

#### PHY1033C/HIS3931/IDH 3931: Discovering Physics:

The Universe and Humanity's Place in It Fall 2016

Prof. Peter Hirschfeld, Physics



#### **Announcements**

HW 6 due today

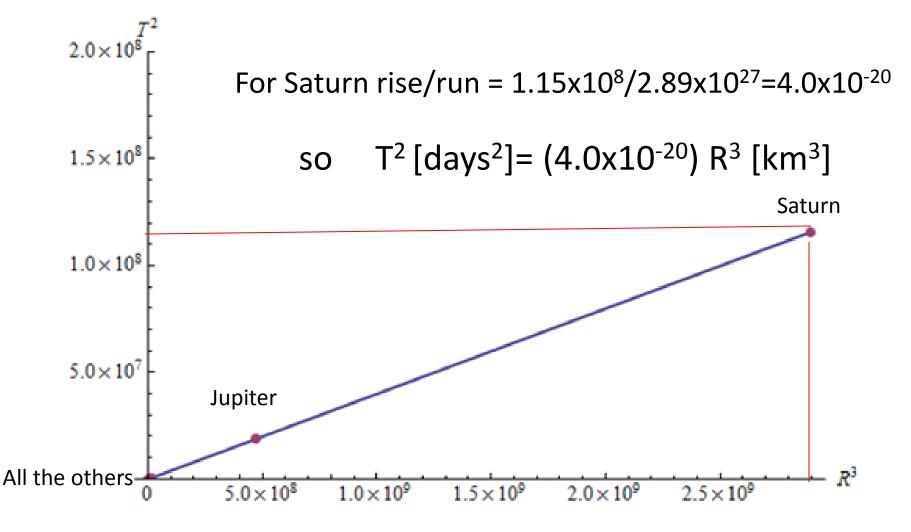
Reading: Gregory, Chapter 9, pp. 177-189; Chapter 7, pp. 140-145.

HW 5 comment

#### Answer:

Planet	Average orbit radius R (10 <sup>6</sup> km)	Orbital period T (Earth days)	R <sup>3</sup> (10 <sup>18</sup> km <sup>3</sup> )	T2(day2)
Mercury	58	88	195112.	7744.
Venus	108	224	1259712.0	50176.
Earth	149	365	3307949.0	133225.
Mars	228	687	1.1852352 × 10 <sup>7</sup>	471969.
Jupiter	778	4333	4.70910952 × 10 <sup>8</sup>	1.8774889 × 10 <sup>7</sup>
Saturn	1426	10,759	2.899736776 × 10 <sup>9</sup>	1.15756081 × 10 <sup>8</sup>
Unknown planet	3300	37,772	3.5937 × 10 <sup>10</sup>	1.426723984 × 10 <sup>9</sup>

Kepler:  $R^3 \propto T^2 \implies R^3 = a T^2$  a = slope of  $R^3$  -  $T^2$  graph



Equally spaced points!!!

#### Clicker question 1

#### Newton's law of universal gravitation

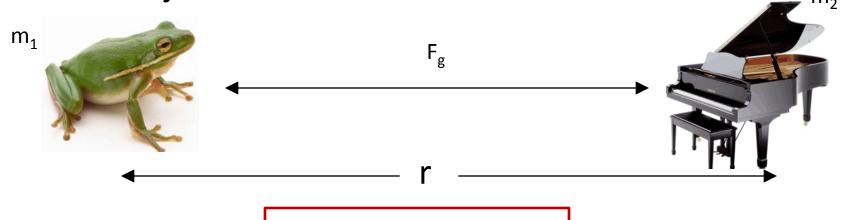
- 1. Says that the grav. force between planets is proportional to  $1/r^3$
- 2. Says that the force between objects is proportional to the masses of the two objects
- 3. Applies only to objects inside the orb of the moon
- 4. Applies only to apples and the moon, but not planets
- 5. Requires two objects to be in direct mechanical contact



#### Last time:

## Newton's law of universal gravitation

 Mutual force of attraction between any two objects with mass in the universe

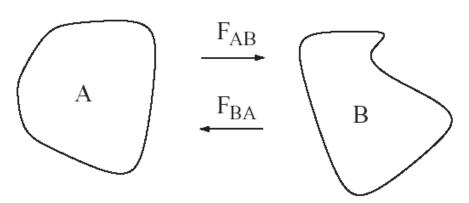


$$F_g = G \frac{m_1 m_2}{r^2}$$



#### Last time: Newton's Laws of motion

- An object moves with a velocity that is constant in magnitude and direction, unless acted on by a nonzero total force (inertia=tendency to stay in same state of motion, measured by mass)
- 2. F=ma Applied total force F is proportional to the acceleration a, with prop. const. (mass) m.
- 3. For every action there is an equal and opposite reaction



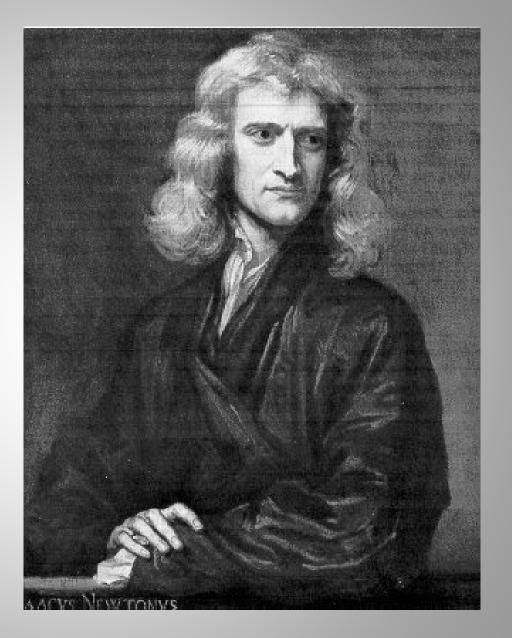
# Discussion question

 A lazy horse argues, "I can't pull the wagon since I can never exert more of a force on it than it exerts on me, according to Newton's 3<sup>rd</sup> law. What is the key reason the cart can really move?

> C=cart H=horse G=ground

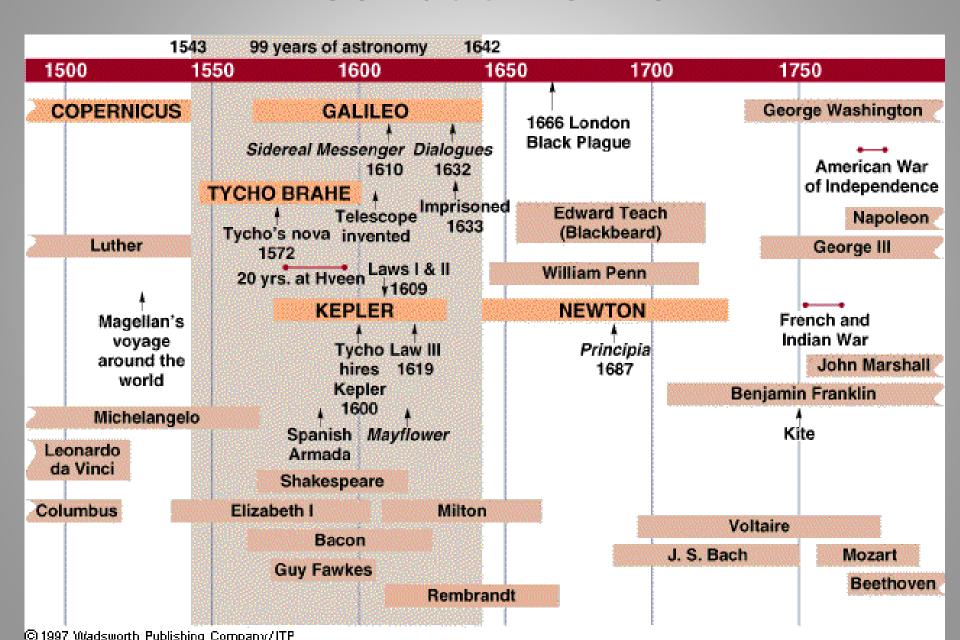


Newton at 46



1643-1727

### Peek at timeline



To Cambridge University as subsizar in 1661

Dominated by Aristotle

Newton read Descartes, Galileo, Kepler and others on his own

Plague forced university to close – Newton goes home to Woolsthorp

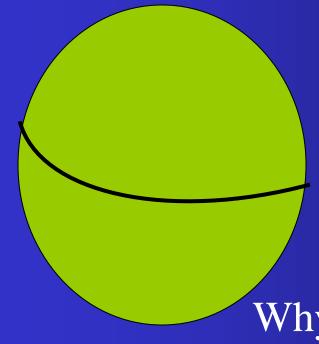
Annus mirabilis of 1666

Calculus

Problem of the Moon

#### The Problem:





Why does the moon orbit the earth?

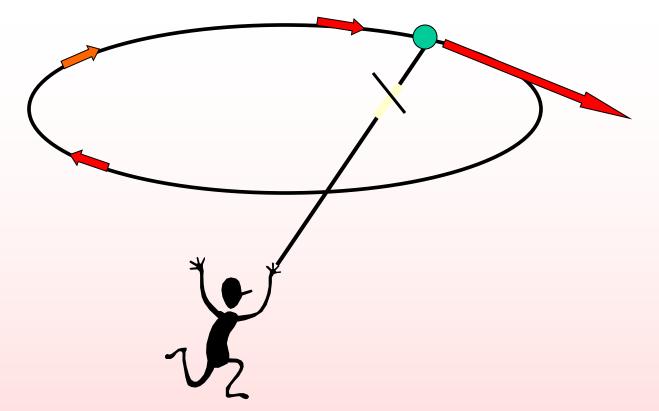
### Complicating the Problem



Galileo's explanation:

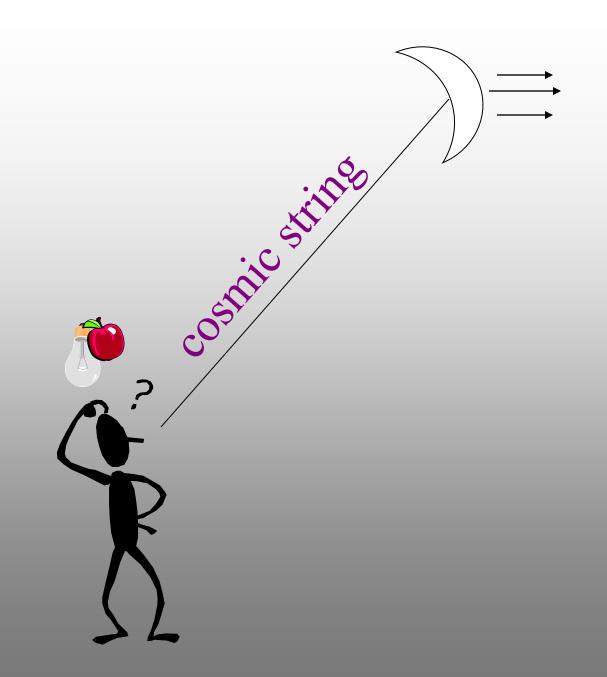
circular inertial motion

For Galileo "natural" motion is uniform and circular



So for Newton straight line motion is "natural" and curved motion is constrained (First Law)

That means to interrupt "natural" motion requires the presence of an unbalanced force (Second Law)



# The legend of the apple

"After dinner, the weather being warm, we went into the garden and drank tea, under the shade of some apple trees, only he and myself. Amidst other discourse, he told me he was in the same situation as when formerly the notion of gravitation came into his mind. It was occasioned by the fall of an apple as he sat in a contemplative mood."

William Stuckley, 1726

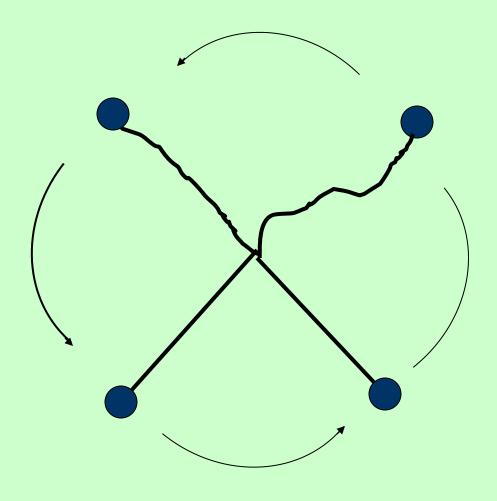
"As he sat alone in a garden, he fell into a speculation on the power of gravity: that as this power is not found sensibly diminished at the remotest distance from the center of the earth to which we can rise, neither at the tops of the loftiest buildings, nor even on the summits of the highest mountains, it appeared to him reasonable to conclude that this power must extend much farther than was usually thought. Why not as high as the moon, said he to himself? And if so, her motion must be influenced by it; perhaps she is retained in her orbit thereby.

# What he already knew

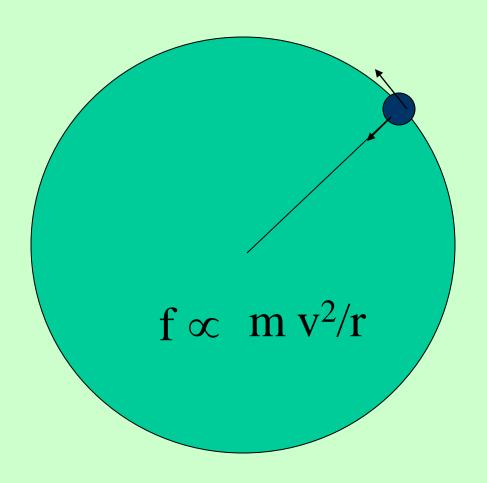
- 1.  $a = 32 \text{ ft/sec}^2$  (by measuring it)
- 2. d = 1/2 a  $t^2$  (from Galileo)
- 3. Inertial motion (his own "corrected" version)
- 4.  $F \propto a$
- 5. f of tension in a string
- 6. The relation between a planet's period and its distance from the sun

To find 5 - the tension in the cosmic string - Newton examined the case of a rock on a string





What factors affect the tension in the string?



Now Newton applies this result to the moon case:

$$f \propto mv^2/r$$

$$v = d/t = 2\pi r/T$$

$$f \propto m(2\pi r/T)^2/r = m(4\pi^2 r^2/T^2) \times 1/r$$

$$\frac{m4\pi^2r^2}{T^2} \quad x \quad \frac{1}{\cancel{1}} \quad = \frac{m4\pi^2r}{T^2}$$

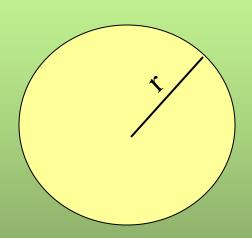
So 
$$f \propto \frac{m4\pi^2 r}{T^2}$$

But, from Kepler's Third Law  $T^2 \propto r^3$ 

$$\frac{m4\pi^{2}r}{T^{2}} = \frac{m4\pi^{2}r}{r^{2}} = \frac{m4\pi^{2}}{r^{2}}$$

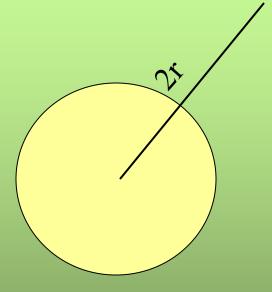
Thus 
$$f \propto \frac{k}{r^2}$$

f is 
$$\propto \frac{1}{r^2}$$



At 1 earth radius:

$$F \propto \frac{1}{r^2}$$
 = some amount  $F_1$ 



At 2 earth radii:

$$F \propto \frac{1}{(2)^2} \times F_1 = \frac{1}{4} F_1$$

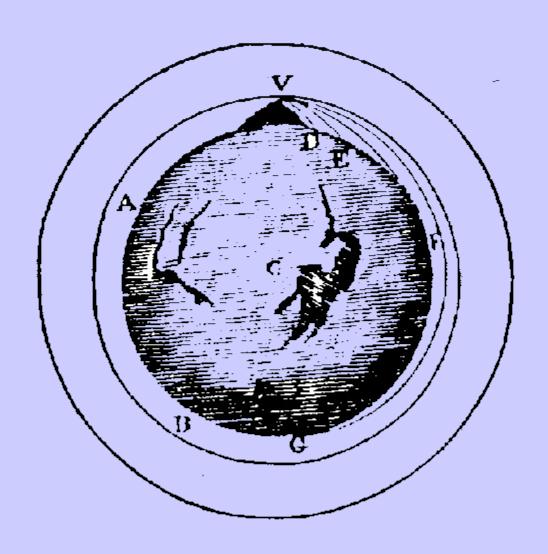
Newton's next move: To show

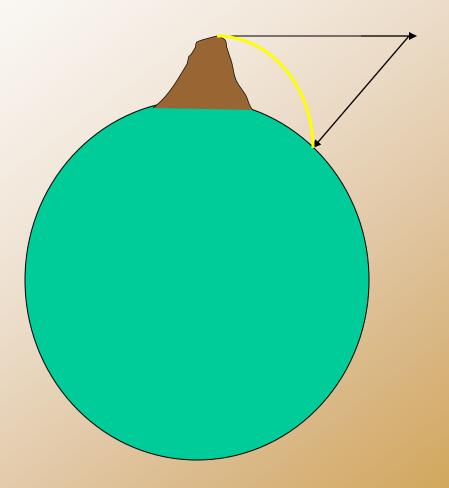
that the moon (like apples)

can be considered a

falling body

#### From Newton's Principia Mathematica

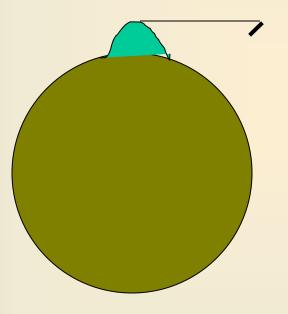


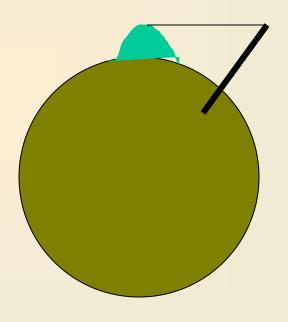


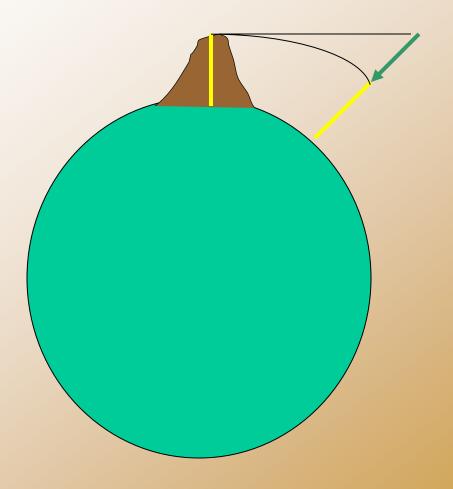
# Two cases of projectile fall

Thrown hard

Thrown slowly



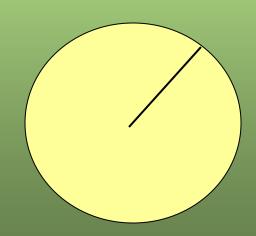




### Since $f \propto a$ , $a \propto f$

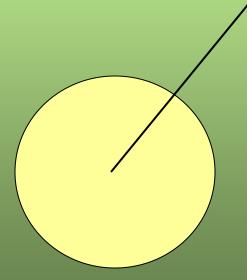
f is also 
$$\propto \frac{1}{r^2}$$

Therefore a is  $\propto \frac{1}{r^2}$ 



At 1 earth radius:

 $a = 32 \text{ ft/sec}^2$ 



At 2 earth radii:

$$a = 1/2^2 \times 32 = 8 \text{ ft/sec}^2$$

### The moon is 60 earth radii away

Therefore at the distance of the moon

$$a = \frac{1}{60^2} \times 32$$

or 
$$\frac{32}{60^2}$$
 ft/sec<sup>2</sup>

In one minute (60 sec) the moon will "fall"

$$d = 1/2 a t^2 = 1/2 x \frac{32}{60^2} x 60^2 =$$

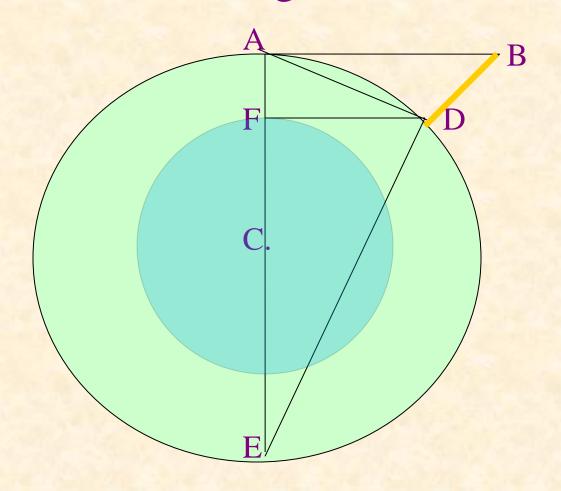
16 feet

- p = The same force that affects apples also affects the moon
- q = The moon falls 16 feet in one minute
  - A. If p then q

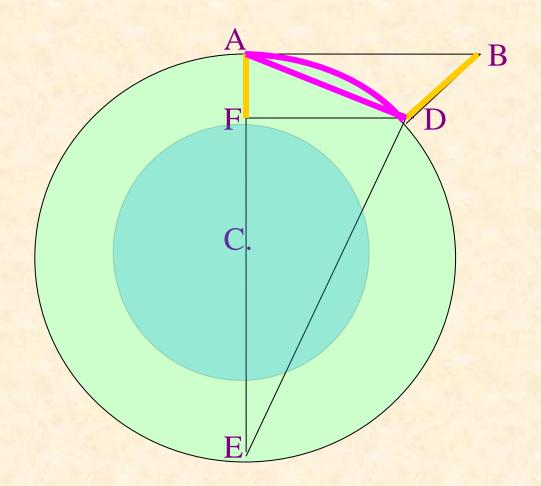
- p = The same force that affects apples also affects the moon
- q = The moon falls 16 feet in one minute
  - A. If p then q

B. Now he has to show q: the Moon actually does fall 16 ft in one minute

His task is to find BD, the distance the moon "falls" in a given amount of time.



### To find BD Newton thinks like an engineer



He is able to show that BD ~ 16 ft. AMAZING!!!!!