11/16/22

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Announcements

PH travel 11/18 11/21 zoom classes
Exam 2 11/30 in-class chs 9-11, 14
Review in class and at 6pm 2205, 11/28
HW 7 due 11/28

Last time

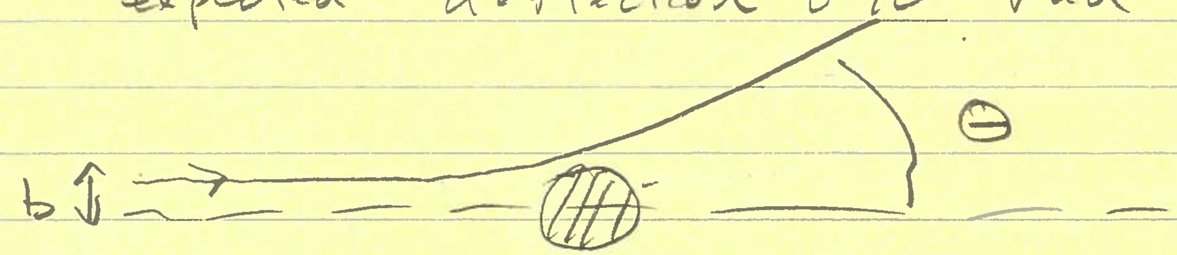
Intro to scattering thy

- Historical 1910
  Rutherford - Geiger - Marsden
  discover atomic nucleus

  backscattering of α's from Au foil

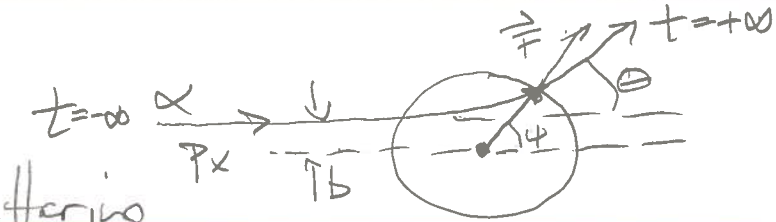
  Thompson "plum pudding" atom:

  expected deflection $\theta \sim 10^{-4}$ rad



final momentum p_y'/p_x' ; $p_x' \simeq p_x$

$$\theta = \tan \frac{p_y}{p_x} \simeq \frac{F_y \Delta t}{mv} \simeq 10^{-4}$$



Rutherford scattering

Now assume + charge inside nucleus of radius 1 fm

Same argument: $\begin{cases} \theta \text{ is asymptotic scattering angle} \\ \psi \text{ is instantaneous direction to particle from nucleus} \end{cases}$

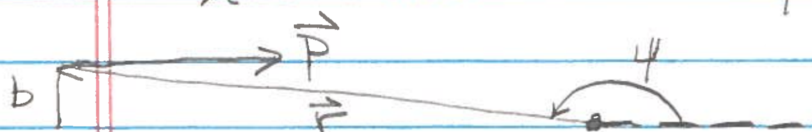
p_y = impulse during scattering $= \int F_y\,dt = \int F \sin\psi\,dt$

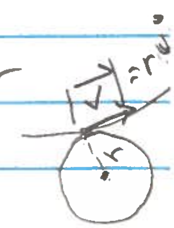
$$= \int_{-\infty}^{\infty} \frac{zZke^2}{r^2} \sin\psi\,dt = zZke^2 \int_{\pi}^{\theta} \frac{\sin\psi}{r^2} \frac{dt}{d\psi}\,d\psi$$

since π is $\psi(t=-\infty)$
θ is $\psi(t=+\infty)$ by definition

also note angular momentum is conserved in central force problem

$$\ell(t=-\infty) = \vec{r}\times\vec{p} = rp\sin(\pi-\psi)$$
$$= pb$$
$$= -mr^2\dot{\psi} \quad \text{later}$$





$\dot{\psi} = \frac{-pb}{mr^2}$

So back to $p_y' = \frac{zZke^2}{-bv} \int_{\pi}^{\theta} \sin\psi\,d\psi$

$$= \frac{zZke^2}{bv} \cos\theta \Big|_{\pi}^{\theta} = \frac{zZke^2}{bv}(1+\cos\theta)$$

final mom. $p_y' = \boxed{p\sin\theta = \frac{zZke^2}{bv}(1+\cos\theta)}$

1/2 angle trig identities

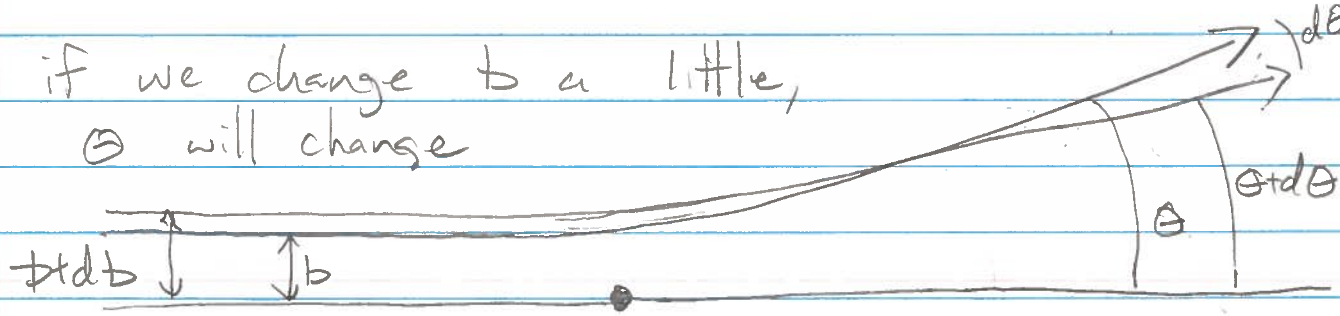
$$\frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\theta/2$$

$$\boxed{\tan\frac{\theta}{2} = \frac{zZke^2}{pvb}} \qquad b = \frac{zZke^2}{pv}\cot\frac{\theta}{2}$$

relates impact parameter, incoming momentum and scattering angle

$$\frac{d\cot\frac{\theta}{2}}{d\theta} = -\frac{1}{2}\csc^2\frac{\theta}{2} = -\frac{1}{2\sin^2\frac{\theta}{2}}$$

if we change b a little, θ will change



If the α is somewhere in a ring of radius b thickness db, it is scattered into a ring of θ's at the detector

$$\text{area} \quad 2\pi b\,db = \left(\frac{zZke^2}{pv}\right)^2 \frac{\pi\cot\theta/2}{\sin^2\theta/2}(-d\theta)$$

Scattering problem for beam + target

PAUSE: this was for 1 α incident on 1 nucleus, diff. θ's

Now let's imagine a finite beam of α's incident on a target of Au foil with many atoms.

Defs.
- total # beam particles N_{inc}
- # scattering centers / area ⊥ to beam n_{tar}
- total # scattered particles N_{sc}
- "area" of target atom σ

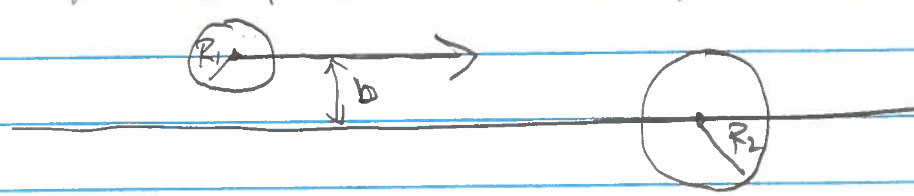
Scattering of a beam cont'd

Basic eqn. $\boxed{N_{sc} = N_{inc}\, n_{tar}\, \sigma}$

Point 1

E.g. shooting at crows in a tree
Each crow has area σ, areal density n_{tar} => $n_{tar}\sigma$ is the fractional area of crows, < 1.

But what is "area" of a gold atom?
Before answering, consider finite size beam particle (sphere radius R_1) incident on target (sphere radius R_2)



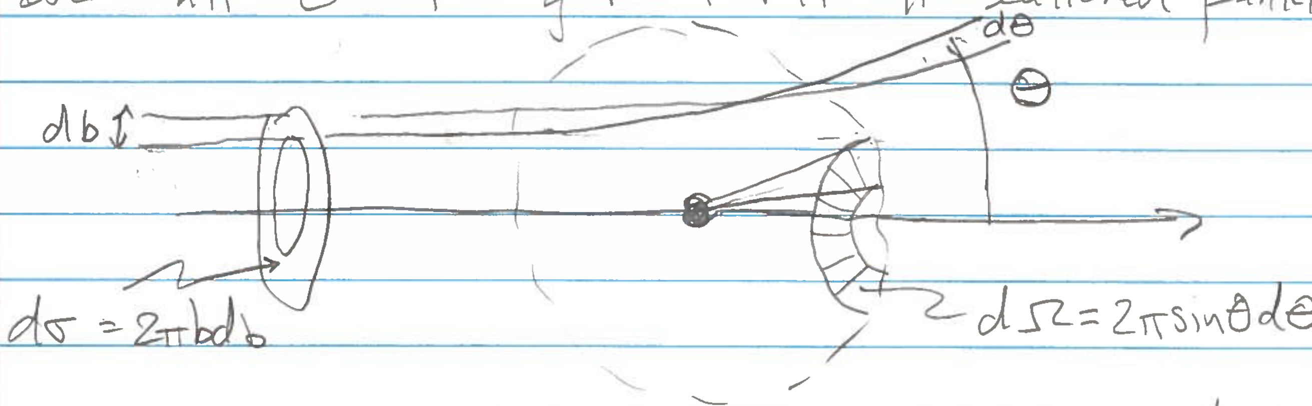
For hard spheres, scattering occurs unless $R_1 + R_2 < b$ => $\sigma = \pi (R_1+R_2)^2$
("effective area")

★ Point: σ depends on both beam particle and target particle, and interaction between them. Goal: calculate σ for model of interaction, & compare with experiment.

More information: what angle θ did scattered particles emerge at, + how does this reflect nature of interactions + scattering particles

Back to Rutherford:

Difference w/ crows: if bullet hit crow it was considered "scattered" w/ probability 1. Now we account for prob. scattering into particular angle – later we will integrate over all θ to get total # scattered particles



We want "differential cross-section" $\frac{d\sigma}{d\Omega}$

$$N_{sc}(\text{into } d\Omega) = N_{inc}\, n_{tar}\, d\sigma(\text{into } d\Omega)$$

$$d\sigma(\text{into } d\Omega) = \frac{d\sigma}{d\Omega}\, d\Omega$$

ie. if you know $\frac{d\sigma}{d\Omega}$, you can figure out the effective $d\sigma$ for scattering into $d\Omega$

$$\boxed{N_{sc}(\text{into } d\Omega) = N_{inc}\, n_{tar} \left(\frac{d\sigma}{d\Omega}\right) d\Omega}$$

$$d\sigma = 2\pi\, b\, db \overset{\text{Rutherford}}{=} \left(\frac{zZke^2}{pv}\right)^2 \frac{\pi \cot\theta/2}{\sin^2\theta/2}\, d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{\left(\frac{zZke^2}{pv}\right)^2 \frac{\pi \cot\theta/2}{\sin^2\theta/2} \cancel{d\theta}}{2\pi \sin\theta \cancel{d\theta}}$$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$= \left(\frac{zZke^2}{pv}\right)^2 \cdot \frac{1}{4\sin^4\theta/2}$$

$pv = 2K$

$K = \frac{1}{2}mv^2$

$$= \left(\frac{zZke^2}{4K\sin^2\theta/2}\right)^2$$

Rutherford Scatt. formula

*

Reminder: in expt we measure

$$N_{sc}\,(\text{into } d\theta) = N_{inc}\, n_{tar} \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

Now we know $d\sigma/d\Omega$; with knowledge of density of target nuclei, masses, thickness of target, we can calculate n_{tar}. With knowledge of N_{inc}, N_{sc} is determined.

Rutherford, Geiger + Marsden verified prediction *:

- by counting # scatterers/minute at different angles, verified $1/\sin^4\theta/2$ dependence.

- using different thicknesses, Au foil, verified $N_{sc} \propto n_{tar} = \underset{\#\text{ density}}{n}\,\underset{\text{thickness}}{t}$

(breaks down if film is too thick – multiple scattering)

- different targets – verified Z^2 dependence.

- reduced KE of α's by interposing sheets of mica in beam to slow them down – checked $1/K^2$

<u>Remarks</u>

a) we looked at differential scattering cross section $d\sigma/d\Omega$

total cross-section $= \int d\Omega \frac{d\sigma}{d\Omega} = \sigma$

b) can define cross-sections for other <u>events</u> by same basic equation

e.g. $N_{abs} = N_{inc}\, n_{tar}\, \sigma_{abs}$ = # particles absorb or captured

ionized, etc.