Chapter 21 – Solutions – Problems 3, 4, 5, 7, 9, 11, 13, 15, 19, 21, 24, 26, 39, 45

3. The circuit is reproduced below.

   a. A current I_2 is going into point A. Coming out from point A are the currents I_1 and I_3. Conservation of current demands that $I_2 - I_1 - I_3 = 0$. This is called the junction equation. Notice that this is the same as saying $I_2 = I_1 + I_3$.

   b. Writing the loop equation implies certain rules that <u>must</u> be followed. Pay close attention to the comments in the "box" of instructions on page 403 of your book.

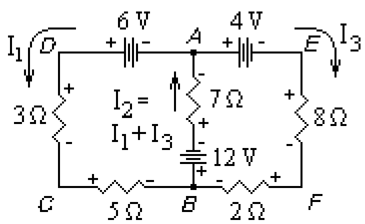
   Rule 1. Assume and label all currents, as well as their directions. Use the prescription I showed you in class. In this problem the author has already done this for you.

   Rule 2. For every battery, mark the + and the – sides to prevent confusion. I have done this for you.

   Rule 3. For every resistor, notice the direction of the assumed current, and mark the where current enters + and where it leaves –. The reason is that the voltage drop across each resistor must be consistent with the direction of the assumed current through that resistor.

   Rule 4. If there are any capacitors in the circuit, you must be aware that no current passes through a capacitor. But there is a voltage drop across a capacitor, and it is charged accordingly.

   Rule 5. Writing the loop equation means writing out the equation "sum of voltage rises and voltage drops around <u>any</u> circuit loop = 0." Taking into account the signs is crucial.



We are now ready to write the loop equation for the left-hand loop. We will go clockwise around the loop starting at point A (the direction does not matter since it only entails a change in all the signs). Remember that the direction of each assumed current in a given resistor determines the correct sign for the voltage expression.

A to $B \rightarrow + 7I_2 + 12$ V

B to $C \rightarrow + 5I_1$

C to $D \rightarrow + 3I_1$

D to $A \rightarrow - 6$ V

The four terms are added to get the loop equation. It is $7I_2 + 8\,I_1 + 6 = 0$.

4. This was not assigned, but let's go ahead and do this one, then solve the circuit for the currents. The loop equation for the loop *AEFBCDA* is as follows (with everything in volts).

A to $E \rightarrow - 4$

E to $F \rightarrow - 8I_3$

F to $B \rightarrow - 2I_3$

B to $C \rightarrow + 5I_1$

C to $D \rightarrow + 3I_1$

D to $A \rightarrow - 6$

These terms added together gives the loop equation for the outer loop. $8I_1 - 10I_3 - 10 = 0$. In order to solve for the currents, we need to solve the three simultaneous equations with the three currents as the unknowns. It is important to realize that the three equations must be the junction equation and any two of the three possible loop equations. YOU CANNOT USE THREE LOOP EQUATIONS SINCE ONLY TWO OF THE THREE ARE INDEPENDENT! As a first step, we eliminate I_2 by substituting $I_2 = I_1 + I_3$ into the first loop equation. We find the two remaining equations to be

$$8I_1 - 10I_3 = 10$$

$$15I_1 + 7I_3 = -6$$

These two equations are solved by multiplying the first one by 7 and the second one by 10, then adding the two. I_3 is eliminated. The result is

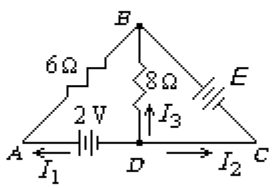
$$I_1 = 0.049 \text{ A}$$

$$I_3 = -0.961 \text{ A}$$

$$I_2 = -0.912 \text{ A}$$

The meaning of the fact that I_2 and I_3 are negative is that the assumed current directions for these two are opposite to the way they actually are. That's no big deal!!!

5. Refer to the figure below. In the figure insert proper + and – signs, consistent with the assumed currents



   a. You should find that

      A to $B \rightarrow -6I_1$

      B to $C \rightarrow -E$

      C to $D \rightarrow 0$

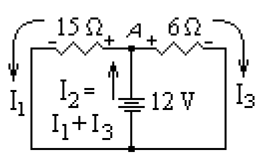
      D to $A \rightarrow +2$, so that $-6I_1 - E + 2 = 0$.

   b. Going into point A is I_1, and going out of point A is I_1. Thus the junction equation is

      $I_1 - I_1 = 0$.

   c. All currents at point D are going out. Thus, $I_1 + I_2 + I_3 = 0$. This, of course, means that at least one of the currents that have been assumed must have the opposite direction.

7. This turns out to be an easy problem because the 15-ohm and 6-ohm resistors are in parallel. Make sure you agree that the two resistors are indeed parallel! Although, the loop and junction equations are not really needed, I will use them to show the logic. The diagram is reproduced with assumed directions for the currents.



   I choose the two loops to be the left-hand and right-hand loops. Going clockwise around the left-hand loop gives the equation $15I_1 - 12 = 0$. The result is $I_1 = 0.8$ A. Going clockwise around the right-hand loop gives the equation $12 - 6I_3 = 0$. The result is $I_3 = 2.0$ A. From the junction equation, $I_2 = I_1 + I_3 = 2.8$ A.

9. Refer to the problem 5 figure above. If $I_1 = 0$, there is no voltage drop across the 6-ohm resistor. The loop equation for the path *ABCDA* is $0 - E + 0 + 2 = 0$. The result is $E = 2$ V. The loop *BCDB* is now chosen, giving $-2 + 0 - 8I_3 = 0$. The result is $I_3 = -0.25$ A, which also implies $I_2 = +0.25$ A. Notice that since I_3 turned out to be negative, the current is actually opposite to what was assumed in the figure.

11. Refer once more to the figure in problem 5 above. If there is a 1.2-volt rise between points *A* and *B*, this implies a current of $I_1 = -0.2$ A through the 6-ohm resistor (opposite to the assumed direction shown). Going around the left-hand loop clockwise, we have the equation

$+2 + 1.2 + 8I_3 = 0$, which tells us $I_3 = -0.4$ A.

Going around the outside loop, we find

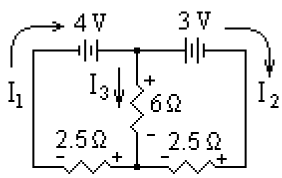
$+2 + 1.2 - E = 0$, which tells us $E = 3.2$ V.

Finally, from the requirement $I_1 + I_2 + I_3 = 0$

$I_2 = -I_1 - I_3 = +0.2 + 0.4$ A $= 0.6$ A.

13. **NOTE: IN SOME BOOKS, THE 4-V BATTERY IS REVERSED IN DIRECTION! YOUR ANSWERS WILL OF COURSE DIFFER**

This is a very basic problem as an application of Kirchhoff's loop and junction rules.



The first step, as usual, is to define the currents and assume current directions. The book has already defined currents and current directions for you. The junction rule demands that at any node, the current in must equal the current out. Thus, $I_1 = I_2 + I_3$. The three currents are the unknowns, and we therefore need three independent equations (of which $I_1 = I_2 + I_3$ is one). The other two are obtained from two (and only two) of the three possible choices of loops. Let us choose the left-hand loop and the outer loop. The three simultaneous equations are then,

(junction equation) $I_1 - I_2 - I_3 = 0$,

(left-hand loop) $-2.5I_1 + 4.0 - 6.0\,I_3 = 0$

(outer loop) $-2.5I_1 + 4.0 - 3.0 - 2.5I_2 = 0$

Solution is facilitated if you are organized. I suggest that you arrange the equations as shown below, then proceed to solve the system by substitution or by adding and subtracting rows. For example, by multiplying row 1 by –2.5 and adding the result to row 3, you can eliminate I_3.

I_1	$-I_2$	$-I_3 =$	0
$-2.5I_1$		$-6.0I_3 =$	–4.0
$-2.5I_1$	$-2.5I_2$	=	–1.0

You should readily obtain the results $I_1 = 0.441$ A, $I_2 = -0.041$ A, $I_3 = 0.483$ A.

15. Using the expression $P = IV = 2500$ W, you can find the current for each voltage.

If $V = 120$ V, $I = 20.8$ A.
If $V = 240$ V, $I = 10.4$ A.

The "lesson" is that the higher the voltage, the lower the current for a given power. This is important because power loss occurs in house wires leading to appliances. The amount of he loss depends on the square of the current. Thus, reducing the current by upping the voltage means less heating in your house wires (and less fire risk).