11/21/22

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Announcements

Ofc hour canceled Tuesday, Wednesday { - sorry - send email }

Test Nov. 30 chs 9-11, 14

Review Mon Nov 28 in-class + 6:00pm NPB 2205

Last time

Rutherford scatt. formula $\frac{d\sigma}{d\Omega} = \left(\frac{zZke^2}{4K\sin^2\theta/2}\right)^2$

Rutherford tested 1) angle dependence ; 2) Z dep.
3) K (kinetic energy of probe particle) dep.

Comparing X-sections in CM + lab frame
(often easier to calculate in CM frame, then transform back to lab)

We decided what changes is $\frac{d\sigma}{d\Omega}$

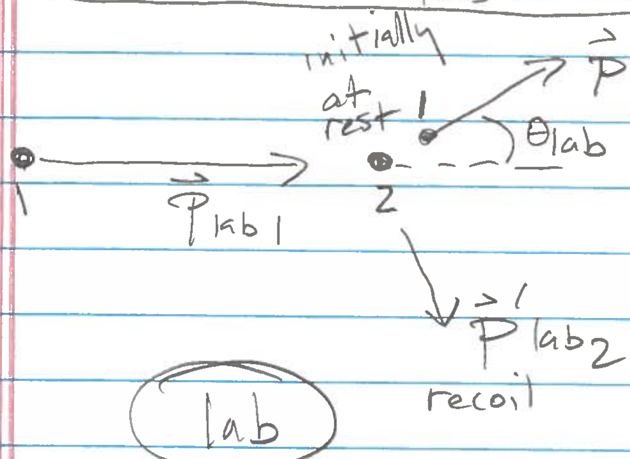
$N_{sc}(\text{into } d\Omega) = N_{inc}\, n_{tar} \frac{d\sigma}{d\Omega} d\Omega$

→ must be same in both

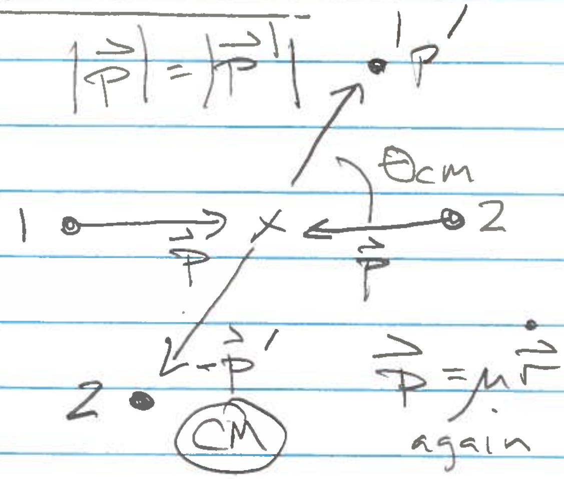
$$\Rightarrow \left.\frac{d\sigma}{d\Omega}\right|_{lab} d\Omega_{lab} = \left.\frac{d\sigma}{d\Omega}\right|_{CM} d\Omega_{CM}$$

$$\Rightarrow \boxed{\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \left.\frac{d\sigma}{d\Omega}\right|_{CM} \left|\frac{d\Omega_{CM}}{d\Omega_{lab}}\right|}$$

Q: How are Θ_{lab} + Θ_{cm} related?



$|\vec{p}| = |\vec{p}\,'|$



$\vec{p} = \mu \dot{\vec{r}}$ again

In lab $\quad \vec{r}_1 = \vec{R} + \frac{m_2}{M}\vec{r} \; ; \quad \vec{r}_2 = \vec{R} - \frac{m_1}{M}\vec{r}$

$$\dot{\vec{r}}_2 = 0 \implies \dot{\vec{R}} = \frac{m_1}{M}\dot{\vec{r}} = \frac{\mu}{m_2}\dot{\vec{r}} = \frac{\vec{p}}{m_2}$$

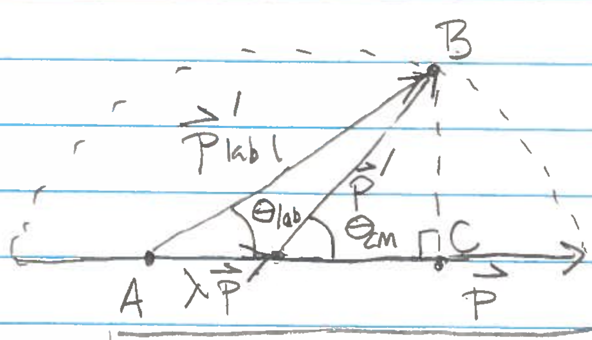
$$\vec{p}_{lab\,1} = m_1 \dot{\vec{r}}_1 = m_1 \dot{\vec{R}} + \frac{m_1 m_2}{M}\dot{\vec{r}} = m_1 \dot{\vec{R}} + \mu \dot{\vec{r}}$$

$$= \frac{m_1}{m_2}\vec{p} + \vec{p} \equiv \lambda \vec{p} + \vec{p}$$

$\lambda = \frac{m_1}{m_2}$ "mass ratio"

Similarly $\vec{p}\,'_{lab\,1} = \lambda \vec{p} + \vec{p}\,'$

$|\vec{p}| = |\vec{p}\,'|$ (elastic scattering)



Consider $\triangle$ ABC

height BC $= p' \sin\Theta_{cm}$

$= p \sin\Theta_{cm}$ $\quad (p = p')$

base $\lambda p + p' \cos\Theta_{cm}$

$= \lambda p + p\cos\Theta_{cm}$

$$\implies \boxed{\tan\Theta_{lab} = \frac{p\sin\Theta_{cm}}{\lambda p + p\cos\Theta_{cm}} = \frac{\sin\Theta_{cm}}{\lambda + \cos\Theta_{cm}}}$$

Explicit example 14.27

equal mass particles $\lambda = 1$

$$\tan\Theta_{lab} = \frac{\sin\Theta_{cm}}{1+\cos\Theta_{cm}}$$

which we recognize as exactly the expression we derived using 1/2-angle trig identites for Rutherford scattering, with substitution

$$\Theta_{lab} = \frac{\Theta_{cm}}{2} \quad \$$$

14.24a Q: How does this result (for special case $m_1 = m_2$) translate into comparison between $\left.\frac{d\sigma}{d\Omega}\right|_{lab}$ + $\left.\frac{d\sigma}{d\Omega}\right|_{cm}$?

- Need $\frac{d\Omega_{cm}}{d\Omega_{lab}}$ for this case

- Note $d\Omega = \sin\theta\, d\theta\, d\phi = -d\cos\theta\, d\phi$

- transformation to lab frame does not affect ϕ

$$\Rightarrow \frac{d\Omega_{cm}}{d\Omega_{lab}} = \left|\frac{d\cos\Theta_{cm}}{d\cos\Theta_{lab}}\right| \overset{\text{from } \$}{=} \left|\frac{d\cos 2\Theta_{lab}}{d\cos\Theta_{lab}}\right|$$

14.24a cont'd

$$= \left| \frac{d\left(2\cos^2\theta_{lab} - 1\right)}{d\cos\theta_{lab}} \right| = \left|4\cos\theta_{lab}\right|$$

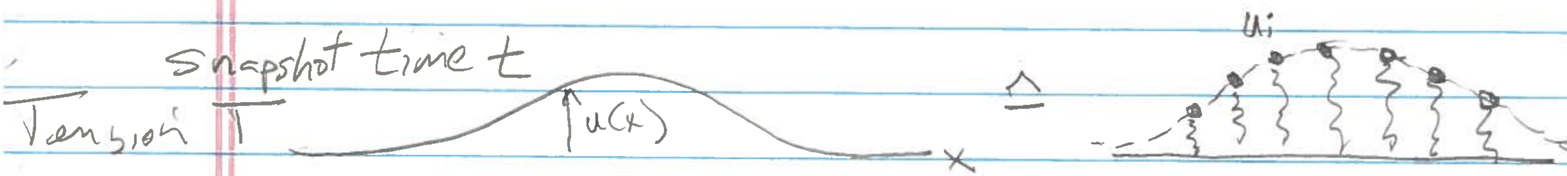
$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \left|4\cos\theta_{lab}\right| \left.\frac{d\sigma}{d\Omega}\right|_{cm}$$

Remark 1) These calculations are important in part because the calculation of $\left.\frac{d\sigma}{d\Omega}\right|_{cm}$ is often much easier → can transform back to lab frame to make predictions for $\frac{d\sigma}{d\Omega}$.

Remark 2) Note all derivations so far assumed nonrelativistic kinematics. Scattering theory concepts are the same for relativistic particles, but expressions will change.

Continuum mechanics (Ch. 16)

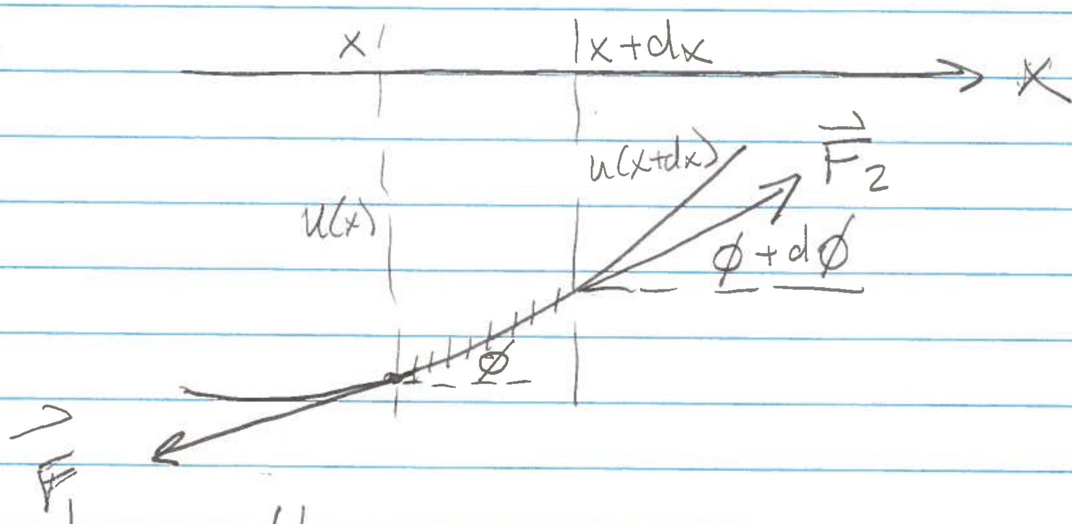
Wave on string $\quad u = \text{displacement} = u(x,t)$



Need partial differential eqn. for wave motion + study normal modes

Consider forces on string "element"

$\phi \ll 1$ small



$$dF_x^{tot} = T\cos(\phi + d\phi) - T\cos\phi$$

$$\cong T\left(\cancel{\cos\phi} - \sin\phi\, d\phi - \cos\phi \frac{d\phi^2}{2}\right) - \cancel{T\cos\phi}$$

$$= -T\left(\sin\phi\, d\phi + \frac{1}{2}\cos\phi\, d\phi^2\right)$$

$$dF_y^{tot} = T\sin(\phi + d\phi) - T\sin\phi$$

$$\cong T\left(\cancel{\sin\phi} + \cos\phi\, d\phi - \sin\phi \frac{d\phi^2}{2}\right) - \cancel{T\sin\phi}$$

$$= T\left(\cos\phi \, d\phi - \sin\phi \, \frac{d\phi^2}{2}\right)$$

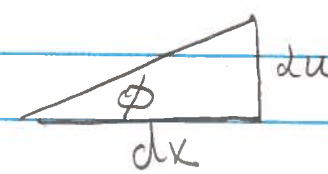
So $d\vec{F}^{tot} \simeq T\left(\sin\phi \, d\phi, \ \cos\phi \, d\phi\right) + \mathcal{O}(d\phi^2)$

$$\simeq T \cos\phi \, d\phi \, \hat{y} \simeq T \, d\phi \, \hat{y}$$

since $\phi \ll 1$ by assumption
(small displacements of string only)

Also note slope of string is

$$\frac{\partial u}{\partial x} = \tan\phi \simeq \phi$$



$$\Rightarrow \quad F_y^{tot} = T \, d\phi = T \frac{\partial \phi}{\partial x} dx$$

$$= T \frac{\partial^2 u}{\partial x^2} dx$$

$$\parallel$$

$$m a_y = m \frac{\partial^2 u}{\partial t^2} = \mu \frac{\partial^2 u}{\partial t^2} dx$$

μ linear density

$$\Rightarrow \quad \boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}}$$

wave equation!

$$c^2 = T/\mu$$