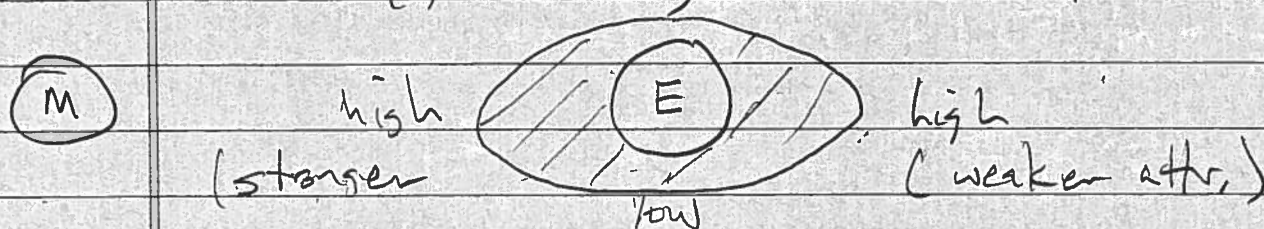
Announcements

PH travel Friday 10/14 and 10/21
Zoom class ONLY both days.

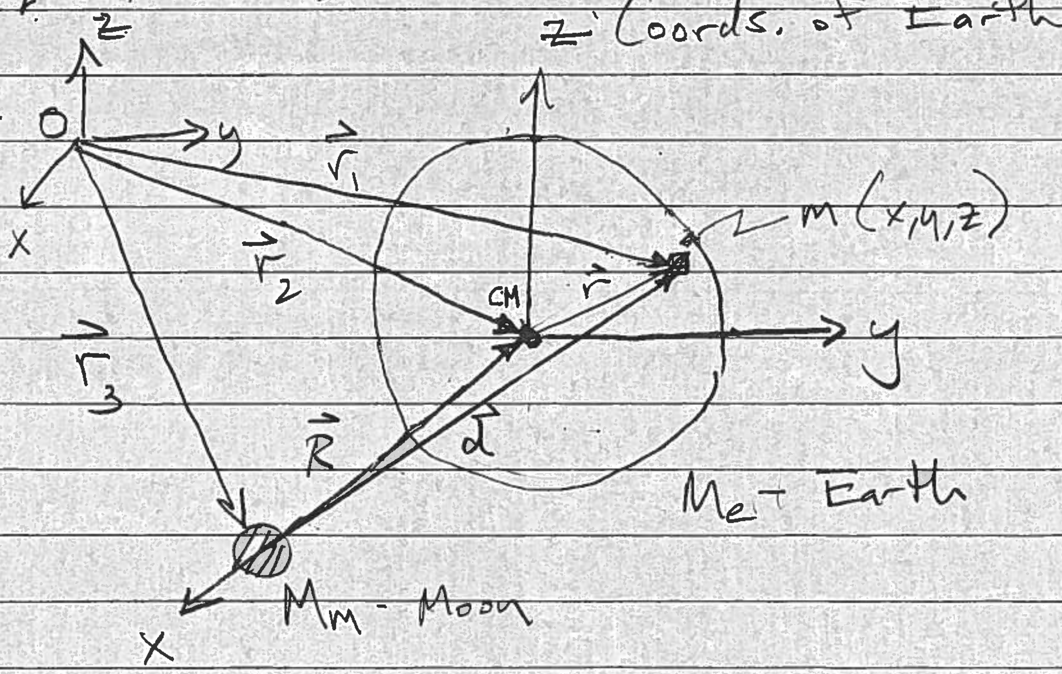
Last time: Accelerated frames.

Linear $\vec{A}$: ball in car, pendulum in car

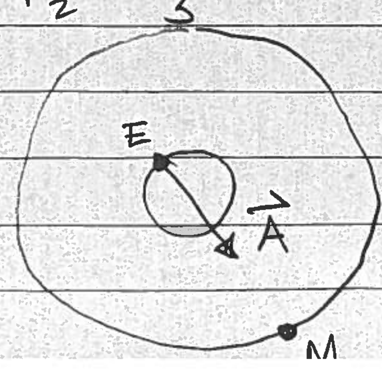
Tides (qualitative): Correct explanation (Newton 1687):



Tides (quantitative):



$$\vec{r} = \vec{r}_1 - \vec{r}_2$$
$$\vec{d} = \vec{r}_1 - \vec{r}_3$$
$$\vec{R} = \vec{r}_2 - \vec{r}_3$$



$\vec{A}$ = acc. of Earth CM
$= v^2/R_e$

R_e = radius of E's circular orbit

$= v^2/R_e$

M

R_e = radius of

packet of water eqn. of motion (1)

$$\boxed{\ddot{\vec{r}} = -\frac{G M_e}{r^2}\hat{r} - G M_m\left(\frac{\hat{d}}{d^2} - \frac{\hat{R}}{R^2}\right)}$$

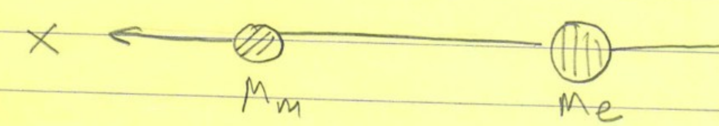
accel. due to E's grav

tidal acc. due to M_m

Next step: express in terms of $\vec{r}$ potential $\Phi(\vec{r}) = U(\vec{r})/m = -\frac{1}{m}\int_0 \vec{F}(\vec{r}\,')\cdot d\vec{r}\,'$

$$\ddot{\vec{r}} = \frac{\vec{F}(\vec{r})}{m} = -\vec{\nabla}\Phi(\vec{r})$$

Chose coord system so that $\hat{R} = \hat{x}$



Guess: $\Phi(\vec{r}) = -\frac{G M_e}{r} - \frac{G M_m}{d} + \frac{G M_m x}{R^2}$

(Can integrate directly but not enlightening)

Just show that $\vec{\nabla}\Phi = \ddot{\vec{r}}$

$$\vec{r} = (x, y, z) \quad |\vec{r}| = r = \sqrt{x^2+y^2+z^2}$$

1st term need $\nabla_{\vec{r}}\frac{1}{r} = \hat{x}\frac{\partial}{\partial x}\frac{1}{r} + \hat{y}\frac{\partial}{\partial y}\frac{1}{r} + \hat{z}\frac{\partial}{\partial z}\frac{1}{r}$

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$\vec{r} = (x, y, z)$

Check: use $\nabla_{\vec{r}}\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$,

(a) $\nabla_{\vec{r}}(x) = \hat{x} = -\hat{R}$ (b)

since $\nabla_{\vec{r}} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

– N.B. $\vec{R}$ (E–M distance) held constant
so since $\vec{d} = \vec{R} + \vec{r}$, $\nabla_{\vec{r}} = \nabla_{\vec{d}}$

$$\Rightarrow \nabla_{\vec{r}}\frac{1}{d} = \nabla_{\vec{d}}\frac{1}{d} = -\frac{\hat{d}}{d^2} \quad (c)$$

Taking (a), (b), (c) together, we verify $\frac{\vec{F}}{m} = -\nabla\Phi$!

Now make use of fact that $r \ll R$ to get simpler approx. expression.

– $d^2 = (R-x)^2 + y^2 + z^2 = R^2 + r^2 - 2Rx$

– So $\frac{1}{d} = \frac{1}{(R^2 - r^2 - 2Rx)^{1/2}} = \frac{1}{R\left(1 - 2x/R + r^2/R^2\right)^{1/2}}$

Taylor expansion in small quantity $\left(\frac{-2x}{R} + \frac{r^2}{R^2}\right) \ll 1$

$$\frac{1}{d} \simeq \frac{1}{R} + \frac{x}{R^2} + \frac{3x^2 - r^2}{2R^3} + \ldots$$

$$\Rightarrow \Phi \simeq -\frac{GM_e}{r} - \underbrace{\cancel{\frac{GM_m}{R}}}_{\text{const!}} - \frac{GM_m}{2R^3}(3x^2 - r^2)$$

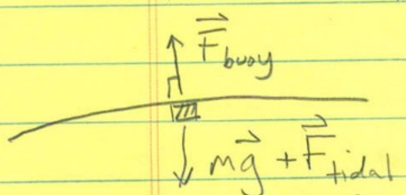
Polar: $x = r\cos\phi\sin\theta$ 3

shift ref. pt., get

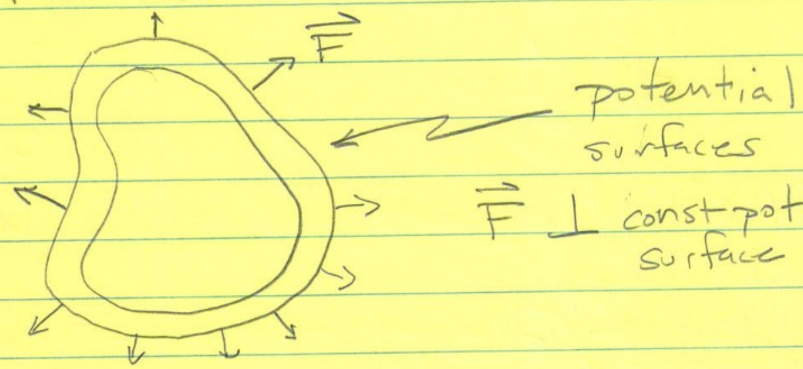
$$\Phi(\vec{r}) = \frac{-GM_e}{r} - \frac{GM_M r^2}{2R^3}\left(3\sin^2\theta\cos^2\phi - 1\right)$$

~~4~~ For equilibrium of ocean's surface, net normal force must vanish

Reminder:



normal to surface



$\vec{F} \perp$ const pot. surface

$\Rightarrow$ surface of ocean is equipotential surface of $\Phi(\vec{r})$

can choose reference const. at will

Pick value $\Phi(\vec{r}) = -\frac{GM_e}{R_e}$ at ocean surf.

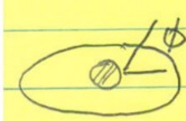
(basically what you would get if moon was absent)

height of tides is then

$$h(\theta,\phi) \equiv r(\theta,\phi) - R_e$$

Set $-\dfrac{GM_e}{R_e} = -\dfrac{GM_e}{r} - \dfrac{GM_m r^2}{2R^3}(\cos^2\phi \sin^2\theta - 1)$

multiply $(-r R_e)$, divide GM_e



$$h = r - R_e = \frac{R_e}{2}\left(\frac{M_m}{M_e}\right)\left(\frac{r}{R}\right)^3 \overbrace{(3\cos^2\phi \sin^2\theta - 1)}^{s(\theta,\phi)}$$

Angular factor $s(\theta,\phi) = \begin{cases} \phi = 0, \pi & \text{max} \\ \phi = \pi/2, 3\pi/2 & \text{min} \end{cases}$

varies w/ θ (latitude)

Note no tides at poles ($\theta = 0$) in this approx.

take $r \simeq R_e$

max-min tide $\Delta h \simeq \dfrac{3}{2} \dfrac{M_m}{M_e} \dfrac{R_e^4}{R^3}$

(km and

$$= \frac{3}{2}\left(\frac{7.3}{590}\right)\left(\frac{6{,}371}{384{,}000}\right)^3 6.371 = 0.56\text{ m}$$

$M_m = 7.3 \times 10^{22}$ kg

$M_e = 5.9 \times 10^{24}$ kg

avg. variation of tides around earth

$R_e = 6371$ km

$R_{e-m} = R = 384{,}000$ km

Note our calculation also works for sun $m \to s$ $\Delta h = \dfrac{3}{2}\dfrac{M_s}{M_e}\dfrac{R_e^4}{R_{s-e}^3}$

find $\dfrac{\Delta h_{moon}}{\Delta h_{sun}} = 2.2$

Sun's effect on tides smaller, not negligible!