Announcements

Quiz today
HW 4 due Friday
Finish reading Ch. 9

Last time

Newton in rotating frame

$$m\ddot{\vec{r}} = \vec{F} + 2m\,\dot{\vec{r}}\times\vec{\Omega} + m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$$

$m\ddot{\vec{r}} = m\frac{d^2\vec{r}}{dt^2}\Big|_S$, $\vec{F} = m\frac{d^2\vec{r}}{dt^2}\Big|_{S_0}$

$2m\,\dot{\vec{r}}\times\vec{\Omega}$: Coriolis force

$m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$: centrifugal force

$$\frac{F_{cor}}{F_{cf}} = \frac{v}{R_e\Omega}$$

$R_e\Omega$ = vel of Earth's surf at equator $\simeq$ 1000 mi/h

so $|\vec{F}_{cor}|$ weak unless $v \sim R_e\Omega$

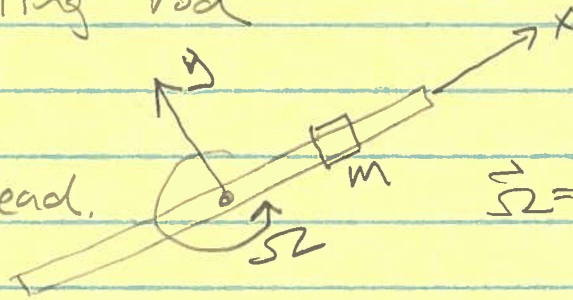
Effective grav. acceleration

$$\vec{g} = \underbrace{\left(-g_0 + \Omega^2 R\sin^2\theta\right)\hat{r}}_{\text{radial}} + \underbrace{\Omega^2 R\sin\theta\cos\theta\,\hat{\theta}}_{\text{tang}}$$

radial corr. at equator
$\theta = 90^\circ$ = 0.034 m/s^2

Example 1: rotating rod

Q: Find eqns. of motion for bead.



$\vec{\Omega} = \Omega \hat{z}$

A: $\vec{v} = \dot{x}\hat{x}$ Forces on bead:

a) $\vec{F}_{cf} = m\Omega^2 x \hat{x}$ $\vec{F}_N = N\hat{y}$

$\vec{F}_{cor} = 2m\vec{v} \times \vec{\Omega} = 2m(\dot{x}\Omega)\underbrace{(\hat{x} \times \hat{z})}_{-\hat{y}}$

$= -2m(\dot{x}\Omega)\hat{y}$

Newton in rot. frame

x: $m\ddot{x} = m\Omega^2 x$

y: $N = F_{cor}$ balance

Soln: $x(t) = Ae^{\Omega t} + Be^{-\Omega t}$

At long times $x = Ae^{\Omega t}$, $\dot{x} = A\Omega e^{\Omega t}$

x, N increase exponentially

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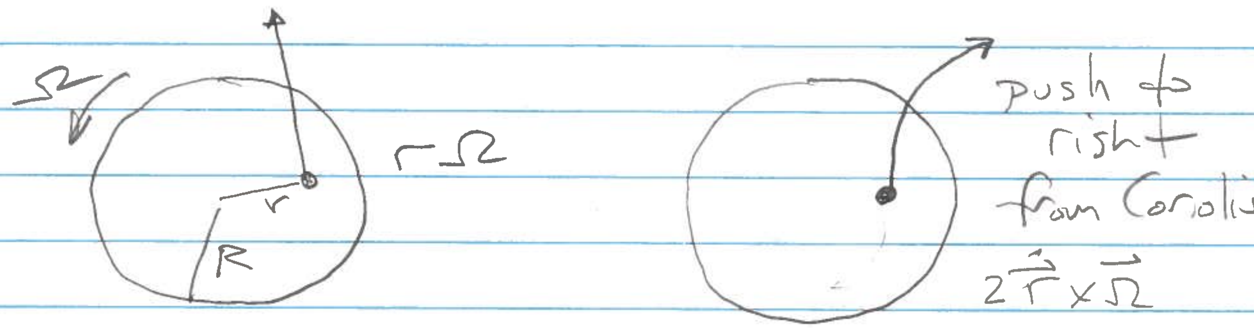
Example problem 2: carousel

frictionless

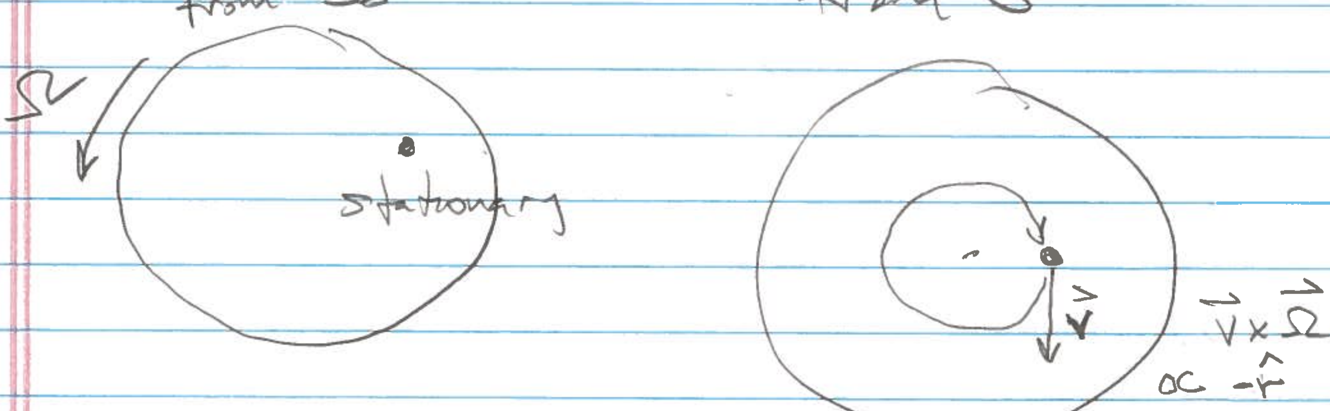
a) release puck on carousel from rest
Describe motion in inertial frame, rotating frame



viewed from S_0 from S

b) release puck at rest in S_0 i.e. lean over the edge and deposit it quickly.

from S_0 from S



$$\vec{F} = 0$$

$$\vec{F}_{cor} = 2m\vec{v}\times\vec{\Omega} = -2m\Omega^2 r\hat{r}$$
$$\vec{F}_{cf} = (\Omega\times\vec{r})\times\vec{\Omega} = m\Omega^2 r\hat{r}$$
$$m\ddot{\vec{r}} = \vec{F}_{cor} + \vec{F}_{cf} = -m\Omega^2 r\hat{r}$$