10/21/22

Announcements

HW4 due today

HW5 posted, due Monday, Oct 31

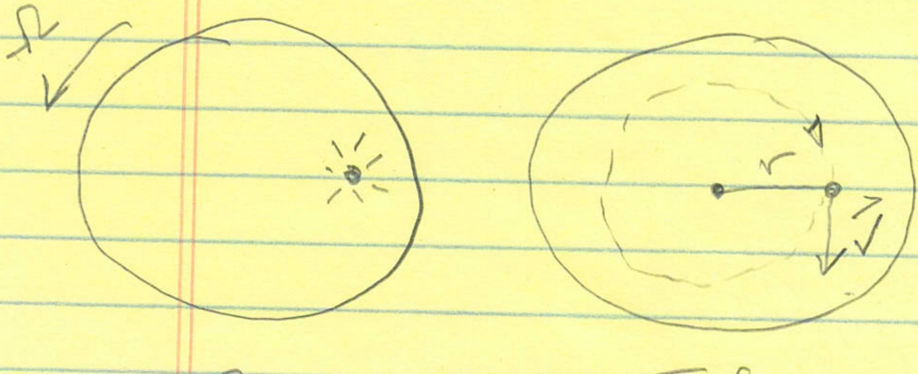
Last time

- quiz remarks

- Example 1 - rotating rod

- Example 2 - puck on frictionless carousel

Puck placed on carousel at rest in S_0, means $v = \Omega r$ in S,



S_0 S frame

(take observer in S at center of carousel)

Coriolis + centr:

$$\left(2\dot{\vec{r}}\times\vec{\Omega} + (\vec{\Omega}\times\vec{r})\times\vec{\Omega}\right)$$
$$= -2\Omega^2 r\hat{r} + \Omega^2 r\hat{r}$$
$$= -\Omega^2 r\hat{r}$$

$\Rightarrow$ executes circular motion in S!

Free fall revisited + Coriolis

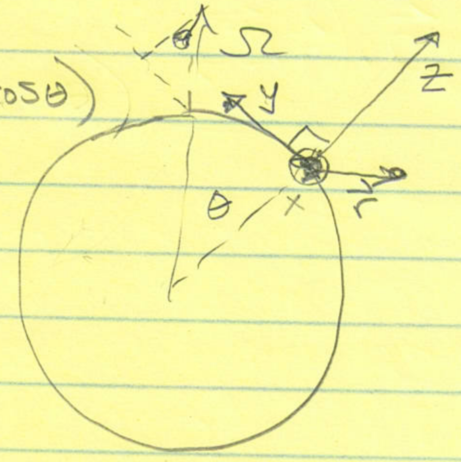
$$m\ddot{\vec{r}} = \underbrace{m\vec{g}_0 + \vec{F}_{cf}}_{m\vec{g}} + \vec{F}_{cor}$$

Force on object released from rest

$$\Rightarrow \boxed{\ddot{\vec{r}} = \vec{g} + 2\dot{\vec{r}} \times \vec{\Omega}} \quad (*)$$

New coords: shift $\vec{r}$ by $\vec{R}_0$

$\vec{\Omega} = (0, \Omega\sin\theta, \Omega\cos\theta)$



$\Rightarrow$ eqn. same!

$\hat{x}$ into paper

$$\dot{\vec{r}} \times \vec{\Omega} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dot{x} & \dot{y} & \dot{z} \\ 0 & \Omega\sin\theta & \Omega\cos\theta \end{vmatrix}$$

$$= \hat{x}(\dot{y}\Omega\cos\theta - \dot{z}\Omega\sin\theta) - \hat{y}\dot{x}\Omega\cos\theta + \hat{z}\dot{x}\Omega\sin\theta$$

$\Rightarrow (*)$ $\ddot{x} = 2\Omega(\dot{y}\cos\theta - \dot{z}\sin\theta)$ $\ddot{y} = -2\Omega\dot{x}\cos\theta$ $\ddot{z} = -g + 2\Omega\dot{x}\sin\theta$

Solve perturbatively

Strategy: Assume Ω is small (check afterwards to see if answers make sense!)

0th order $\Omega = 0$

includes centrifugal force

$$\Rightarrow \ddot{x} = 0, \quad \ddot{y} = 0, \quad \ddot{z} = -g$$

notation: x_0, y_0, z_0 means $\Omega = 0$

$$\Rightarrow x_0 = 0, \quad y_0 = 0, \quad z_0 = h - \tfrac{1}{2}gt^2 \quad \text{usual}$$

1st order: evaluate 1st order terms using 0th order soln

$$\vec{r} = \vec{r}_0 + \underbrace{\vec{r}_1}_{\text{small}}$$

e.g.

$$\underbrace{\ddot{x}_0}_{0} + \ddot{x}_1 = 2\Omega\left[(\underbrace{\dot{y}_0}_{0} + \dot{y}_1)\cos\theta - (\underbrace{\dot{z}_0}_{-gt} + \dot{z}_1)\sin\theta\right]$$

$$\ddot{x}_1 = 2\Omega\left[\dot{y}_1\cos\theta + (gt - \dot{z}_1)\sin\theta\right]$$

but $\dot{y}_1, \dot{z}_1$ are $\mathcal{O}(\Omega)$

$\dot{z}_0 = -gt$ is $\mathcal{O}(1)$ (i.e., 0th order)

so $\ddot{x}_1 \simeq 2\Omega g t \sin\theta$

$\mathcal{O}(\Omega^2)$

similarly $\ddot{y}_1 = -2\Omega \dot{x}_1 \cos\theta \simeq 0$; $\ddot{z} = -g + 2\Omega \dot{x} \sin\theta \simeq -g$

So to 1st order in Ω,

$$\ddot{x}_1 \cong 2\Omega g t \sin\theta \qquad \ddot{y} = 0 \qquad \ddot{z} = -g$$

$x(0) = 0$ $\Rightarrow$ $x(t) \simeq \frac{1}{3}\Omega g t^3 \sin\theta, \; y = 0, \; z = h - \frac{1}{2}gt^2$

Q: when (if at all) is this approx. valid?

$t \to 0$ We assumed $\dot{x}_1, \dot{y}_1, \dot{z}_1 \ll gt$

e.g. $\dot{x}_1 \simeq \Omega g t^2 \sin\theta \ll gt$

time to fall $t \simeq \sqrt{\frac{2h}{g}}$

$\Rightarrow \underset{\text{take } \sin\theta \simeq 1}{\Omega \ll t^{-1}} \simeq \sqrt{\frac{g}{2h}}$

Ang. vel. of E: $\frac{2\pi}{24\,\text{hrs}} = 7 \times 10^{-5}\,\text{s}^{-1} = \Omega$

Drop ball from $\underline{1\,\text{km}}$: $t_{fall} = \sqrt{\frac{2000}{9.8}} = 14.3\,\text{s}$

$t^{-1} = 0.07\,\text{s}^{-1}$ So $\Omega \ll t^{-1}$ $\underline{\text{OK}}$

Can trust $x_1 = \frac{1}{3}\Omega g t^3 \sin\theta$

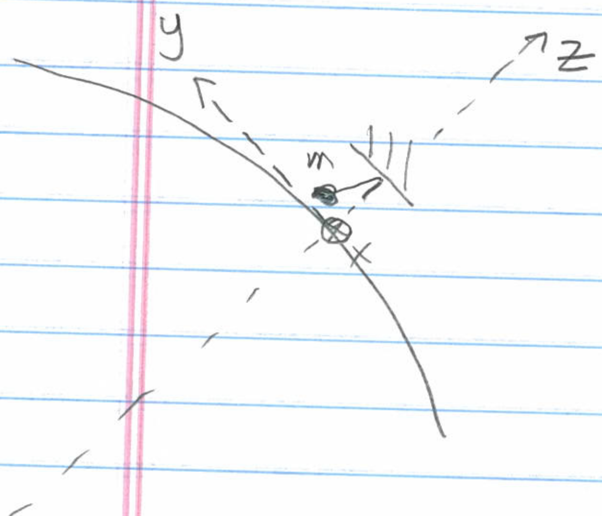
$= (0.67\,\text{m}) \sin\theta$

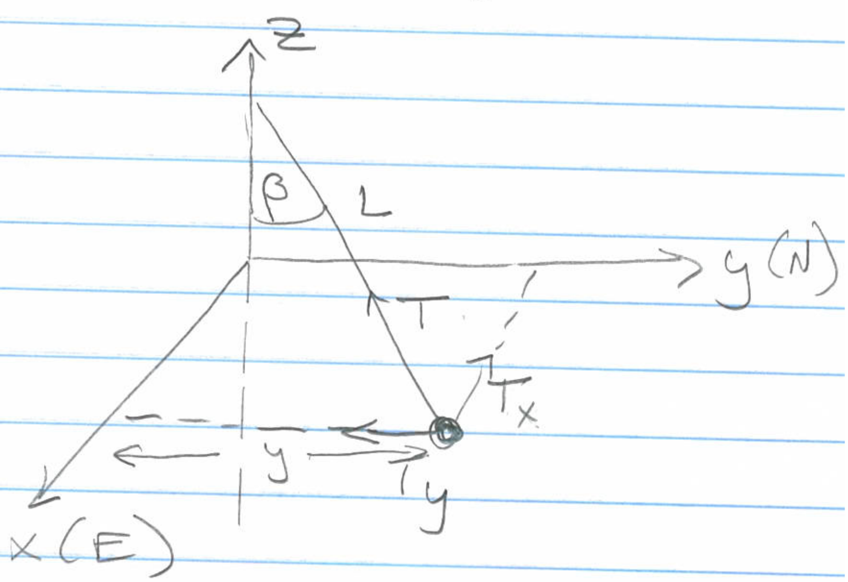
Coriolis force causes 0.1% deflection for $h = 1\,\text{km}$

Foucault pendulum (Jean Foucault)

$$m\ddot{\vec{r}} = \vec{T} + \underbrace{m\vec{g}_0 + m(\vec{\Omega}\times\vec{r})\times\vec{r}}_{m\vec{g}} + 2m\dot{\vec{r}}\times\Omega$$

small oscillations $\quad \vec{T} \simeq mg\hat{z}$ zeroth order





$$\vec{L} = x\hat{x} + y\hat{y} + z\hat{z} \qquad L = \sqrt{x^2+y^2+z^2}$$

$$\vec{T} = T_x\hat{x} + T_y\hat{y} + T_z\hat{z}$$

$$\frac{T_x}{T} = \frac{-x}{L} \qquad \frac{T_y}{T} = \frac{-y}{L}$$

$T \simeq T_z = mg$ (small oscillations)

$$\Rightarrow T_x = -mgx/L \qquad T_y = -mgy/L$$

Check that soln (#) works to 1st order

$$\dot{y} = -A\Omega_z \cos\Omega_z t \cos\omega_0 t$$

$$+ A\omega_0 \sin\Omega_z t \sin\omega_0 t$$

$$\simeq A\omega_0 \sin\Omega_z t \sin\omega_0 t$$

$$\Rightarrow \ddot{y} \simeq + A\omega_0^2 \sin\Omega_z t \cos\omega t$$

1st order term in Ω_z $\quad + 2A\Omega_z \omega_0 \cos\Omega_z t \sin\omega_0 t$ $\quad \ddot{y}_1$

$$\dot{x}_0 = -A\omega_0 \cos\Omega_z t \sin\omega_0 t \qquad \text{0th order}$$

So in $\ddot{y}$ eqn,

$$\ddot{y}_1 \simeq +2A\Omega_z \omega_0 \cos\Omega_z t \sin\omega_0 t$$

cancels

$$+2\Omega_z \dot{x}_0 \simeq -2\Omega_z (A\omega_0 \cos\Omega_z t \sin\omega_0 t)$$

Similarly for $\ddot{x}_1$ and $\dot{y}_0$