Ch. 14 Scattering Thy

Background:

- 1910 – structure of atom not known

- Thompson + Kelvin: electrons embedded in spherical ball of positive charge ∴ "plum pudding"

- Rutherford, Geiger Marsden fired α-particles (^{4}He nuclei) from radioactive source into thin Au foil.

- w/in Thompson model, average θ deflection of α-particle is

$$\theta_{av} \simeq \frac{z Z e^2}{16\pi\varepsilon_0 R K}$$

from + charge only since e^-'s are light

z = chg of probe; $z = 2$ for α's
Z = atomic # of target; $Z = 79$ for Au
R = radius of atom
K = KE of α

For Rutherford expt, $\theta_{av}^{\text{Thompson}}$ would be 10^{-4} rad $\cong 0.01°$

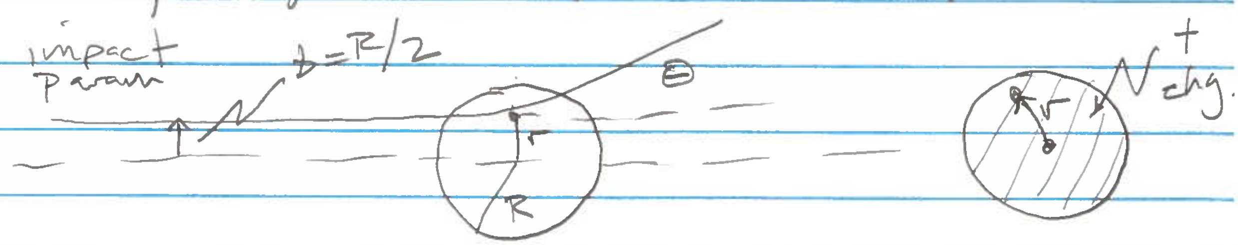
- Instead, small # of α's scattered backwards, 90 - 180 degrees

Rutherford "It was quite the most incredible event that ever happened to me in my life. It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you"

$\longrightarrow$ Rutherford deduces existance of atomic nucleus.

So goal of scattering thy is to deduce structure of target, as well as nature of interactions between probe and target.

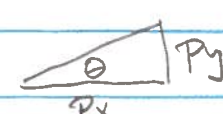
- Scattering angle estimate for Thompson atom:



Impulse in y-direction $p_y \simeq F_y \Delta t$

$$F = \frac{zZe^2}{4\pi\varepsilon_0 R^3}\, r$$ (Physics II Gauss law problem - linear restoring F

Take $r \simeq R/2$; $\theta_{av} \simeq \tan\theta_{av} = \frac{p_y}{p_x}$



$$\Delta t \cong \frac{R}{v}$$

$$\tan\theta_{av} = \frac{P_y}{P_x} = \frac{\frac{zZe^2}{8\pi\varepsilon_0 R^2}\cdot\left(\frac{R}{v}\right)}{mv} = \frac{zZe^2}{16\pi\varepsilon_0 RK}$$

we will calculate Rutherford + other scattering processes in same spirit, but somewhat more precisely.

for Rutherford put in

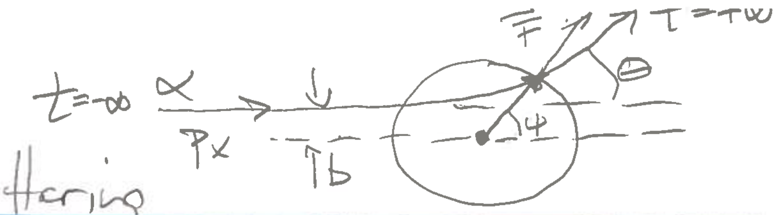
$$z = 2 \quad Z = 72 \quad R \sim 10^{-10}\,m$$

$$m = 4 \text{ nucleons} \simeq 7 \times 10^{-27}\,kg$$

$$K = 3 \times 10^{6}\,eV$$

$$\Rightarrow \quad \theta_{avg} \sim 10^{-4} \ll 1$$

but some α's came back $\theta_{avg} \simeq \pi$



Rutherford scattering

Now assume + charge inside nucleus of radius 1 fm

Same argument: $\left\{ \begin{array}{l} \theta \text{ is asymptotic scattering angle} \\ \psi \text{ is instantaneous direction to particle from nucleus} \end{array} \right.$

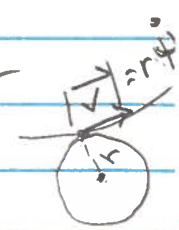
p_y = impulse during scattering $= \int F_y\,dt = \int F\sin\psi\,dt$

$$= \int_{-\infty}^{\infty} \frac{zZke^2}{r^2}\sin\psi\,dt = zZke^2\int_{\pi}^{\theta}\frac{\sin\psi}{r^2}\frac{dt}{d\psi}\,d\psi$$

since π is $\psi(t=-\infty)$
θ is $\psi(t=+\infty)$ by definition

also note angular momentum is conserved in central force problem

$$l(t=-\infty) = \vec{r}\times\vec{p} = rp\sin(\pi-\psi)$$
$$= pb$$
$$= -mr^2\dot{\psi} \text{ later}$$



$\dot{\psi} = \frac{-pb}{mr^2}$

So back to $p_y = \frac{zZke^2}{-bv}\int_{\pi}^{\theta}\sin\psi\,d\psi$

$$= \frac{zZke^2}{bv}\cos\theta\Big|_{\pi}^{\theta} = \frac{zZke^2}{bv}(1+\cos\theta)$$

$$p_y = \boxed{p\sin\theta = \frac{zZke^2}{bv}(1+\cos\theta)}$$

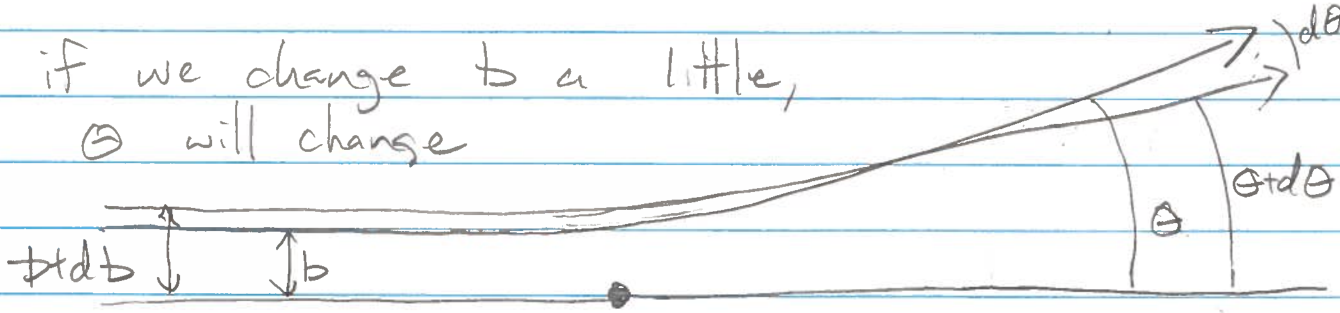
1/2 angle trig identities

$$\frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\theta/2$$

$$\boxed{\tan\frac{\theta}{2} = \frac{zZke^2}{pvb}} \qquad b = \frac{zZke^2}{pv}\cot\frac{\theta}{2}$$

relates impact parameter, incoming momentum and scattering angle

if we change b a little, θ will change



If the α is somewhere in a ring of radius b thickness db, it is scattered into a ring of θ's at the detector

$$\text{area} \quad 2\pi b\,db = \left(\frac{zZke^2}{pv}\right)^2 \frac{\pi\cot\theta/2}{\sin^2\theta/2}\,d\theta$$

Scattering problem for beam + target

PAUSE: this was for 1 α incident on 1 nucleus. Now let's imagine a finite beam of α's incident on a target of Au foil with many atoms.

Defs.
- total # beam particles N_{inc}
- # scattering centers / area $\perp$ to beam n_{tar}
- total # scattered particles N_{sc}
- "area" of target atom σ