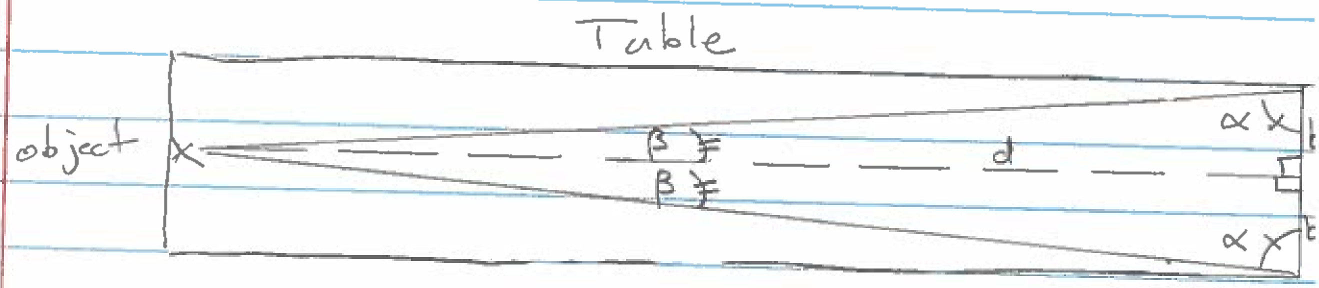
Math / Physics review

Rather than try to review everything, let's go back to a few places where math came up:

I. Parallax – how to measure astronomical distances

Lab:



We measured α and baseline b to deduce distance of object d. Strictly speaking, we need trigonometry:

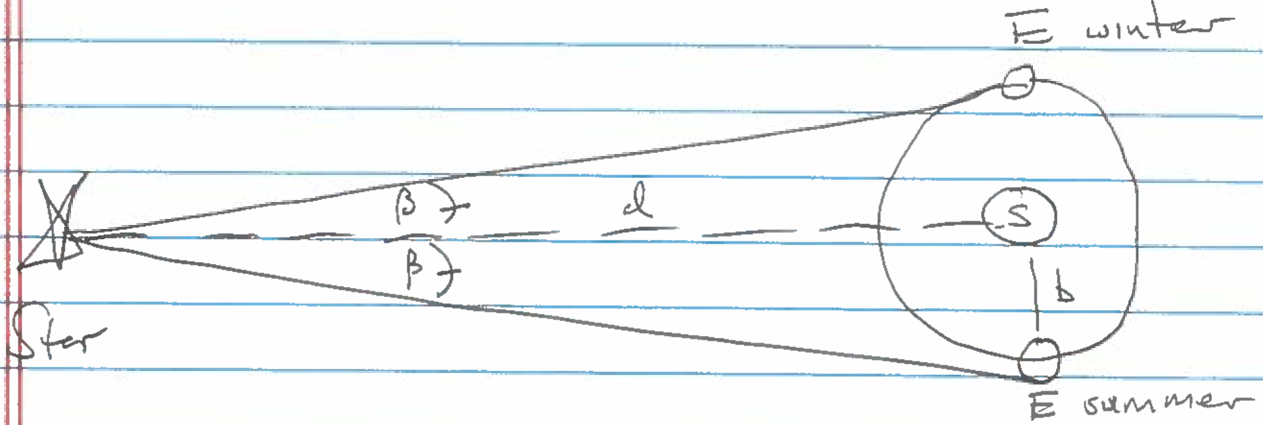
$$\boxed{\begin{aligned}\beta &= 90^\circ - \alpha \\ &= \tfrac{\pi}{2}\,\text{rad} - \alpha\end{aligned}}$$

$$\tan\beta = \frac{\text{opp}}{\text{adj}} = \frac{b}{d} \Rightarrow d = \frac{b}{\tan\beta}$$

but we can also get away with the small angle approximation. Since β is very small because $d \gg b$, if we measure it in radians we can say

$$\tan\beta \simeq \beta \quad \Rightarrow \quad d = b/\beta$$

This approx. is very good particularly for astronomy where things are far away.



Here's how to measure the distance to Alpha Centauri,

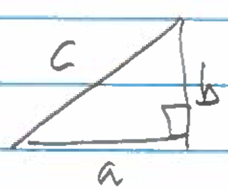
$$d \simeq b/\beta = \frac{9.3 \times 10^{7} \text{ miles}}{3.6 \times 10^{-6} \text{ radians}}$$

$$= 2.6 \times 10^{13} \text{ miles} = 4 \text{ light-years}$$

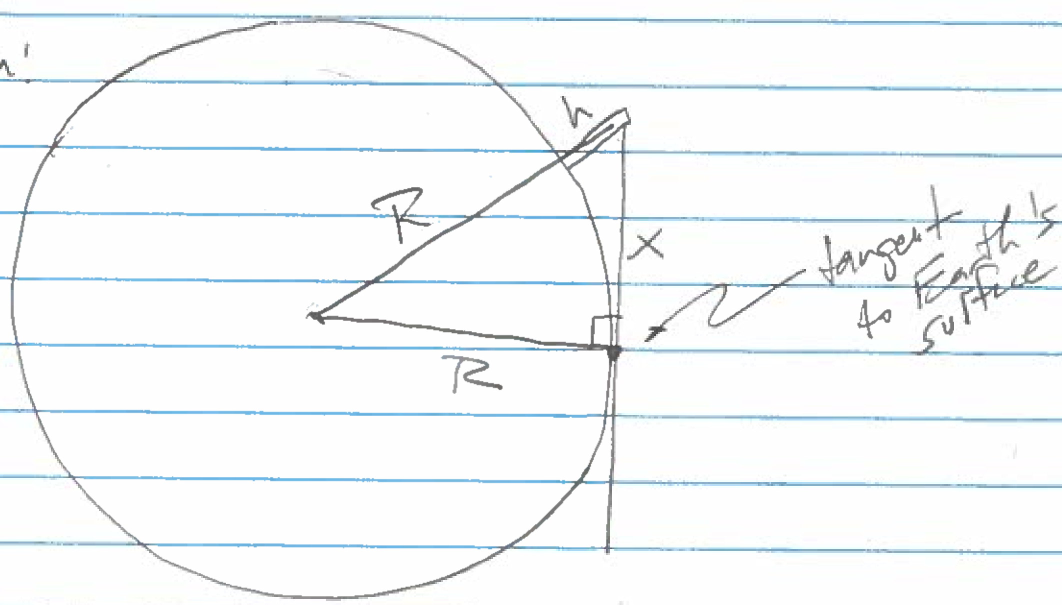
So even with huge baseline you need to resolve 0.2 seconds of arc.

II. Pythagorean thm: Galileo problem hint

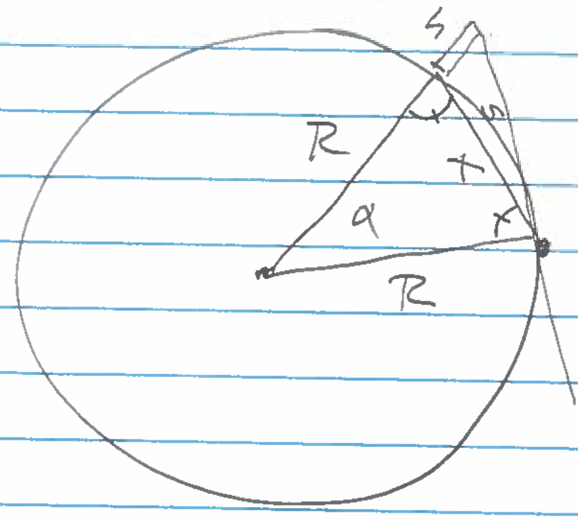
Pythagorean thm:



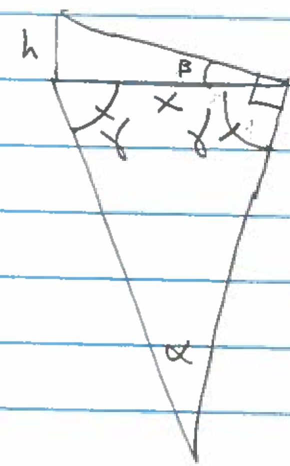
$a^2 + b^2 = c^2$



or use small angles:



$$\alpha = \frac{s}{R} \simeq \frac{x}{R}$$



angle γ in icoseles Δ

$$\alpha = \pi - 2\gamma$$

$$\gamma = \frac{\pi}{2} - \frac{\alpha}{2} \implies \beta = \frac{\pi}{2} - \gamma = \frac{\alpha}{2}$$

again small angles, $\frac{\alpha}{2} = \frac{h}{x} = \frac{h}{\alpha R}$

$$\implies \alpha = \sqrt{\frac{2h}{R}}$$

$$\implies x = \sqrt{2hR}$$

III. Equations of motion for const. velocity and/or acceleration in <u>1 dimension</u>

At any time, object's motion characterized by its position, velocity, acceleration, ... x, v, a

x = position
v = rate of change of position
a = rate of change of v

In principle, all these quantities are vectors meaning they have a magnitude (size) and a direction.

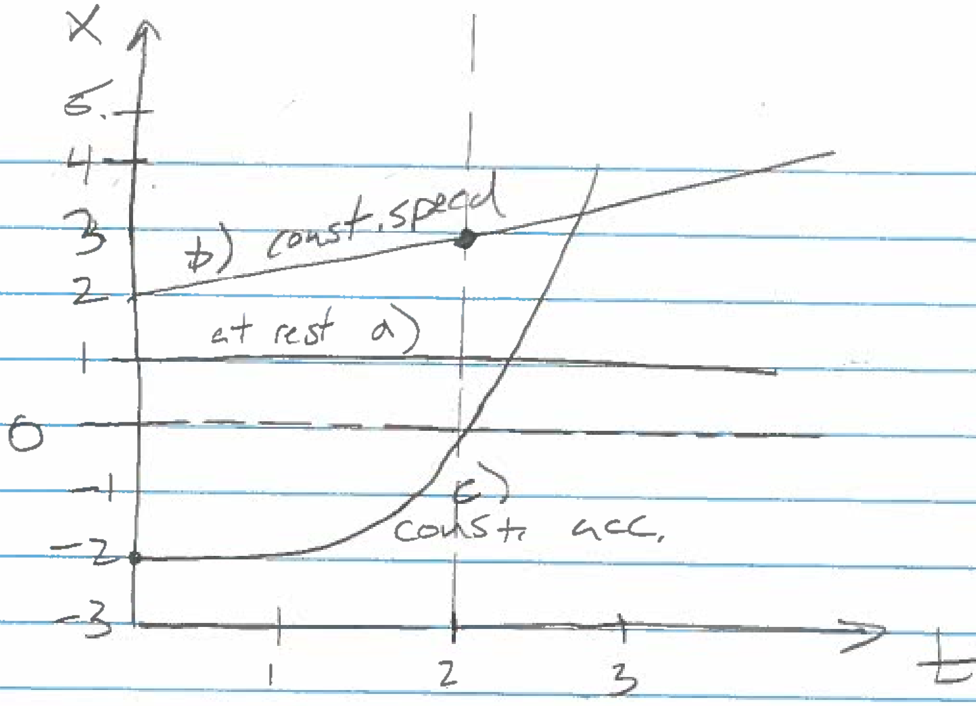
magnitude of velocity is called <u>speed</u>

average velocity $\vec{v}_{avg} = \dfrac{\Delta \vec{x}}{\Delta t}$

" " acceleration $\vec{a}_{avg} = \dfrac{\Delta \vec{v}}{\Delta t}$

over some time interval $\Delta t \equiv t_f - t_i$ (final - initial values).

You can visualize different kinds of motion on a graph:



These correspond roughly to the cases from HW 4:

a) $x = 1$

b) $x = 2 + \frac{1}{2}t$

$\uparrow$ initial position

c) $x = -2 + \frac{1}{2}t^2$

Compare to general equations

$$\boxed{\begin{aligned} x &= x_0 + v_0 t + \tfrac{1}{2}at^2 \\ v &= v_0 + at \\ a &= \text{const} = a \end{aligned}}$$

So for car a), $x_0 = 1$, $v_0 = a = 0$

b), $x_0 = 2$, $v_0 = \frac{1}{2}$, $a = 0$

c), $x_0 = -2$, $v_0 = 0$, $a = \frac{1}{2}$

Note the 1 dimension of motion can be up/down as well; replace $x \leftrightarrow y$

<u>Example</u>: How long does it take for rock to fall from 100-ft cliff if released from rest?

$$y = y_0 - \frac{1}{2} g t^2 \qquad g = |a| = 32 \frac{ft}{s^2}$$

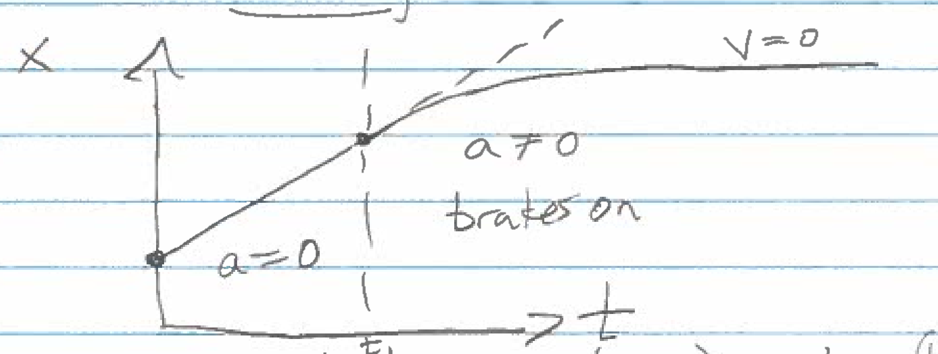
$$= 100 - 16 t^2$$

When rock hits ground, its height y is zero, so

$$0 = 100 - 16 t^2$$

$$t = \sqrt{\frac{100}{16}} = 2.5 \text{ sec}$$

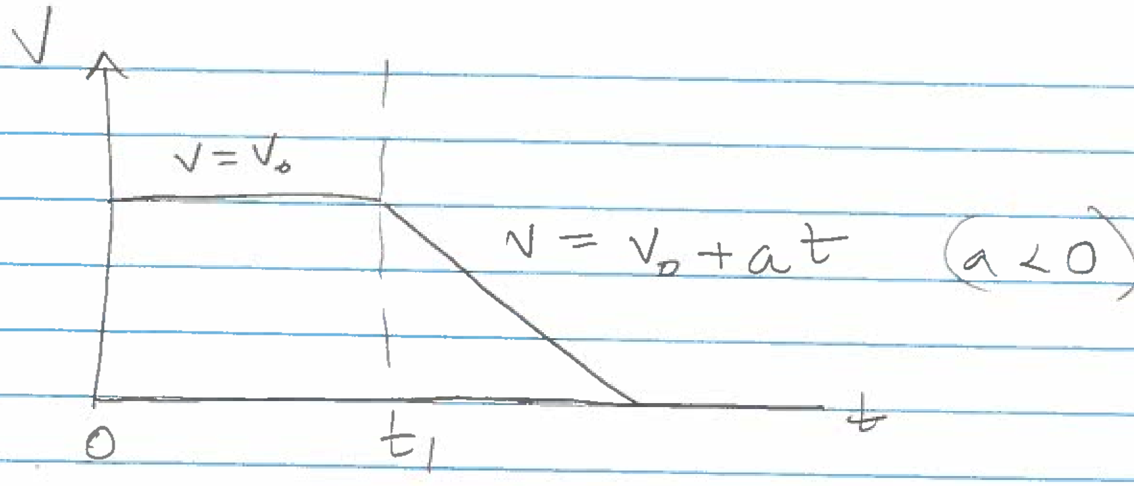
<u>Question</u>: what does the motion of a <u>braking</u> car look like?



$x = v_0 t \qquad x = v_0 (t - t_1) + \frac{1}{2} a (t - t_1)^2$

$(a < 0)$

Same situation



A car is driving at 40 ft/sec when the driver slams on the brakes, which slow the car down at a rate of 10 ft/s². How far does the car go in this time?

time to stop $\quad v = 40 - 10t$

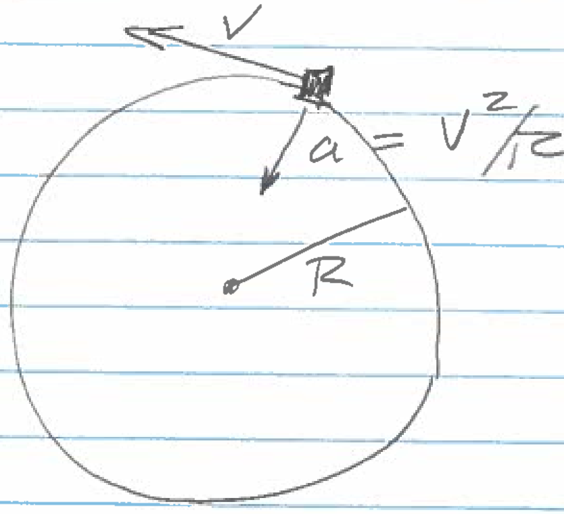
$$\Rightarrow t = 4 \text{ sec}$$

$$x = v_0 t + \tfrac{1}{2} a t^2$$

$$= 40t - 5t^2$$

$$= 40(4) - 5(4)^2 = 80 \text{ ft}$$

IV Uniform Circular motion



any object moving in a circle at const. speed is accelerating towards the center with acceleration $a = v^2/R$

NB: speed is not changing, but velocity is, because the direction of motion is changing

Example: Object on Earth's equator: what is its centripetal acc. due to Earth's rotation?

$$v = \frac{2\pi R}{T}$$

$2\pi R$ ← circumference

T ← period 1 day

$$= \frac{2\pi\,(6400\text{ km})}{(24\cancel{\text{hrs}})\left(60\frac{\cancel{\text{min}}}{\cancel{\text{hr}}}\right)\left(60\frac{\text{sec}}{\cancel{\text{min}}}\right)} = 465\ \frac{m}{s}$$

$$a = \frac{v^2}{R} = \frac{(465\ m/s)^2}{(6.4 \times 10^6\ m)} = 0.036\ \frac{m}{s^2}$$

Compare to gravitational acc. at Earth's surface of $9.8\ m/s^2$ — very small, one reason why effects of Earth's motion not noticed.