of our original equations yields:

$$\begin{aligned} I_1 &= 0.2A \\ I_2 &= 0.4A \\ I &= \mathbf{0.6A}. \end{aligned}$$

9. This problem only asks for the magnitude of the voltage difference between A and B, $|V_A - V_B|$. To determine $V_A - V_B$, compute the change in the voltage starting at B and going to A. We go first through the right $2V$ battery and then through the top 1Ω and 2Ω resistors:

$$+2V - (1\Omega)I_2 + (2\Omega)I_1 = +2V - 0.4 + 0.4 = \mathbf{2V}.$$

10. A long time after the switch is closed, the charge Q on the capacitor does not change with time. This means that the current, which is the rate of change of the charge, is zero. Consequently, the following is not true: **A long time after the switch is closed the current is V/R.**

11. To determine the direction of rotation, suppose that the positively charged particle is moving upwards. The right-hand rule says that if B is into the page (fingers) and the velocity of the positively charge particle is up (thumb), then the force will be to the particle's left resulting in counterclockwise motion. Rotating your hand so that your thumb points to the right, the force on the wire will be upward. The correct answer is **(i) Counter-clockwise, (ii) towards the top of the page.**

12. With the way the x and z axes are oriented, the positive y axis is into the page. The torque is equal to

$$\tau = IAB\sin(\theta),$$

where θ is the angle between the magnetic moment of the loop and the magnetic field. $\sin(\theta)$ is a maximum when $\theta = 90^0$, so the torque is a maximum when the magnetic moment of the loop is perpendicular to the magnetic field. This means the magnetic field can be either in the $+$ or $-$ x direction, or in the $+$ or $-$ z direction. The potential energy of the loop is equal to

$$U = -IAB\cos(\theta),$$

where θ is the same as before. The potential energy will be a maximum when $\cos(\theta)$ is a minimum: $\theta = 180^0$. Thus, the magnetic moment must be antiparallel to the magnetic field. Since the loop's magnetic moment is in the $-y$ direction, the magnetic field must point in the $+y$ direction. The correct answer is **The torque is a maximum with B in the x direction, and the energy is a maximum with B in the y direction.**

13. The magnetic force on a current carrying wire is given by $F = IlB\sin(\theta)$. We are given the force per unit length, F, but we are not given the angle θ. **Not enough information given.**

14. The time to complete an orbit is the distance traveled in one orbit, $2\pi r$, divided by the speed of the electron, v.

$$T = \frac{2\pi r}{v}.$$

Since the electron is moving in a circular orbit perpendicular to a magnetic field,

$$F = \frac{mv^2}{r} = qvB \rightarrow \frac{v}{r} = \frac{eB}{m}.$$

Thus, the time to complete an orbit is

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{eB} = \mathbf{7 \times 10^{-12}s}.$$

15. A charged particle moving perpendicular to a magnetic field will travel in a circle. The force on the particle is $F = qvB\sin(\theta) = qvB$. Since we know $|q| = 1.6 \times 10^{-19}C$ and $v = 10^5 m/s$, we need to find F in order to find B. Furthermore, since $F = ma$ and $m_e = 9.11 \times 10^{-31}kg$, if we can find a, we'll be able to solve for B.

Although the electron is moving in a circle, it is only deflected by $1cm$ in traveling a distance of $1m$. Thus, if the electron is initially traveling in the x-direction, it will only move a small amount in the y-direction and it is a good approximation to assume that the acceleration is only in the y-direction.

$$\begin{aligned} \Delta x &= 1m = vt \\ \Delta y &= 0.01m = \frac{1}{2}at^2 \end{aligned}$$

These two equations can be solved for the acceleration

$$a = \frac{2 \times 0.01m}{t^2} = \frac{2 \times 0.01m}{(1m/v)^2} = 2 \times 10^8 m/s^2.$$

This means that the force is $F = m_e a = 18.11 \times 10^{-23}N$ and the magnetic field is

$$B = \frac{F}{qv} = \mathbf{10^{-8}T}.$$

16. Fig. 8.2 is incorrect because the magnetic field goes in closed loops. Fig. 8.3 is incorrect because field lines go from the N to the S pole of a magnet. Thus, the correct answer is **Figs. 8.2 and 8.3**

17. Because P is equidistant from all of the wires and the wires have the same current, the magnitude of the four contributions to the magnetic field will be the same. The directions of the fields at P can be determined by the right hand rule where your thumb points in the direction of the current and your fingers wrap around the wire (B direction).

wire	angle of B at P
top left	225
top right	135
bottom left	135
bottom right	225

Adding these fields as vectors, the net B field is at $\mathbf{180^0}$.

18. The B field at Q due to the $2A$ wire is

$$B = \frac{\mu_o I}{2\pi r} = \frac{4\pi \times 10^{-7} Wb \cdot A^{-1} m^{-1} (2A)}{2\pi (0.001m)} = 4 \times 10^{-4} T \text{ out of the page.}$$

The B field at Q due to the current loop is

$$B = \frac{N\mu_o I}{2R} = \frac{4\pi \times 10^{-7} Wb \cdot A^{-1} m^{-1} (3A)}{2(0.001m)} = 1.88 \times 10^{-3} T \text{ into the page.}$$

Using the superposition principle, the net field into the page is

$$-4 \times 10^{-4} T + 1.88 \times 10^{-3} T = \mathbf{1.5 \times 10^{-3} T \text{ into the page}}.$$

19. The top wire is attracted to the middle wire and repelled from the bottom wire. The net force per unit length on it is

$$\frac{F}{l} = \frac{\mu_o I^2}{2\pi d} \text{ down } + \frac{\mu_o I^2}{2\pi (2d)} \text{ up } = \frac{\boldsymbol{\mu_o I^2}}{\mathbf{4\pi d}} \textbf{down}.$$

20. The force due to the top and bottom segments of the rectangular loop will cancel out. The left segment of the rectangular loop is repelled from the wire, and the right segment of the rectangular loop is attracted to the wire. Since the left segment is closer to the wire, the force on it is larger. Consequently, the net force is repulsive. **The loop feels a force away from the long wire.**