PHY 1033C - Lab 3
How Eratosthenes Measured the Circumference of the Earth

In today's lab we will show how Eratosthenes, a Greek mathematician, was able to estimate the circumference of the Earth using a sundial and some information about shadows in a 'nearby' city.

1. To start out, place a toothpick at the peak of the 'sphere' (hemisphere) leaving about 3cm exposed, being careful to make sure the toothpick is perpendicular to the sphere's surface.

2. Place another toothpick in the sphere around 3 to 7 cm away from the first, again leaving about 3cm exposed and making sure it is perpendicular to the surface.

Now we need to align our 'Sun.' Eratosthenes knew that at noon on the summer solstice in the nearby city of Swenet, shadows appeared directly below the object casting them. What he knew specifically was that the shadow of a person looking down a deep well at this time would block the reflection of the Sun that should have appeared in the water at the bottom. To recreate this, align the sphere *on the ground* so that the shadow of the toothpick at the peak is directly underneath it (i.e. the toothpick is parallel to the incoming light).

In Alexandria, Eratosthenes used the gnomon of a sundial, the part that casts the shadow, to determine that the angle of the Sun at local noon on the summer solstice was about 1/50 of a circle, or 7.20°. Our second toothpick will play the part of his sundial's gnomon.

3. Measure and record the length of the second toothpick's shadow.

4. Measure and record how much of that toothpick is exposed.

5. These two lengths act like the legs of a right triangle. Using some trigonometry, we see that

$$\theta = \arctan\left(\frac{\text{length of toothpick's shadow}}{\text{exposed length of toothpick}}\right) = \arctan\left(\frac{L_s}{L_t}\right)$$

As it turns out, this angle, which is the angle between our Sun's rays and the toothpick (see Figure 1(b)), is also the angle between the two toothpicks as measured from the center of the sphere.

6. Redraw Figure 1(a) in your lab book and show that these two angles are the same by drawing on the figure.

Using this knowledge, we're almost ready to compute the circumference of the Earth. The last bit of information that we need is the distance between the two toothpicks.

7. To measure this distance, use a ruler.

8. Using this distance and the angle found earlier, compute the circumference, C, of the sphere by

$$C = [\text{distance between toothpicks}] \cdot \left(\frac{360°}{\text{angle between toothpicks in degrees}}\right) = L_d \cdot \left(\frac{360°}{\theta}\right)$$

and the diameter, d, of the sphere by

$$d = \frac{C}{\pi}$$

9. Compute the percent error in your estimate by directly measuring the diameter of your sphere, d_m, with a ruler.

$$\% \text{ Error} = \frac{|d - d_m|}{d_m} \cdot 100\%$$

Eratosthenes didn't have a pocket calculator available to help him compute the angle; however, because of the small angle approximation,

$$\tan \theta_r \approx \theta_r \quad , \quad \theta_r = \theta \cdot \left(\frac{\pi}{180°} \right)$$

where θ_r is a 'small' angle measured in radians, he was able to easily estimate the circumference without such a calculator.

10. Convert the angle Eratosthenes measured, $7.20°$, to radians and show that use of the small angle approximation was valid in his case.

11. Eratosthenes didn't have to directly measure the distance between Swenet and Alexandria, as he had been informed that it was around 5000 stadia. A stade is a unit of length that corresponds to about 185 meters. Use the experimental values Eratosthenes had to make an estimate for the circumference of the Earth and compare it to the currently known value of 40,074,275 m.
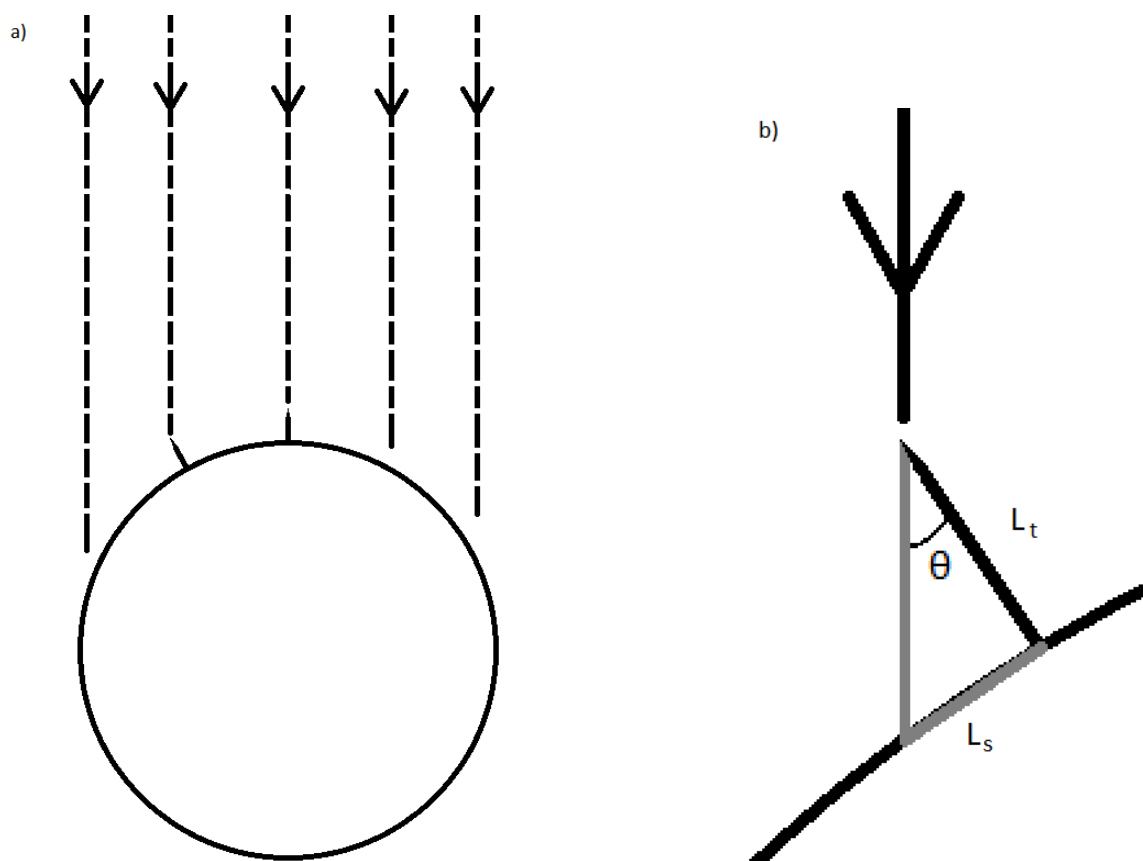
Example Figures

Figure 1: a) Diagram of lab situation, showing light as dashed lines and toothpicks sticking out from surface of sphere. b) Close up of the toothpick that casts a shadow.