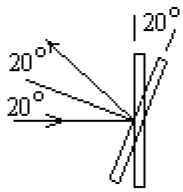
**Chapter 26 – Problems 3, 7, 9, 11,15, 18, 19, 40, 41, 25, 27, 31, 33, 37**

3. The sketch below shows the geometry involved as a result of rotating the mirror by $20^o$. Notice that the axis of the mirror also rotates by $20^o$, so that the incident ray is now $20^o$ from the new axis. But, as a result, the reflection angle is also $20^o$ from the rotated axis. The total angle between the incident and reflected rays is $40^o$.




7. The purpose of this problem is to show how the image changes as the object is moved. You will find that when the object is far beyond *R*, the image is inverted and is between *R* and *f*, but close to *f*. As the object approaches *R* from the left, the inverted image approaches *R* from the right. When the object is between *R* and *f*, the inverted image is beyond *R*. The closer the object is to *f*, the farther the image is from the mirror. Finally, as the object passes to between *f* and the mirror, the image becomes upright and virtual. You should sketch each of the four object positions to get a feel for how they look. I have sketched only part c) because the sketches are time consuming to draw.

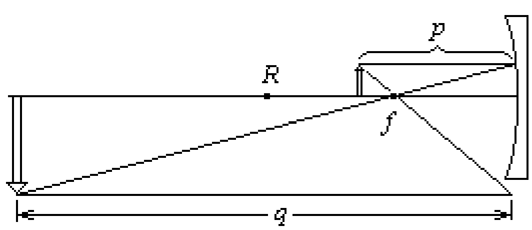
   a. Let $R$ = 60 cm, so that $f$ = 30 cm. With $p$ = 120 cm, we find

   $$\frac{1}{120}+\frac{1}{q}=\frac{1}{30}\rightarrow\frac{1}{q}=\frac{4}{120}-\frac{1}{120}\rightarrow\frac{1}{q}=\frac{3}{120}=\frac{1}{40}$$ . The result is $q$ = 40 cm

   Notice the way I have kept the calculations in fraction form, finding a common denominator to obtain 1/q. I recommend you do this, assuming the numbers is easy to work with. Since $q$ is positive, the image is real and inverted. Also, the object height is 2 cm. The height of the image relative to the height of the object is in the same ratio as $q$ is to $p$. Thus $h_i = h_o(40)/(120)$ = 2/3 cm.

   b. Next, let p = 80 cm. Following the exact procedure as in part a), the result is $q$ = 48 cm. As in a), the image is real and inverted. The image height is $h_i = h_o(48)/(80)$ = 6/5 cm.

   c. When $p$ = 40, we should expect to get the exact opposite of a). There we found $p$ = 120 and $q$ = 40. Here, since $p$ = 40, we should expect $q$ = 120. Indeed, this is the correct answer as given by the mirror equation. Also, the image height is now $h_i = h_o(120)/(40)$ = 6 cm. The sketch is shown below. Only two of the four usual rays have been drawn. Can you draw the other two?




   d. This time the object is inside the focal length of the mirror at $p$ = 20 cm. The sketch is similar to the sketch drawn in problem 19. Substituting into the mirror equation gives

   $$\frac{1}{20}+\frac{1}{q}=\frac{1}{30}\rightarrow\frac{1}{q}=\frac{2-3}{60}=-\frac{1}{60}$$ . The result is $q$ = -60 cm.

   The minus sign tells us the image is 60 cm behind the mirror. It is therefore virtual and upright. The image height is 3 times the object height and is thus 6 cm.

9. We now have a convex rather than concave mirror. There are several useful general features worth remembering. Of course, you must first remember that the focal length of a convex mirror is a negative number. Starting with the object far from the mirror, the image will be close to but just inside *f*. The image is behind the mirror and is thus virtual and upright. It is also very small compared to the object. As the object gets closer to the mirror from the left, the image also gets closer to the mirror from the right. As a result it remains virtual and upright, gradually getting larger but always remaining smaller than the object. Now let's check these general features with examples.Assume a convex mirror having a focal length of 30 cm (radius of curvature of 60 cm). In the mirror equation, *f* must be negative. This means we use $f$ = -30 cm.

   a. Let $p$ = 120 cm for an object 2 cm high. Substituting into the mirror equation gives

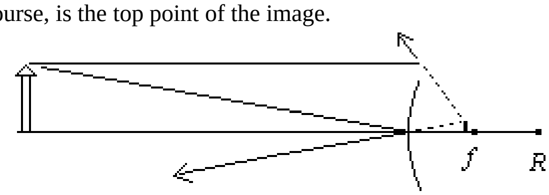
   $$\frac{1}{120}+\frac{1}{q}=-\frac{1}{30}\Rightarrow\frac{1}{q}=-\frac{1}{30}-\frac{1}{120}=-\frac{5}{120}.$$ The result is $q$ = -24 cm.

   The magnification is given by $h_i = h_o(24)/120 = h_o(0.20)$cm = (2)(0.20) or 0.40 cm, and is virtual and upright. Your book puts a minus sign in the magnification formula. I usually just forget about it and rely on the sketch to tell me whether the image is inverted or upright. Your book would say $h_i = -h_o(q/p) = -2.0(-24)/120 = +0.40$ cm. The + sign signifies the image is upright.

b. This time, let $p = 80$ cm. Then, substituting into the mirror equation gives

$$\frac{1}{80}+\frac{1}{q}=-\frac{1}{30}\Rightarrow\frac{1}{q}=-\frac{1}{30}-\frac{1}{80}=\frac{-8-3}{240}=-\frac{11}{240}$$ . The result is $q = -21.8$ cm.

The magnification is given by $h_i = h_o(240)/[(11)(80)] = 6/11$ cm, or 0.545 cm, and is virtual and upright. A sketch of the example is shown below. Two of the four usual rays are drawn. Notice how the two rays appear to come from a common point, which, of course, is the top point of the image.




c. With $p = 40$ cm, I find $q = -120/7 = -17.1$ cm. The image height is 0.855 cm.

d. With $p = 20$ cm, I find $q = -60/5 = -12.0$ cm. The image height is 1.20 cm.

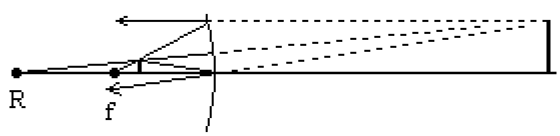
11. This problem is a repetition of problem 9 in approach. You should be able to carry out the solutions easily' so that I will not work them. Just be sure you can sketch them with reasonable accuracy, especially for part c), where the object is inside the focal length, producing a virtual upright image.

15. The data give you important clues. The facts that the mirror is concave, that the image is real and that $q$ is much larger than $p$ implies that the object is between $R$ and $f$, but close to $f$. The sketch is similar to the one shown in problem 7. The image is seen to be inverted. Also the image is (200)/(25) times as large as the object, making it 4.0 cm high.

18. This problem becomes easy, provided you have taken to heart the general behaviors discussed in earlier problems. Since the image is 4 times as large as the object, we can set $q = 4p$. Although you can now blindly substitute, you should also recognize that the object must be between $f$ and $R$, in order to produce a real inverted image with $q$ larger than $p$. Now, substitute into the mirror equation, setting $q = 4p$. This gives us $1/p + 1/4p = 1/30$. Solving for $p$ leads to the result $p = 150/4 = 37.5$ cm and $q = 150$ cm, a result that makes sense according to the expected general behavior.

19. From general knowledge, we know that since the image is virtual, the object must be inside the focal length. We also know that the value of $q$ is 5 times the value of $p$. However, the $q$ must be negative since it describes the position of a virtual image. Refer to the sketch below. Notice the three rays used, and notice how they are used to form the image location as the point from which all three rays seem to come. The numerical solution is found by substituting $q = -5p$ in the mirror equation.

$$\frac{1}{p}-\frac{1}{5p}=\frac{1}{45}\rightarrow\frac{4}{5p}=\frac{1}{45}$$ . Solving for $p$ gives $p = 36$ cm. Therefore $q = -180$ cm.




40. The focal length of the mirror is 4 ft. Since the cat is 3 ft from the mirror, it is inside the focal length. Thus, with no further ado, the cat's image is known to be virtual and upright. Substituting $p = 3$ ft and $f = 4$ ft into the mirror equation gives $q = -12$ ft. The sketch is very much like the sketch for problem 19, which we just did. The magnification is defined as the ratio of image height to object height. But this is identical to the ratio of $q$ to $p$, which we see is $12/3 = 4$. Note that the expression for magnification in your text is $-(q/p)$ in order to automatically tell you the orientation of the image. I prefer to ignore the sign and rely on the sketch I always make either in my head or on paper. In any event, the cat's image height is 4 times the cat's real height. Thus, $h_i = 2$ ft.

41. No matter what the object distance is, a convex mirror will always produce a virtual upright image of a real object. (We will discuss virtual objects in a later chapter.) Using the mirror equation, with $p = 6$ inches and $f = -5$ inches, we easily find that $q = -(30/11) = -2.73$ inches. The magnitude of the image height is , $h_i = h_o(-(q/p)) = $ (3 inches)(30/11)/6 = 1.4 inches

25. Assume that the wavelength of the light in a vacuum is $589 \times 10^{-9}$ m. Since the frequency is $f = c/$(lambda), we find that $f = (3.00 \times 10^{8}$ m/s)/($0.589 \times 10^{-6}$ m) = $5.1 \times 10^{14}$ Hz. When the light enters the glass, the frequency cannot change, because the oscillation rate must be continuous across the boundary. However, once the ray enters the glass the speed is reduced to $v = c/$(mu). The index of refraction for this particular glass is mu = 1.55. Thus,

$v = (3.00 \times 10^{8}$ m/s)/(1.55) = $1.94 \times 10^{8}$ m/s.

Finally, since the speed of the light is reduced, the wavelength must be reduced in the same proportion. The wavelength in the glass is

lambda = (589 nm)/1.55 = 380 nm.

27. The refractive index of water is 1.33, and we assume (to excellent approximation) that the refractive index of air is 1.00. Snell's law then

tells us that given the incident angle of $70^o$, the angle of refraction, $z_r$, satisfies the equation

$1.00\sin(70^o) = 1.33\sin(z_r)$. The answer is $z_r = 45^o$.

Using the same law, and assuming the angle of refraction is to be $z_r = 20^o$, the angle of incidence, $z_i$, is found from

$1.00\sin(z_i) = 1.33\sin(20^o)$. The answer is $z_i = 27^o$.

31. The critical angle phenomenon can only occur when the light beam “tries” to pass from the material of higher refractive index to the material of lower refractive index. In this problem, the ray passes from the glass (mu = 1.55) to the oil (mu unknown). As the incident angle in the glass approaches the critical angle of $59^o$, the angle of refraction in the oil approaches $90^o$. At the critical angle, then,

    $1.55\sin(59^o) = mu_{oil}\sin(90^o)$. Thus, $mu_{oil} = 1.55\sin(59^o) =1.33$.

33. When a ray passes through a glass pane, the exit ray is exactly parallel to the entering ray, although the ray has been shifted a small amount. The amount of shift is proportional the thickness of the glass pane. The problem assumes the entering ray to be incident at an angle of $70^o$ and the refractive index of the glass pane to be 1.55. Therefore,

    $1.00\sin(70^o) = 1.55\sin(z_r)$, which leads to $z_r = 37.3^o$.

    The exit angle is, of course, $70^o$.

37. This is a problem in which the issue is the critical angle. From the point of view of the fish, the world above the water surface is visible from horizon to horizon in all directions within a cone limited by the critical angle, as explained in class. Knowing the refractive index of water to be 1.33, this critical angle is determined by

    $1.33\sin(z_{critical}) = 1.00\sin(90^o)$. This tells us that $z_{critical} = 41^o$ and that $z_i$ is less than or equal $41^o$.