11/18/22

Announcements

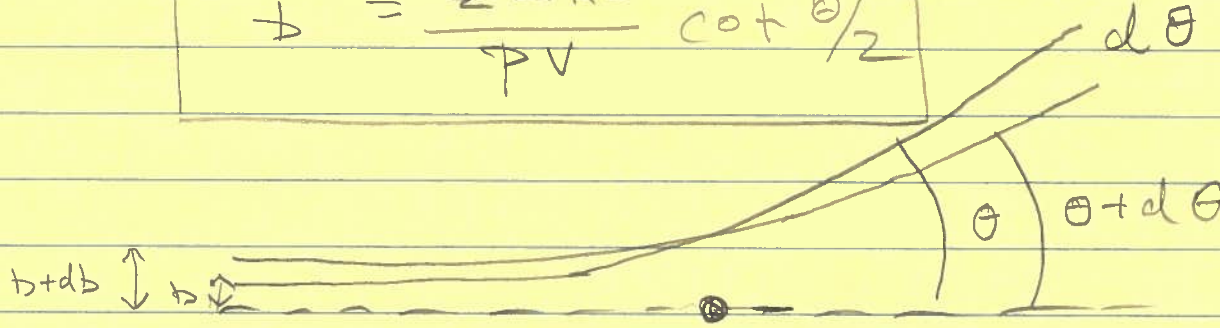
- Class zoomed Monday
- Test 2 11/30 chs 9-11, 14
  - Practice test posted
  - Review in-class 11/28, 6pm 2205 11/28
- HW 6 solns posted, 3(c) notes

Last time

- Rutherford problem

relation between impact parameter b and scattering angle θ:

$$\boxed{b = \frac{zZke^2}{pv}\cot\theta/2}$$



area of incoming particles $\quad 2\pi b\,db = \left(\frac{zZe^2k}{pv}\right)^2 \frac{\pi\cot\theta/2}{\sin^2\theta/2}(-d\theta)$

- scattering cross section σ (area)

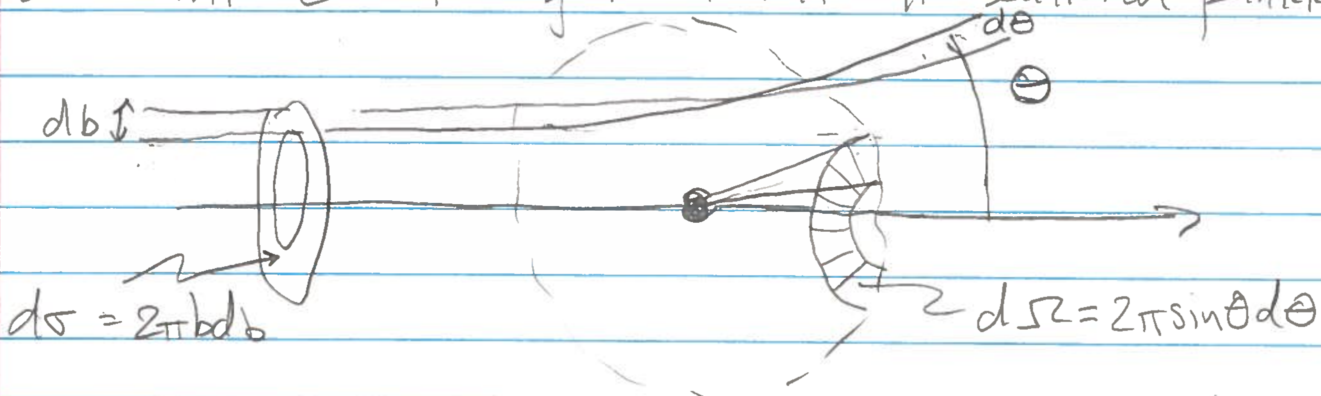
$$N_{sc} = N_{inc}\, n_{tar}\, \sigma$$

N_{sc} = # scattered
N_{inc} = # incident
n_{tar} = # targets/area

- differential x-sec. $dN_{sc} = N_{sc}(\text{into } d\Omega) = N_{inc}\, n_{tar}\left(\frac{d\sigma}{d\Omega}\right)d\Omega$

Back to Rutherford:

Difference w/ crows: if bullet hit crow, it was considered "scattered" w/ probability 1. Now we account for prob. scattering into particular angle - later we will integrate over all θ to get total # scattered particles



We want "differential cross-section" $\frac{d\sigma}{d\Omega}$

$$N_{sc}(\text{into } d\Omega) = N_{inc}\, n_{tar}\, d\sigma(\text{into } d\Omega)$$

$$d\sigma(\text{into } d\Omega) = \frac{d\sigma}{d\Omega}\, d\Omega$$

ie. if you know $\frac{d\sigma}{d\Omega}$, you can figure out the effective $d\sigma$ for scattering into $d\Omega$

$$\boxed{N_{sc}(\text{into } d\Omega) = N_{inc}\, n_{tar} \left(\frac{d\sigma}{d\Omega}\right) d\Omega}$$

$$d\sigma = 2\pi b\, db \overset{\text{Rutherford}}{=} \left(\frac{zZke^2}{pv}\right)^2 \frac{\pi \cot\theta/2}{\sin^2\theta/2}\, d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{\left(\frac{zZke^2}{pv}\right)^2 \frac{\pi \cot\theta/2}{\sin^2\theta/2}\, d\cancel{\theta}}{2\pi \sin\theta\, d\cancel{\theta}}$$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$= \left(\frac{zZke^2}{pv}\right)^2 \cdot \frac{1}{4\sin^4\theta/2}$$

$pv = 2K$

$K = \frac{1}{2}mv^2$

$$= \left(\frac{zZke^2}{4K\sin^2\theta/2}\right)^2$$

Rutherford Scatt. formula (*)

Reminder: in expt we measure

$$N_{sc}\,(\text{into } d\theta) = N_{inc}\, n_{tar} \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

Now we know $d\sigma/d\Omega$; with knowledge of density of target nuclei, masses, thickness of target, we can calculate n_{tar}. With knowledge of N_{inc}, N_{sc} is determined.

Rutherford, Geiger + Marsden verified prediction (*) :

- by counting # scatterers/minute at different angles, verified $1/\sin^4\theta/2$ dependence.

- using different thicknesses Au foil, verified $N_{sc} \propto n_{tar} = n t$ (# density, thickness)
(breaks down if film is too thick – multiple scattering)

- different targets – verified Z^2 dependence.

- reduced KE of α's by interposing sheets of mica in beam to slow them down – checked $1/K^2$

Remarks

a) we looked at differential scattering cross section $d\sigma/d\Omega$

total cross-section $= \int d\Omega \frac{d\sigma}{d\Omega} = \sigma$

b) can define cross-sections for other events by same basic equation

e.g. $N_{abs} = N_{inc}\, n_{tar}\, \sigma_{abs}$ = # particles absorbed or captured

ionized, etc.

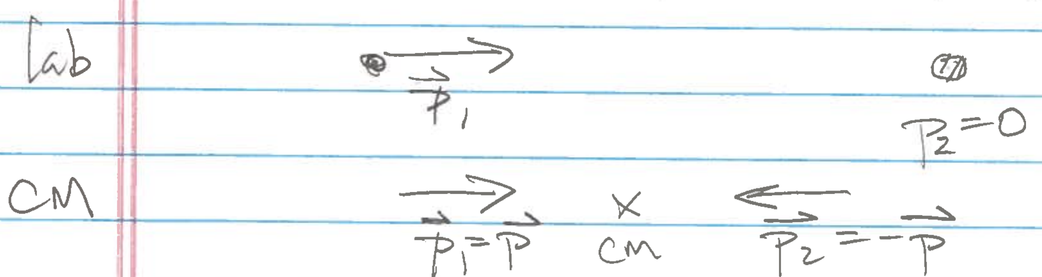
Cross sections in different frames (sec 14.7)

Some statements proved in text but which should be intuitive:

- Momentum for the relative motion of 2 particles (generalized momentum $\frac{\partial \mathcal{L}}{\partial \dot{\vec{r}}}$, $\vec{r} = \vec{r}_1 - \vec{r}_2$)

  is $\vec{p} = \mu \dot{\vec{r}}$ $\quad \mu$ = reduced mass $= \frac{m_1 m_2}{m_1 + m_2}$

In CM frame, $\vec{p}_1 = \vec{p}$, $\vec{p}_2 = -\vec{p}$



Q: how do x-sections transform between lab + CM frames?

Since the basic scattering eqn is true in any frame (it just represents particle conservation)

$$N_{sc}^{cm} = N_{inc}^{cm}\, n_{tar}^{cm}\, \sigma_{cm}$$

$$N_{sc}^{lab} = N_{inc}^{lab}\, n_{tar}^{lab}\, \sigma_{lab}$$

but # incident particles same
scattered " "
areal density of target same

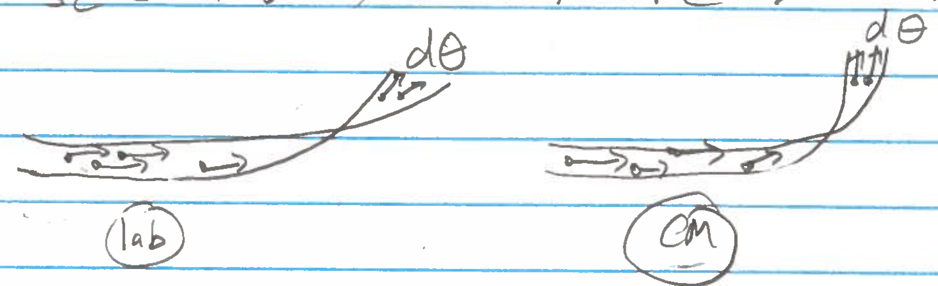
$$\Rightarrow \sigma_{cm} = \sigma_{lab}$$

Individual scattering events may look different in both frames, e.g. $b(\theta)$ will be different, so $\frac{d\sigma}{d\Omega}$ will be different.

But $\sigma = \int d\Omega \frac{d\sigma}{d\Omega}$ will be the same, since physically the same # of particles will be scattered or not.

$$N_{sc}(\text{into } d\Omega) = N_{inc}\, n_{tar} \frac{d\sigma}{d\Omega} d\Omega$$

$N_{sc}(\text{into } d\Omega)$ must be same in both



and again, n_{tar} and N_{inc} are the same

$$\Rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{lab} d\Omega_{lab} = \left.\frac{d\sigma}{d\Omega}\right|_{cm} d\Omega_{cm}$$

$$\Rightarrow \boxed{\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \left.\frac{d\sigma}{d\Omega}\right|_{cm} \left|\frac{d\Omega_{cm}}{d\Omega_{lab}}\right|}$$

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