10/10/22

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Announcements

1) All grades posted on Canvas
All solutions posted incl. test.
2) Term paper (optional) "abstract" due Oct. 12
1 page, example on course page.
must give 3 references you have consulted
5–10 pp.
Due after T'giving
Ideas for topics on course page
3) Read Ch. 9, Noninertial frames

Chapter 9

Inertial (nonaccelerating) ref frames.
Newton: $\exists$ one absolute coordinate system S for universe S' constant velocity wrt S $\Longrightarrow$ S' is inertial frame.

"Galilean relativity" physical laws are same in all inertial frames, i.e. result of expt. same.

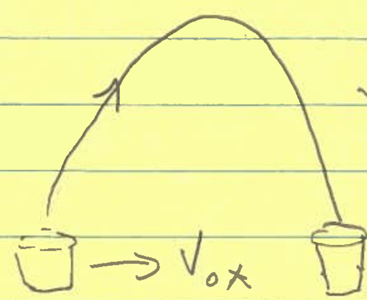
Example: Throw a ball straight up while driving in car at const. $\vec{V}$.

$m\ddot{\vec{r}} = \vec{F}$ for both
$\dot{\vec{r}}_0 = \dot{\vec{r}} + \vec{V}$
$\ddot{\vec{r}}_0 = \ddot{\vec{r}}$ since $\dot{\vec{V}} = 0$

To you S

$y = v_{0y}t - \frac{1}{2}gt^2$
$x = 0$
ball lands in cup

To observer on side of road S_0



$y = V_{0y}t - \frac{1}{2}gt^2$
$x = V_{0x}t$

Historical note: we give Galileo credit for this, but it was really Newton. G believed that physical laws didn't change for frames moving at the earth's surface, i.e. it was important for him that Earth was rotating.

G's Ex.: tennis game in a moving ship.

Q what if car is accelerating?

$$\dot{\vec{r}}_0 = \dot{\vec{r}} + \vec{V}(t)$$

$$\ddot{\vec{r}}_0 = \ddot{\vec{r}} + \vec{A}$$

To you (S) To observer on side of road (S_0)

Acceleration evidently equivalent to an external force exerted opposite to $\vec{a}$

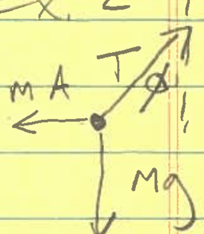
Eqn. of motion in accelerated frame

$$m\ddot{\vec{r}} = m\ddot{\vec{r}}_0 - \underbrace{m\vec{A}}_{\substack{\text{inertial} \\ \text{force}}}$$

(Newton's law in noninertial frame)

$$= \vec{F}_{tot} - \vec{F}_{inertial}$$

Ex. 2 Pendulum in accelerating frame



$$m\ddot{\vec{r}} = \vec{T} - mg\hat{y} - m\vec{A} \equiv \vec{T} + m\vec{g}_{eff}$$

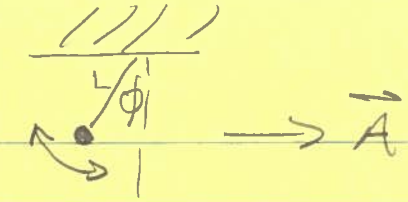
$$\vec{g}_{eff} = -g\hat{y} - \vec{A}$$

Equilibrium $\ddot{\vec{r}} = 0 \quad \vec{T} = -m\vec{g}_{eff}$

$g_{eff} = \sqrt{g^2 + A^2}$

$$T_y = T\cos\phi = mg \qquad \tan\phi = A/g$$
$$T_x = T\sin\phi = mA$$

Small oscillations



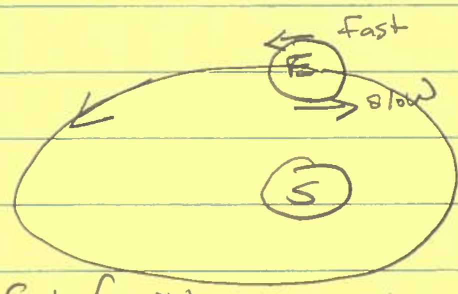
$$\delta\phi = \phi - \phi_{eq} \qquad \ddot{\delta\phi} = -\frac{g_{eff}}{L}\sin\delta\phi \simeq -\frac{g_{eff}}{L}\delta\phi$$

$$\Rightarrow \omega^2 = \frac{g_{eff}}{L} = \frac{\sqrt{g^2+A^2}}{L}$$

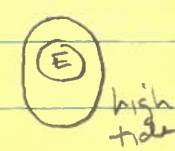
<u>Tides</u> Why? Why 2/day? Height?

Important historically; correct explanation given by Newton (Principia 1687)

Galileo believed tides occurred because of "interference" of Earth's rotation + revolution around sun



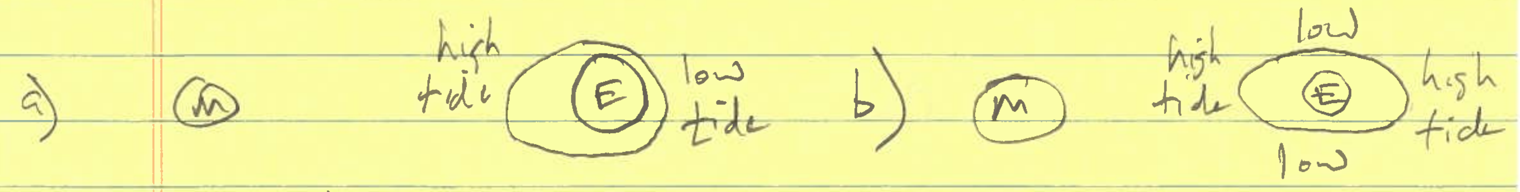
water higher when v = slow



Easily falsifiable: ⇒ only 1 tide/day, doesn't explain variations with lunar motion. But Galileo felt it was <u>proof that Earth moved!</u> Went to Rome to explain to pope Urban VII, led to trial for heresy by inquisition.

Suppose sloshing idea of G was correct, but it was due to <u>moon</u> (believed by ancients)

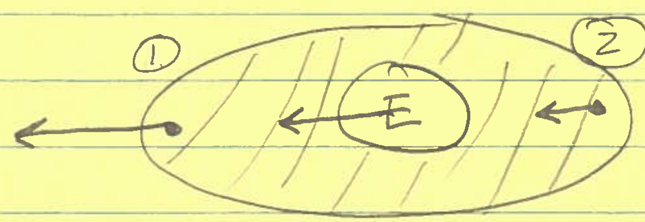
Then we might have picture like a)



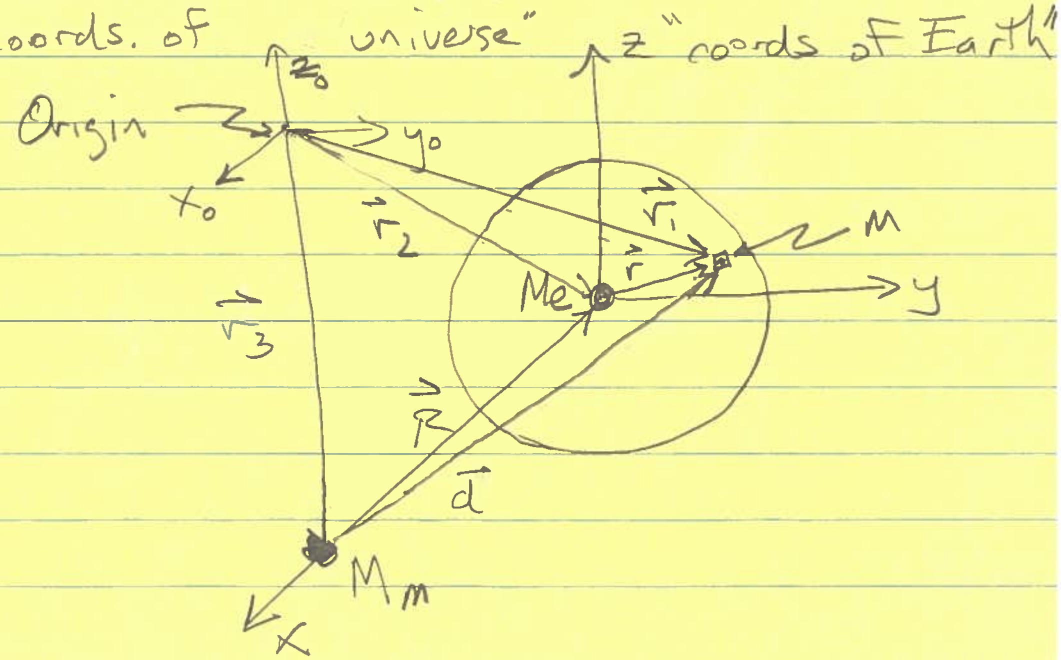
still wrong

right

Newton's Theory gives b): $1/r^2$ force is larger (more attractive) at points on ocean closest to moon, weakest on far side, "average" in middle



Every point mass is accelerating towards moon, but ① is largest, ② weakest.



$\vec{r} = \vec{r}_1 - \vec{r}_2$

$\vec{d} = \vec{r}_1 - \vec{r}_3$

$\vec{R} = \vec{r}_2 - \vec{r}_3$

Motion of water (m)

$$m\ddot{\vec{r}}_1 = -\underbrace{\frac{GmM_e}{r^2}\hat{r}}_{\text{Earth}} - \underbrace{\frac{GmM_m}{d^2}\hat{d}}_{\text{Moon}}$$

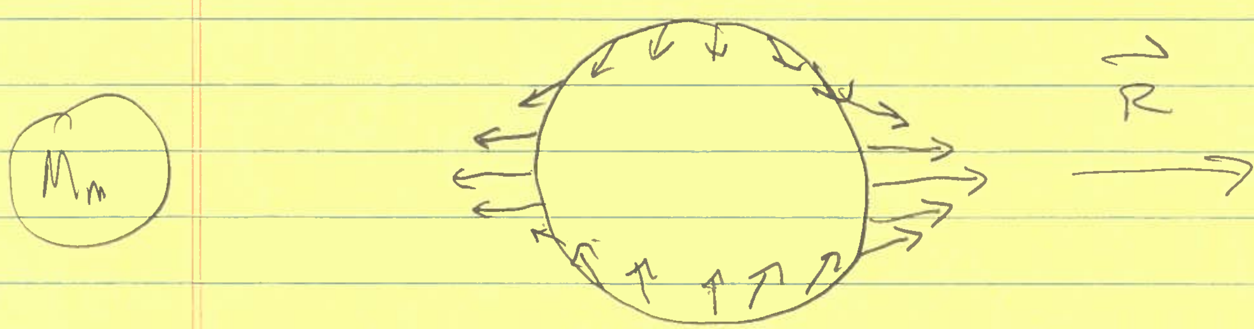
Motion of Earth:

$$M_e\ddot{\vec{r}}_2 = -\frac{GM_eM_m}{R^2}\hat{R}$$

Relative motion of water (m) + Earth:

$$(\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2) = \ddot{\vec{r}} = \frac{-GM_e}{r^2}\hat{r} \underbrace{- \frac{GM_m}{d^2}\hat{d} + \frac{GM_m}{R^2}\hat{R}}_{\text{tide generating force}}$$

(Last term not a force on m, but rather represents 'inertial force')



tide-generating force

We want force on small packet of water m:

$$① \quad m\ddot{\vec{r}}_1 = -\frac{GmM_e}{r^2}\hat{r} - \frac{GmM_m}{d^2}\hat{d}$$

but Earth itself is attracted, too:

$$② \quad M_e\ddot{\vec{r}}_2 = -\frac{GM_eM_m}{R^2}\hat{R}$$

①/m, ②/Me, subtract:

$$\boxed{\ddot{\vec{r}} = \underbrace{-\frac{GM_e}{r^2}\hat{r}}_{\text{accel. due to E's grav.}} \underbrace{- GM_m\left(\frac{\hat{d}}{d^2} - \frac{\hat{R}}{R^2}\right)}_{\text{tidal accel. due to } M_m}}$$

Express in terms of potential $\left(\frac{PE}{mass}\right)$

$$U(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{F}\cdot d\vec{r}\,'$$

$$\Phi(\vec{r}) = \frac{U(\vec{r})}{m} = -\frac{1}{m}\int_{\infty}^{\vec{r}} F(\vec{r})\cdot d\vec{r}\,'$$

$$\ddot{\vec{r}} = \frac{\vec{F}(r)}{m} = -\vec{\nabla}\Phi(\vec{r})$$

"Calculate" Φ, noting $-\hat{R} \parallel \hat{x}$ "without loss of generality"

$x \leftarrow M — E$ Guess: $\boxed{\Phi(\vec{r}) = -\frac{GM_e}{r} - \frac{GM_m}{d} + \frac{GM_m x}{R^2}}$