PHY3054 - Exam 2 - Solutions - Fall 1995 - Selman Hershfield

1. The resistance of a wire is

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2} = \frac{\rho l}{\pi (d/2)^2} = \frac{4\rho l}{\pi d^2}.$$

Since the wires are all made of the same material, the resistivity, ρ, is the same. Therefore, we only have to compare l/d^2 for the different wires.

$$\begin{aligned}
\frac{l}{d^2} \text{ for Wire \#1} &= \frac{1m}{(0.001m)^2} = 1 \times 10^6 m^{-1} \\
\frac{l}{d^2} \text{ for Wire \#2} &= \frac{2m}{(0.002m)^2} = 5 \times 10^5 m^{-1} \\
\frac{l}{d^2} \text{ for Wire \#3} &= \frac{2m}{(0.001m)^2} = 2 \times 10^6 m^{-1} \\
\frac{l}{d^2} \text{ for Wire \#4} &= \frac{1m}{(0.0005m)^2} = 4 \times 10^6 m^{-1}
\end{aligned}$$

Wire 4 has the largest resistance.

2. The parallel combination of the 2Ω resistors is

$$\left(\frac{1}{2\Omega} + \frac{1}{2\Omega}\right)^{-1} = 1\Omega.$$

Placing this in series with the two 1Ω resistors produces an equivalent resistance of 3Ω for the top branch of the circuit. The remaining parallel combination with the 1.5Ω resistor gives an equivalent resistance of

$$\left(\frac{1}{3\Omega} + \frac{1}{1.5\Omega}\right)^{-1} = \mathbf{1\Omega}.$$

3. To find the current through the 1Ω resistor, we need to know the voltage across it, which is also the voltage across the entire circuit. Since we know the current going through the entire circuit (1A), we can calculate the voltage by using the equivalent resistance of the circuit. Using $R_{eff} = 0.8\Omega$, the voltage across the circuit is $V = (1A)(0.8\Omega) = 0.8V$ and the current through the 1Ω resistor is **0.8A**.

4. All of the bulbs have the same resistance R. Bulbs A, B, and C are in parallel. They each have voltage V, and current V/R. Bulbs D and E are in series. The current through each of them is $V/(2R)$. Since for identical bulbs, the brightness is proportional to the current, the correct answer is **A, B, and C** are the brightest.

5. Adding resistors in series increases the effective resistance, and adding resistors in parallel decreases their effective resistance. The smallest resistance is obtained adding the resistors in parallel:

$$\left(\frac{1}{1\Omega} + \frac{1}{3\Omega} + \frac{1}{6\Omega}\right)^{-1} = \frac{2}{3}\Omega = \mathbf{0.67\Omega}.$$

6. Use the loop rule starting at the 100Ω resistor on the left and going in the direction of I:

$$\begin{aligned} 0 &= -(100\Omega)I - 1.5V - (100\Omega)I - (100\Omega)I + 1.5V + 1.5V \\ 0 &= -(300\Omega)I + 1.5V \rightarrow \mathbf{5mA}. \end{aligned}$$

7. Let I_1 be clockwise in the left loop, and I_2 be counterclockwise in the right loop. Applying the loop rule in the left loop starting at point A and going clockwise yields

$$\begin{aligned} 0 &= -(2\Omega)I_1 - (1\Omega)I_3 - 2V + 1V \\ 0 &= -(3\Omega)I_1 - (1\Omega)I_2 - 1V. \end{aligned}$$

Using the right loop starting at point B and going counterclockwise gives

$$\begin{aligned} 0 &= -(2\Omega)I_2 - (1\Omega)I_3 - 2V + 3V - (1\Omega)I_2 \\ 0 &= -(4\Omega)I_2 - (1\Omega)I_1 + 1V \rightarrow I_2 = -0.25I_1 + 0.25A. \end{aligned}$$

Substituting this expression for I_2 into the first equation,

$$0 = -(3\Omega)I_1 - (1\Omega)(-0.25I_1 + 0.25V) - 1V \rightarrow I_1 = \mathbf{-0.45A}.$$

8. We know from above that $I_1 = -0.45A$ and $I_2 = -0.25(-0.45A) + 0.25A = 0.3625A$. To find $V_A - V_B$ compute the change in voltage starting at B and going to A.

$$-(2\Omega)I_2 + (2\Omega)I_1 = -1.625V \approx \mathbf{-1.64V}.$$

9. The real batteries can be represented as ideal batteries in series with their internal resistances. Thus, the current through the circuit is

$$I = \frac{V}{R_{net}} = \frac{3V}{120\Omega} = 25mA.$$

The energy dissipated through the 100Ω resistor is $P = (25mA)^2(100\Omega) = 62.5mV$. Each battery supplies half of this power (31.25mV) because they are identical. Each battery also dissipates $P = (25mA)^2(10\Omega) = 6.25mW$ internally because of the internal resistance. However, when a current is flowing, the voltage across each battery is $1.5V - (25mA)(10\Omega) = 1.25V$. **Statement 4 is not true: The voltage across each battery is 1.5V while the current is flowing.**

10. A voltmeter measures the potential difference between two points in a circuit, so Fig. 7.2 is correct. The voltmeter should have a large resistance because if it were low, current would flow through the meter and the voltage across the resistor, $V = IR$, would be changed. The correct answer is **(i) Fig. 7.2 (ii) large resistance**.

11. **Done correctly.**

12. (i) Using the right hand rule with the thumb in the direction of I and fingers in the direction of B, the force (palm) faces into the page. (ii) Since $F = qvB\sin(\theta)$ and $\theta = 0^o$ (v is parallel to B), the force is zero. Thus, the correct answer is **(i) into the page (ii) no force on the electron.**

13. Magnetic field lines go from the N pole to the S pole of a magnet. Since the compass needle lines up with the magnetic field lines, the correct answer is **(i) north (ii) south** .

14. For a charged particle traveling perpendicular to a magnetic field the magnetic field can be determined by
$$qvB = \frac{mv^2}{r} \to B = \frac{mv}{qr}.$$
Since the proton completes one circumference ($2\pi r$) in 1 millisecond, its velocity is
$$v = \frac{2\pi r}{0.001s}.$$
Substituting this into the first equation yields
$$B = \frac{2m\pi r}{qr(0.001s)} = \mathbf{0.66 \times 10^{-4} Tesla}.$$

15. The magnetic field of the large loop is out of the page at loop 1 and into the page at loop 2. The potential energy of the small loops is
$$U = -MB\cos(\theta).$$
U is minimized when $\cos(\theta) = 1$ or $\theta = 0^0$, so the magnetic moments of the loops must be parallel to the field they are in. The magnetic moment of loop 1 should be out of the page (counterclockwise current). The magnetic moment of loop 2 should be into the page (clockwise current). The correct answer is **Counterclockwise in loop 1 and clockwise in loop 2.**

16. Add the magnetic fields due to the loop and the wire.
$$\begin{aligned} B_{loop} &= \frac{\mu I}{2R} \text{ into page} \\ B_{wire} &= \frac{\mu I}{2\pi R} \text{ out of page} \\ B_{net} &= (1 - \frac{1}{\pi})\frac{\mu I}{2R} \text{ into page} = \mathbf{8.6 \times 10^{-7} T \text{ into the page}} \end{aligned}$$

17. Let B_i be the magnetic field at P due to the wire carrying i Amperes. Add the fields due to each wire:
$$B_1 = \frac{\mu_o(1A)}{2\pi(1m)} = 2 \times 10^{-7} T \text{ into the page}$$

$$
\begin{aligned}
B_2 &= \frac{\mu_o(2A)}{2\pi(1m)} = 4\times10^{-7}T \text{ out of the page} \\
B_3 &= \frac{\mu_o(3A)}{2\pi(2m)} = 3\times10^{-7}T \text{ out of the page} \\
B_4 &= \frac{\mu_o(4A)}{2\pi(2m)} = 4\times10^{-7}T \text{ out of the page} \\
B_{net} &= -|B_1| + |B_2| + |B_3| + |B_4| \text{ out of the page} \\
&= \mathbf{0.9\times10^{-6}\ Tesla\ out\ of\ the\ page}.
\end{aligned}
$$

18. The $2A$ wire is attracted to the $1A$ wire (force up) and attracted to the $3A$ wire (force down). Thus, the force per unit length on the $2A$ wire is

$$
\begin{aligned}
\frac{F}{l} &= \frac{\mu_o(2A)(1A)}{2\pi(1m)} \text{ up} + \frac{\mu_o(2A)(3A)}{2\pi(1m)} \text{ down} \\
&= \mathbf{8\times10^{-7}N/m\ down}.
\end{aligned}
$$

19. Since the force on one of the wires is zero, the field is parallel to that wire and is thus at a 40^0 angle with the other wire. Using $F = IlB\sin(\theta)$,

$$
B = \frac{F}{Il\sin(\theta)} = \frac{5N/m}{(2A)\sin(40^0)} = \mathbf{3.9\ Tesla}.
$$

20. The forces on the top and bottom of the loop are equal and opposite so they cancel. The forces on the left and right sides of the loops are opposite, but not equal since they are at different distances from the long wire. In particular the force of the left side is larger because it is closer to the long wire. Thus, the net force should be to the right since the wires with current in opposite directions repel. The net force is

$$
\begin{aligned}
F \text{ on left wire} &= \frac{(1A)(2m)\mu(1A)}{2\pi(0.5m)} = 8\times10^{-7}N \text{ to the right} \\
F \text{ on right wire} &= \frac{(1A)(2m)\mu(1A)}{2\pi(1m)} = 4\times10^{-7}N \text{ to the left} \\
F_{net} &= \mathbf{4\times10^{-7}N\ to\ the\ right}.
\end{aligned}
$$