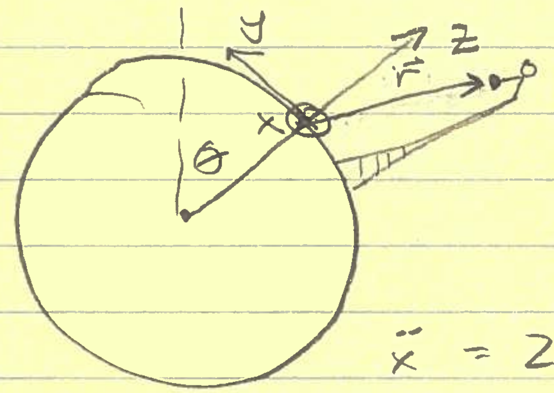
Announcements

HW 5 posted due Oct 31

Last time

- Free fall at E's surface incl. Coriolis



$$\vec{\Omega} = (0, \Omega\sin\theta, \Omega\cos\theta)$$

$$\ddot{\vec{r}} = \vec{g} + 2\dot{\vec{r}} \times \vec{\Omega}$$

$$\ddot{x} = 2\Omega(\dot{y}\cos\theta - \dot{z}\sin\theta)$$
$$\ddot{y} = -2\Omega\dot{x}\cos\theta$$
$$\ddot{z} = -g + 2\Omega\dot{x}\sin\theta$$

Solved perturbatively $\vec{r} = \underset{(\Omega=0)}{\vec{r}_0} + \underbrace{\vec{r}_1}_{\text{small}}$

1st order
$$\begin{cases} x \simeq 2\Omega g t^3 \sin\theta \\ y \simeq 0 \\ z \simeq h - \frac{1}{2}gt^2 \end{cases}$$

- Foucault pendulum: $m\vec{g}$ + Coriolis

$\Omega_z = \Omega\cos\theta$

$w_0 = \sqrt{g/l}$

$$\left.\begin{aligned} x &= A\cos\Omega_z t\cos w_0 t \\ y &= -A\sin\Omega_z t\cos w_0 t \end{aligned}\right\}$$ slowly rotating plane of oscillation at w_0

Ch. 10 Motion of Rigid Bodies

Linear motion of many particles:
Reminder: CM moves in response to total ext. force

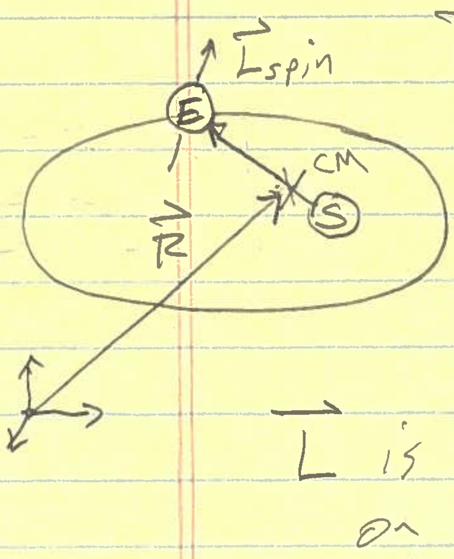
$$\vec{F}_{tot} = M\vec{a}_{cm} \qquad M = \sum_\alpha m_\alpha$$

What about angular momentum of CM of a) rigid body; b) bunch of particles?

Turns out (see book), if we put $\vec{r}_\alpha = \vec{R} + \vec{r}_\alpha'$ for each particle

can show $\vec{L} = \vec{L}_{\text{motion of cm}} + \vec{L}_{\text{motion relative to cm}}$

Most common example: rotating planet revolving around the sun

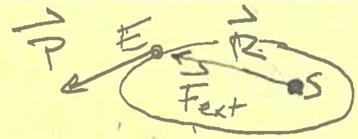


$$\vec{L} = \vec{L}_{orb} + \vec{L}_{spin}$$

$$\underbrace{\vec{R} \times \vec{P}}_{CM} \qquad \underbrace{\sum_\alpha \vec{r}_\alpha{}' \times m_\alpha \dot{\vec{r}}_\alpha{}'}_{\text{relative motion}}$$

$\vec{L}$ is conserved if no external $\vec{\tau}$ on system

but also (approximately) $\vec{L}_{orb}$, $\vec{L}_{spin}$ conserved independently



Ind. cons. of $\vec{L}_{orb}$, $\vec{L}_{spin}$ approx. since { planets, sun not pts. / sun not infinitely massive

$$\dot{\vec{L}}_{orb} = \underbrace{\dot{\vec{R}}\times\vec{P}}_{0} + \vec{R}\times\dot{\vec{P}} \simeq \vec{R}\times\vec{F}_{ext} \underset{\text{only if } F \text{ central}}{\simeq} 0$$

$\vec{R} \parallel \vec{F}_{ext} = M\ddot{\vec{R}}$

(grav. force of sun on planet central if it can be treated as pt. (sphere))

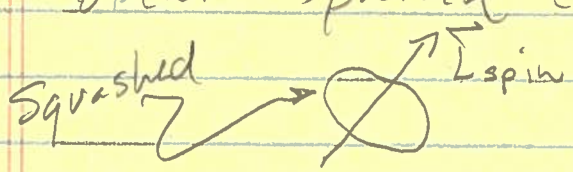
$$\dot{\vec{L}}_{spin} = \dot{\vec{L}} - \dot{\vec{L}}_{orb}$$

$$= \underbrace{\sum_\alpha \vec{r}\,'_\alpha \times \vec{F}_{ext} + \vec{R}\times\vec{F}_{ext}}_{\dot{\vec{L}}} - \underbrace{\vec{R}\times\vec{F}_{ext}}_{\dot{\vec{L}}_{orb}}$$

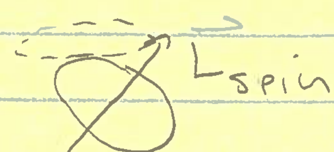
$$= \sum_\alpha \vec{r}\,'_\alpha \times \vec{F}_{ext} = \vec{\tau}^{\,CM}_{ext}$$

i.e. torque of external force about CM

If Earth were perfect uniform sphere, this would be zero, but it's an oblate spheroid (tiny effect)



⇒ "precession of equinoxes"



period 26,000 years

Current pole star is Polaris

NB ancient Greeks were aware of precession of the equinoxes, which is flat out amazing. In 3000 BC pole star was Thuban, 5 times fainter than Polaris.

Kinetic energy of rotating body

again sum of CM motion and relative motion

$$T = T_{\text{motion of CM}} + T_{\text{motion relative to CM}}$$
$$= \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} \sum_\alpha m_\alpha \dot{\vec{r}}_\alpha^{\,2}$$

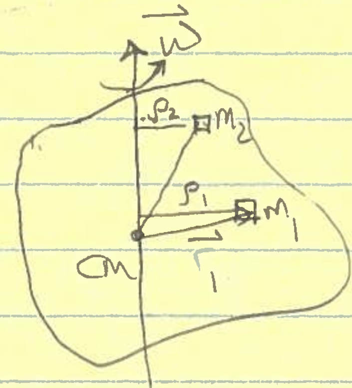
for system of particles. For rigid body only possible relative motion is rotation

Potential energy $\quad U = U_{ext} + U_{int}$

U_{int} is sum of all pairwise interactions of particles, assumed to be conservative + depend only on distance, but the distances $\vec{r}_{\alpha\beta}$ are fixed, so $U_{int} = \text{const}$

Consider only external forces.

Rotation about fixed axis



$\uparrow z$ along $\vec{\omega} = (0, 0, \omega)$

(use $\vec{r}_\alpha$ for dist. from CM now)

$$\vec{L} = \sum_\alpha \vec{r}_\alpha \times m_\alpha \dot{\vec{r}}_\alpha$$

Ch. 9

$$\vec{v}_\alpha = \vec{\omega} \times \vec{r}_\alpha = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ x_\alpha & y_\alpha & z_\alpha \end{vmatrix} = \omega(-\hat{x} y_\alpha + \hat{y} x_\alpha)$$

$$\vec{\ell}_\alpha = m_\alpha \vec{r}_\alpha \times \vec{v}_\alpha = m_\alpha \omega \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x_\alpha & y_\alpha & z_\alpha \\ -y_\alpha & x_\alpha & 0 \end{vmatrix}$$

$$= \omega \left[-\hat{x} z_\alpha x_\alpha - \hat{y} z_\alpha y_\alpha + \hat{z}(x_\alpha^2 + y_\alpha^2) \right]$$

$\rho = \sqrt{x^2 + y^2}$

$$L_z = \sum_\alpha m_\alpha (x_\alpha^2 + y_\alpha^2)\omega = \sum_\alpha m_\alpha \rho_\alpha^2 \omega$$

$$\equiv I_z \omega \quad ; \quad I_z \equiv \sum_\alpha m_\alpha \rho_\alpha^2$$

N.B. each little mass α contributes $m_\alpha \rho_\alpha^2$ to total <u>moment of inertia</u> I_z around z axis.

$\star$ In this situation KE is $T = \frac{1}{2}\sum_\alpha m_\alpha (\underbrace{\rho_\alpha \omega}_{v_\alpha})^2 = \frac{1}{2} I_z \omega^2$