PHY3054 - Exam 3 - Solutions - Fall 1996 - Selman Hershfield

1. This problem simply checks to see that you know what the speed of light is:

$$d = vt \Rightarrow t = \frac{3.85 \times 10^8 m}{3 \times 10^8 m/s} = 1.28s \approx \mathbf{1s}.$$

2. If you look at an AM/FM radio, you will find that the AM dial is in kilo-Hertz (KHz), which is $10^3 s^{-1}$, and the FM dial is in mega-Hertz (MHz), which is $10^6 s^{-1}$.

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{3.3m} = 90.9 \times 10^6 s^{-1} \approx 91MHz \text{ or } \mathbf{91FM}$$

3. When light at incoming polarization, θ_i, passes through a polarizer at angle, θ_p, the intensity is reduced by $\cos^2(\theta_p - \theta_i)$ and the outgoing light is at polarization θ_p. For this problem just multiply together the reduction in the intensity as the light passes through each polarizer:

$$I = I_0 \cos^2(0) \cos^2(60) \cos^2(70) = \mathbf{0.030 I_0}.$$

4. When two polarizers have polarization axes perpendicular to each other, no light gets through. By placing a third polarizer (sun glasses) in between the first two, you can have light pass through the three polarizers. This was a demonstration done in class. Suppose that the first polarizer is at 0^o, the second polarizer is at 90^o, and the sunglasses are at θ. Assuming that the incident light is unpolarized with intensity I_o, the intensity of the exiting light is

$$I = \frac{1}{2} I_0 \cos^2(\theta - 0) \cos^2(\theta - 90).$$

If you plot this, you will find that this has a maximum at 45^o, 135^o, 225^o, and 315^o. We also demonstrated this in class. The angle between adjacent maxima is $\mathbf{90^o}$.

5. According to Snell's Law,

$$\begin{aligned} n_A \sin\theta_A &= n_B \sin\theta_B \\ n_C \sin\theta_C &= n_B \sin\theta_B \\ \Rightarrow n_A \sin\theta_A &= n_B \sin\theta_B = n_C \sin\theta_C. \end{aligned}$$

Since these are all equal, the material with the largest index of refraction will have the smallest angle: $\theta_C < \theta_A < \theta_B$ implies $\mathbf{n_C > n_A > n_B}$.

6. The light ray is not bent at the second interface because the indices of refraction are the same ($n = 1.5$). This means that we only need to worry about the first interface. Light is incident at 29^o below the horizontal according to the little axes at the left hand side of the figure. The incident light makes an angle relative to the normal to the interface of $29 + 30 = 59$ degrees.

$$1.7 \sin(59^o) = 1.5 \sin\theta_2 \Rightarrow \theta_2 \approx 76^o.$$

The angle of 76^o is below the normal and hence makes an angle of $76 - 30 =$ **-46 degrees** relative to the horizontal.

7. The furthest distance the boater can see occurs when the light exits at 90^o relative to the normal into the air. This means that the angle, θ, of the incoming light is

$$1.4 \sin\theta = 1.0 \sin 90 \Rightarrow \theta = 45.58^o.$$

Since the distance to the bottom is 3 meters, the furthest horizontal distance is 3 m $\times$ $\tan(\theta) = 3.06$ m. The boater can see **A, B, and C**.

8. You are given that the shark *appears* to be 10 m away. Using the apparent depth equation, one can determine how far away the shark actually is:

$$\frac{10}{d} = \frac{d'}{d} = \frac{n_2}{n_1} = \frac{1}{1.33} \Rightarrow d = 13.3m.$$

Since $d - d' = 3.3m$, **the shark is 3.3m further away than you think.**

9. **Done correctly.**

10. The image is at **C**. It is real and inverted.

11. The focal length of the mirror is $f = -2$m because it is convex and the radius is 4 m. The object distance is 5 m. The lens equation,

$$-\frac{1}{2} = \frac{1}{5} + \frac{1}{d_i},$$

implies that $d_i = -1.4$ and $m = -d_i/d_o = 0.3$. Since $|m| < 1$, the image appears **smaller** than the object. Because $m > 0$, the image is **right-side-up**.

12. The large mirror at the left produces an image of the back of your head which is 1.5 m behind the mirror. This image is 3 m from you and 3.3 m from the smaller mirror. Consequently, the image of the back of your head in the small mirror is 3.3 m behind the small mirror which is **3.6 m** from you.

13. The image is at **B**. It is virtual and right-side-up.

14. From the figure you can see that the sum of the object distance (slide to lens) and image distance (lens to screen) is 5 m.

$$d_o + d_i = 5m$$

Also, the height of the object is 35 mm = 0.035 m, and the height of the image is -2 m ($h_o < 0$ because it is inverted). Consequently, the magnification is

$$m = \frac{-2}{0.035} = -\frac{d_i}{d_o} \Rightarrow d_i = 57d_o.$$

Substituting into the first equation, then gives $d_i = 0.086$m $\approx$ **85mm**.

15. From the figure we can see that the magnification is

$$m = \frac{3}{2} = -\frac{d_i}{d_o}.$$

Since the distance to the paper is $d_o = 10\text{cm}$, $d_i = m\, d_o = -15\text{cm}$. Using the lens equation then gives

$$\frac{1}{f} = \frac{1}{10cm} + \frac{1}{-15cm} \Rightarrow f = \mathbf{30cm}.$$

16. The distance from the converging lens to the object is $d_o = 14$ cm. Using the lens equation with $f = 7$ cm, the image produced by the first lens is at $d_i = 14$ cm, which is the same as $x = -3 + 14 = 11$ cm. This image is on the outgoing light side of the second lens at $x = 6$ cm. Consequently, $d_o = -5$ cm, and from the lens equation with $f = -11\text{cm}$, we find that $d_i = 9.17$ cm, which is the same as $x = 15.17 \approx$ **15 cm**.

17. The angular size of a person 1.8 m tall and 100 m away is

$$\theta = \frac{1.8m}{100m} = 0.018 \text{ radians} = 0.018 \times \frac{180}{\pi} \text{ degrees} \approx \mathbf{1degree}.$$

The angular magnification of a telescope is $M = -f_o/f_e = -1/0.4 = -2.5$. Thus, the angular size of the person when viewed through the telescope is **2.5 degrees**.

18. The object distance for the first lens is $d_o = 0.51$ cm. Since the focal length of the first lens is 0.5 cm, the image distance for the first lens is $d_i = 25.5$ cm, which is at $x = 25.5 - 10 = 15.5$ cm. This image is $d_o = 20 - 15.5 = 4.5$ cm from the second lens. Consequently, the image distance for the second lens is $d_i = -45$ cm. The net magnification is the product of the magnifications, $m = -d_i/d_o$, for the two lens.

$$m_{net} = \left(\frac{25.5 \text{ cm}}{0.51 \text{ cm}}\right)\left(\frac{-45 \text{ cm}}{4.5 \text{ cm}}\right) = \mathbf{-500}$$

19. The angular magnification depends for a microscope depends on the separation between the lenses, $L = 30$ cm, the focal lengths of the objective and eyepiece, and the near point, which is taken to be $N = 30$ cm here.

$$M = \frac{-(L - f_e)N}{f_e f_o} = -300 \Rightarrow |M| = \mathbf{300}$$

20. Since the person can see clearly objects at 1m, we want the glasses to bring objects in from $d_o = \infty$ to $d_i = -(1 - 0.005) = -0.995$ m. The image distance is negative because the image is not on the side of the outgoing light.

$$\frac{1}{f} = \frac{1}{\infty} + \frac{1}{-0.995 \text{ m}} \Rightarrow f = -0.995 \text{ m} \approx -1 \text{ m}$$

The minus sign tells us we need a **diverging lens with f = 1 m**.