10/17/22

Announcements

HW 4 due Oct. 21
Lecture 10/21 zoomed.
Put HW in Chao Zhang mailbox

Last time Newton's laws in rotating frame

- Ang momentum vector $\vec{\omega} = \omega\hat{\omega}$
  Direction $\hat{\omega}$ given by rh rule
- Addition of angular velocities $\vec{\omega}_{31} = \omega_{32} + \underbrace{\vec{\Omega}}_{\vec{\omega}_{12}}$
- Time derivative in rotating frame

$$\left.\frac{d\vec{Q}}{dt}\right|_{S_0} = \left.\frac{d\vec{Q}}{dt}\right|_{S} + \vec{\Omega}\times\vec{Q} \qquad \text{arb. vector } \vec{Q}$$

inertial rotating

Apply to vector $\vec{r}$ - position of particle in S

showed

$$\left.\frac{d^2\vec{r}}{dt^2}\right|_{S_0} = \left.\frac{d}{dt}\right|_S\left(\left.\frac{d\vec{r}}{dt}\right|_S + \vec{\Omega}\times\vec{r}\right) + \vec{\Omega}\times\left(\left.\frac{d\vec{r}}{dt}\right|_S + \vec{\Omega}\times\vec{r}\right)$$

$$= \left.\frac{d^2\vec{r}}{dt^2}\right|_S + 2\Omega\times\left.\frac{d\vec{r}}{dt}\right|_S + \vec{\Omega}\times\vec{\Omega}\times\vec{r}$$

Use notation $\dot{\vec{Q}} = \left.\frac{d\vec{Q}}{dt}\right|_S$ Multiply $\times\, m$

$$\underbrace{m\left.\frac{d^2\vec{r}}{dt^2}\right|_{S_0}}_{\text{Newt.}} = \boxed{\vec{F}_{S_0} = m\ddot{\vec{r}} + \underbrace{2m\vec{\Omega}\times\dot{\vec{r}}}_{-\text{Coriolis Force}} + \underbrace{m\vec{\Omega}\times(\vec{\Omega}\times\vec{r})}_{-\text{Centrifugal F}}}$$

So $m\ddot{\vec{r}}$ in rotating frame is equal to the total force calculated in inertial frame, plus Coriolis + centrifugal terms

N.B. Coriolis force $\propto \dot{\vec{r}}$, so it vanishes if object is at rest in S.
Ignoring angles, rough magnitude of terms are

$$F_{Cor} \sim m v \Omega \quad , \quad F_{cf} = m r \Omega^2$$

For object moving at Earth's surface, Origin of coordinates at CM : $r = R$

$$\frac{F_{cor}}{F_{cf}} = \frac{m v \Omega}{m r \Omega^2} = \frac{v}{(R\Omega)}$$

$V = R\Omega$ vel. of pt. at E's equator

$$V \cong 1000\,mi/h \simeq 0.44\,km/s$$

=> anything at Earths surface going $\ll 1000 \frac{mi}{h}$ doesn't feel much Coriolis force

— Effects of centrifugal force (neglect Coriolis)

1 Free fall on Earth

$$m\ddot{\vec{r}} = \vec{F}_{grav} + \vec{F}_{cf} = m g_0 + m\Omega^2 R \sin\theta\, \hat{\rho}$$

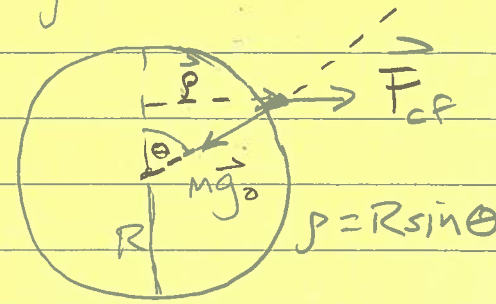
$\rho = r\sin\theta$

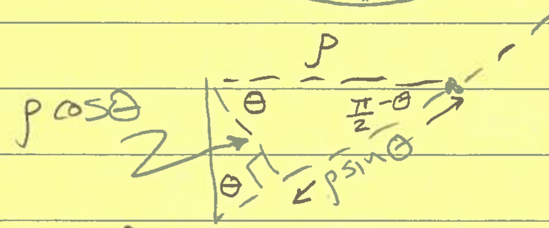
$(\vec{\Omega} \times \vec{r}) \times \vec{\Omega} = r\Omega^2 \sin\theta\, \hat{\rho} = r\Omega^2 \rho\, \hat{\rho}$

so effective $\vec{g} = \vec{g_0} + \Omega^2 \vec{\rho}$

$\vec{\rho} = \rho \sin\theta \, \hat{r} + \rho \cos\theta \, \hat{\theta}$

$= R\left(\sin^2\theta \, \hat{r} + \sin\theta\cos\theta \, \hat{\theta}\right)$





So radial component of $\vec{g}$ is

$$\vec{g} = \underbrace{\left(-g_0 + \Omega^2 R \sin^2\theta\right)\hat{r}}_{\text{"g" at E'surface reduced}}$$

$$+ \underbrace{\Omega^2 R \sin\theta\cos\theta \, \hat{\theta}}_{\text{tangential component}}$$

At equator, cf is max, grad min,
and $g_{tang} = 0$

poles, $g = g_0$

45° g_{tang} is max

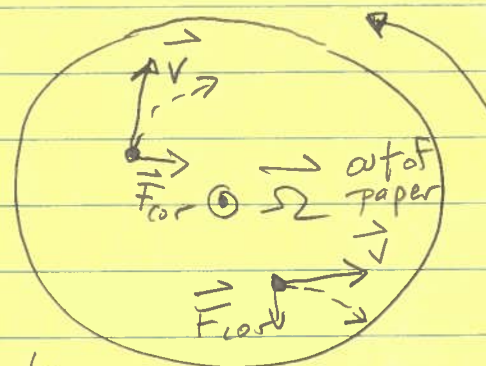
— Coriolis force

$$\vec{F}_{cor} = 2m\dot{\vec{r}} \times \vec{\Omega}$$

* similarity with Lorentz force on charged particle $q\vec{v} \times \vec{B}$?

* acts at right angles to $\dot{\vec{r}}$ and $\vec{\Omega}$

Turntable analogy: ball rolling or sliding on moving disk



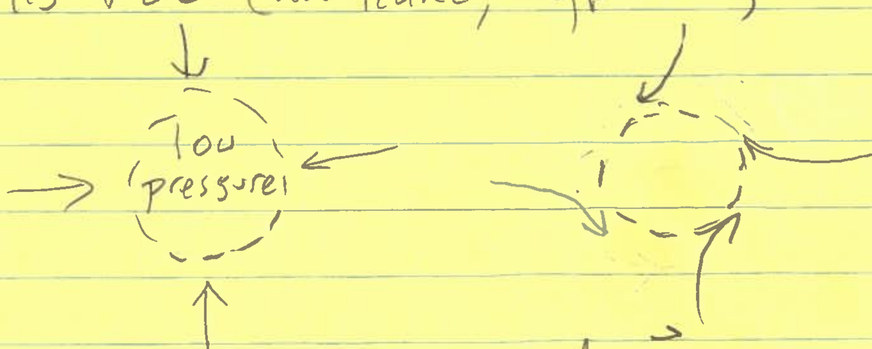
Nearly same as Earth viewed from above N. Pole

in N. hemisphere $\vec{F}_{cor}$ pushes objects moving nearly tangentially to the right

" " S. hemisphere " " left

cyclonic motion can originate from coriolis force (hurricane, typhoon)

shown in N hemisphere



w/o $\vec{F}_{cor}$ with $\vec{F}_{cor}$