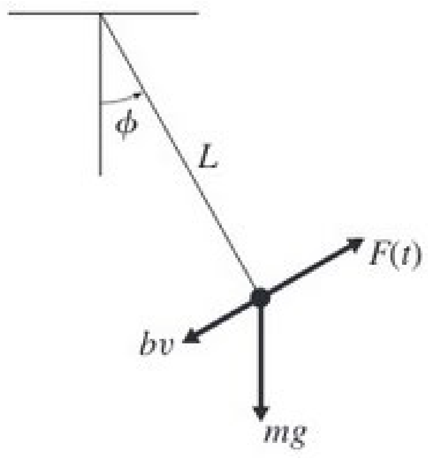
Announcements

- NO IN-PERSON CLASS ON MONDAY, ZOOM ONLY, LINK ON COURSE ANNOUNCEMENTS PAGE
- Reading: Secs. 12.1-12.6,12.9s
- Test 1 on Friday 9/30 in class covers Chs. 8,12,13
- Format: ~1/3 short answer/identification, ~2/3 easy HW-type problems
- BUT: hurricane coming???
- Resources for test studying:
  - a) GRE prep https://www.asc.ohio-state.edu/physics/ugs/livesite/ugs_gre.php
  - b) UF graduate prelim exam https://www.phys.ufl.edu/wp/index.php/graduate/preliminary-exam/
- QUIZ 3

Last time: Chaos through example of Driven Dissipative Pendulum

General: nonlinear, inhomogeneous ODE’s

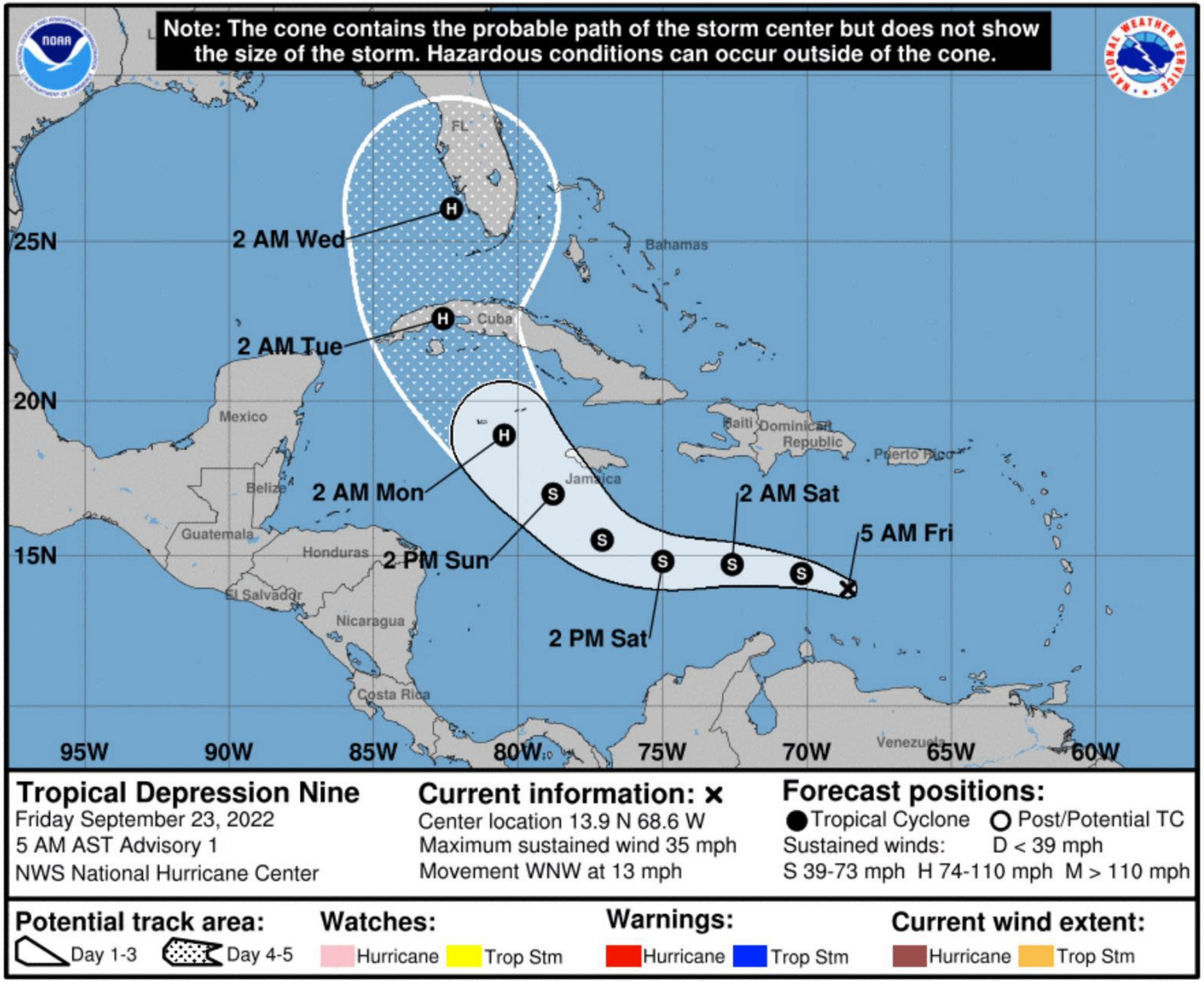
Eqn. in, e.g. x(t):
- nonlinear means no powers of x, x’, x’’ other than 1
- inhomogeneous: term that doesn’t depend on x. E.g. “driving force”



damping, driving force

$$\ddot{\phi} + \frac{b}{m}\dot{\phi} + \frac{g}{L}\sin\phi = \frac{F_0}{mL}\cos\omega t.$$

restoring force


Note: The cone contains the probable path of the storm center but does not show the size of the storm. Hazardous conditions can occur outside of the cone.
FL
Bahamas
Cuba
Mexico
Belize
Guatemala
Honduras
El Salvador
Nicaragua
Costa Rica
Haiti
Dominican Republic
Puerto Rico
Jamaica
Venezuela
2 AM Wed
2 AM Tue
2 AM Mon
2 PM Sun
2 PM Sat
2 AM Sat
5 AM Fri
25N
20N
15N
95W
90W
85W
80W
75W
70W
65W
60W
Tropical Depression Nine
Friday September 23, 2022
5 AM AST Advisory 1
NWS National Hurricane Center
Current information: ×
Center location 13.9 N 68.6 W
Maximum sustained wind 35 mph
Movement WNW at 13 mph
Forecast positions:
Tropical Cyclone
Post/Potential TC
Sustained winds: D < 39 mph
S 39-73 mph H 74-110 mph M > 110 mph
Potential track area:
Day 1-3
Day 4-5
Watches:
Hurricane
Trop Stm
Warnings:
Hurricane
Trop Stm
Current wind extent:
Hurricane
Trop Stm

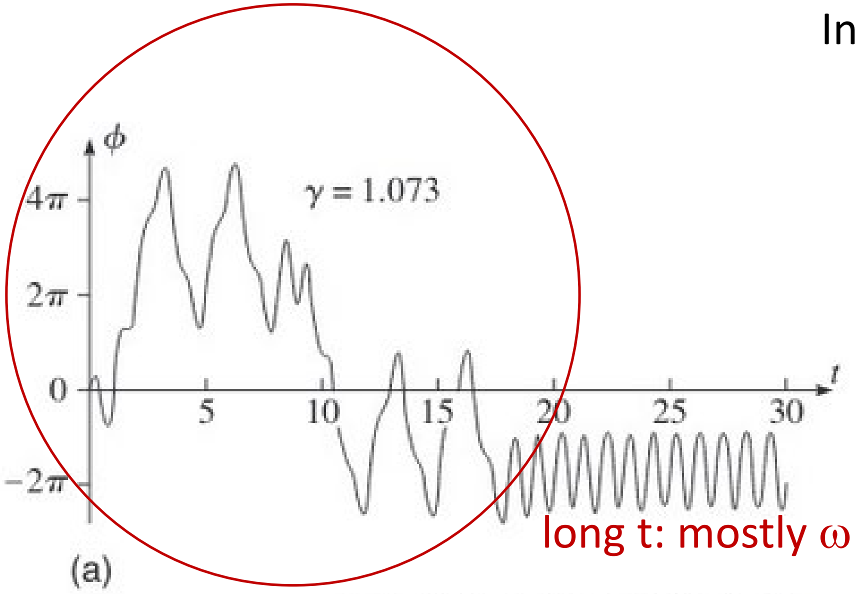
Period doubling route to chaos in dissipative driven pendulum (DDP)

$$\ddot{\phi} + 2\beta\dot{\phi} + \omega_o^2 \sin\phi = \gamma\omega_o^2 \cos\omega t.$$

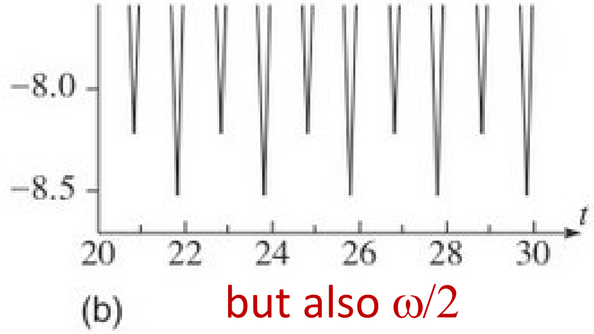
γ = dimensionless "driving force"

Initial conditions: $\phi(0)=0$, $\phi'(0)=0$

short t: "transient" region

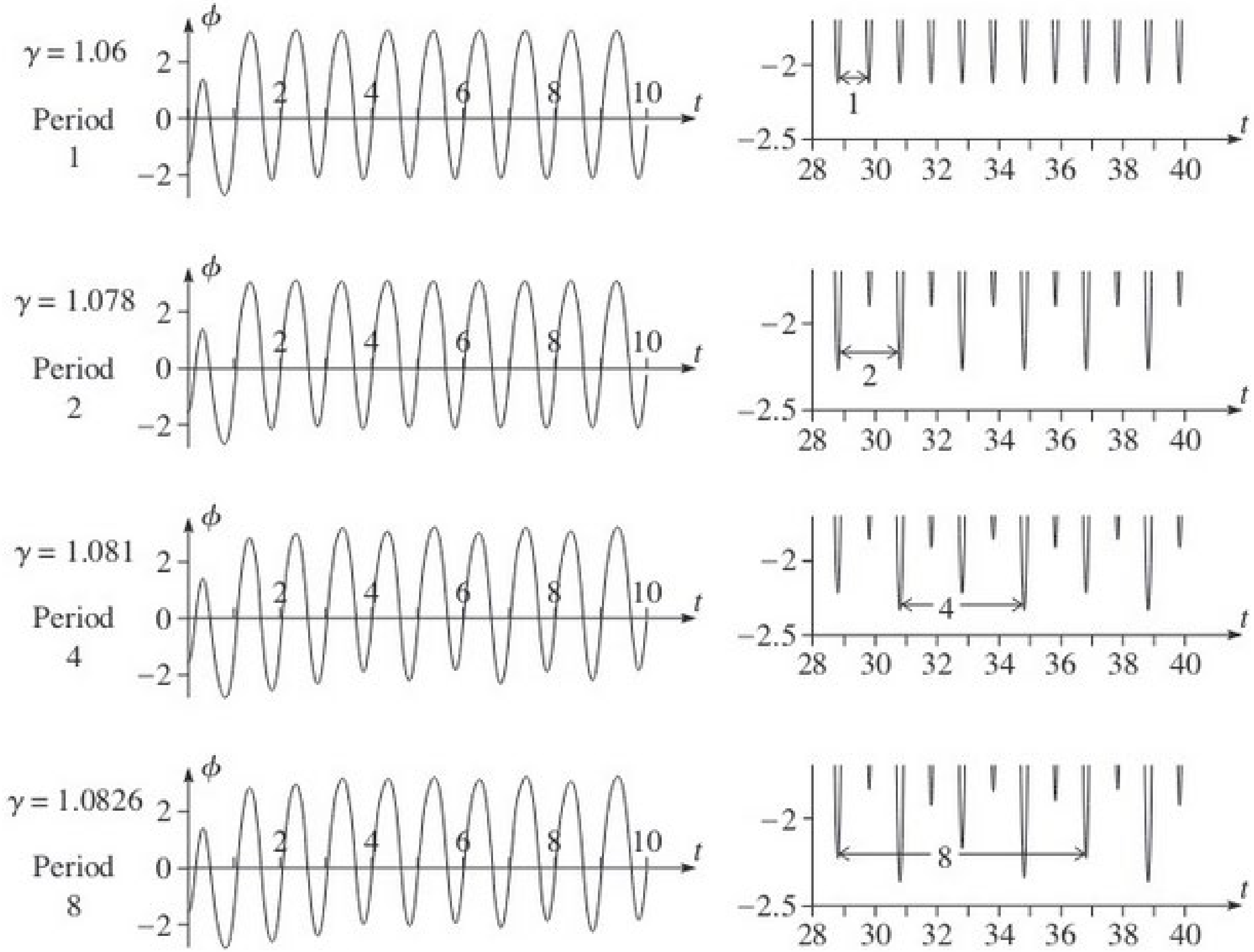


long t: mostly ω

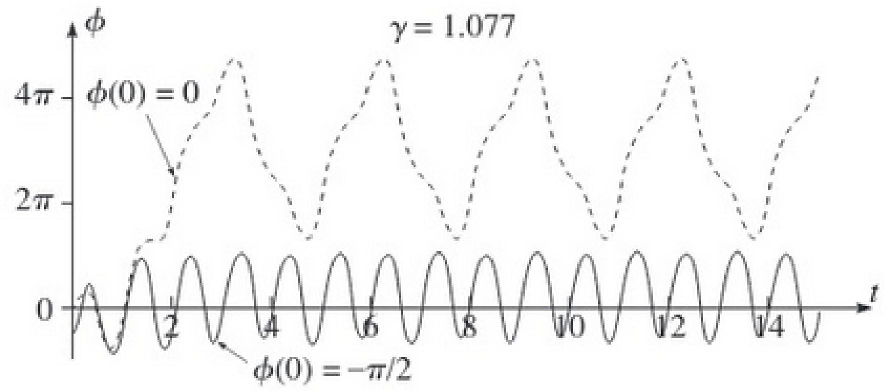


but also $\omega/2$

Increasing drive strength

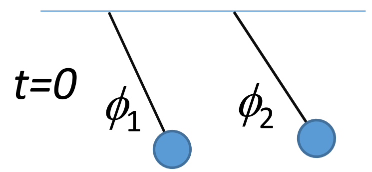


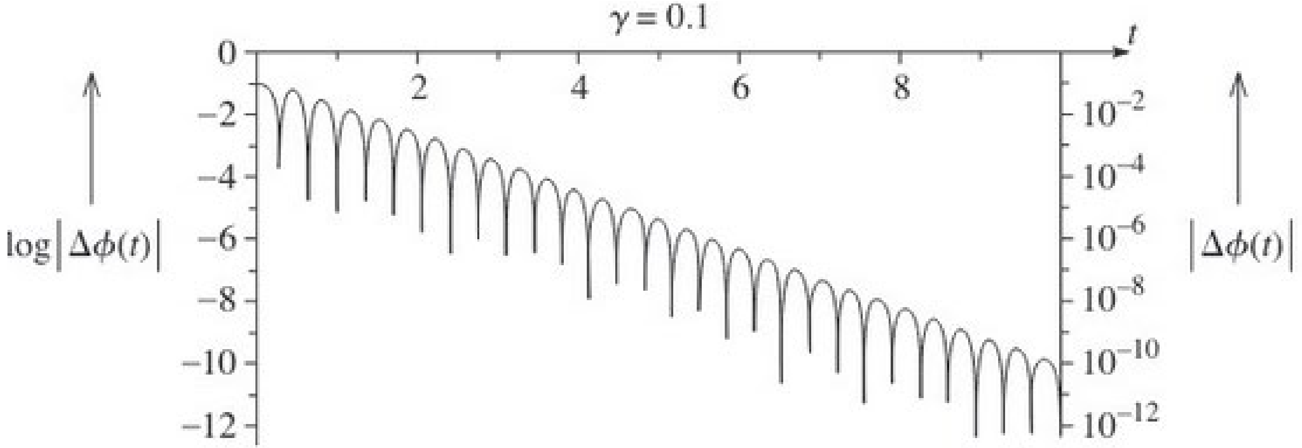
Sensitivity to initial conditions in range of large drive γ

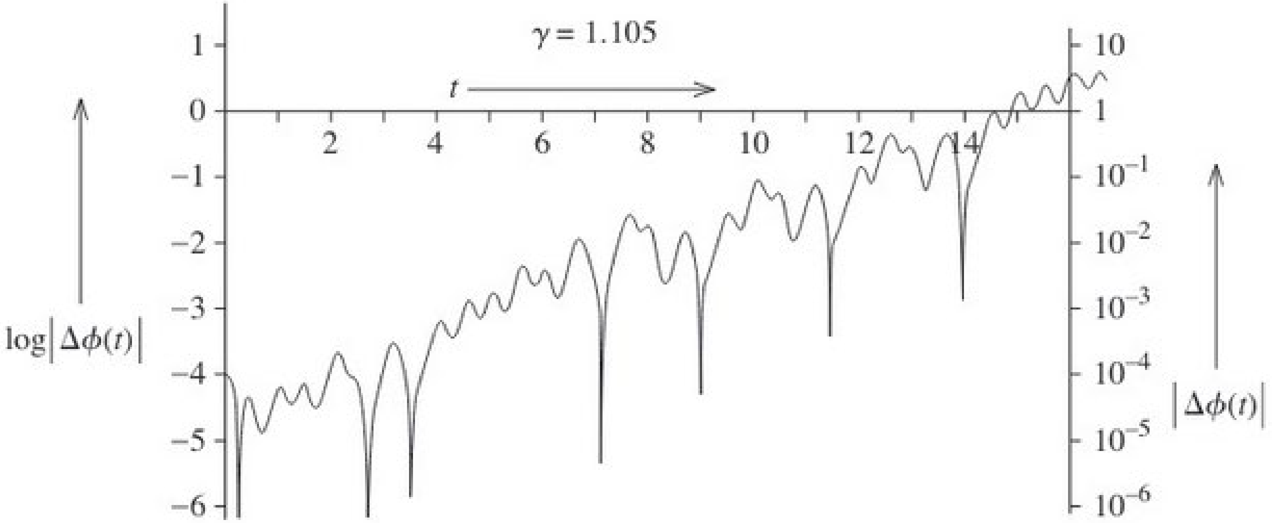


Two solutions for the same DDP, with the same drive strengths, but different initial conditions [$\phi(0) = \dot{\phi}(0) = 0$ for the dashed curve, but $\phi(0) = -\pi/2$ and $\dot{\phi}(0) = 0$ for the solid curve]. Even after the transients have died out, the two motions are totally different.

Two identical DDPs, with initial angle separated by 10^{-4} radians







"Universality" of transitions to chaos

Record values of g where period doubles: "bifurcation points"

n	period	γ_n	interval
1	$1 \to 2$	1.0663	
			0.0130
2	$2 \to 4$	1.0793	
			0.0028
3	$4 \to 8$	1.0821	
			0.0006
4	$8 \to 16$	1.0827	

$$(\gamma_{n+1} - \gamma_n) \approx \frac{1}{\delta}(\gamma_n - \gamma_{n-1})$$

$$\delta = 4.6692016$$

Feigenbaum number

Called "universal" because it's the same number that shows up in many applications of period doubling:

a) Oscillating chemical reactions:

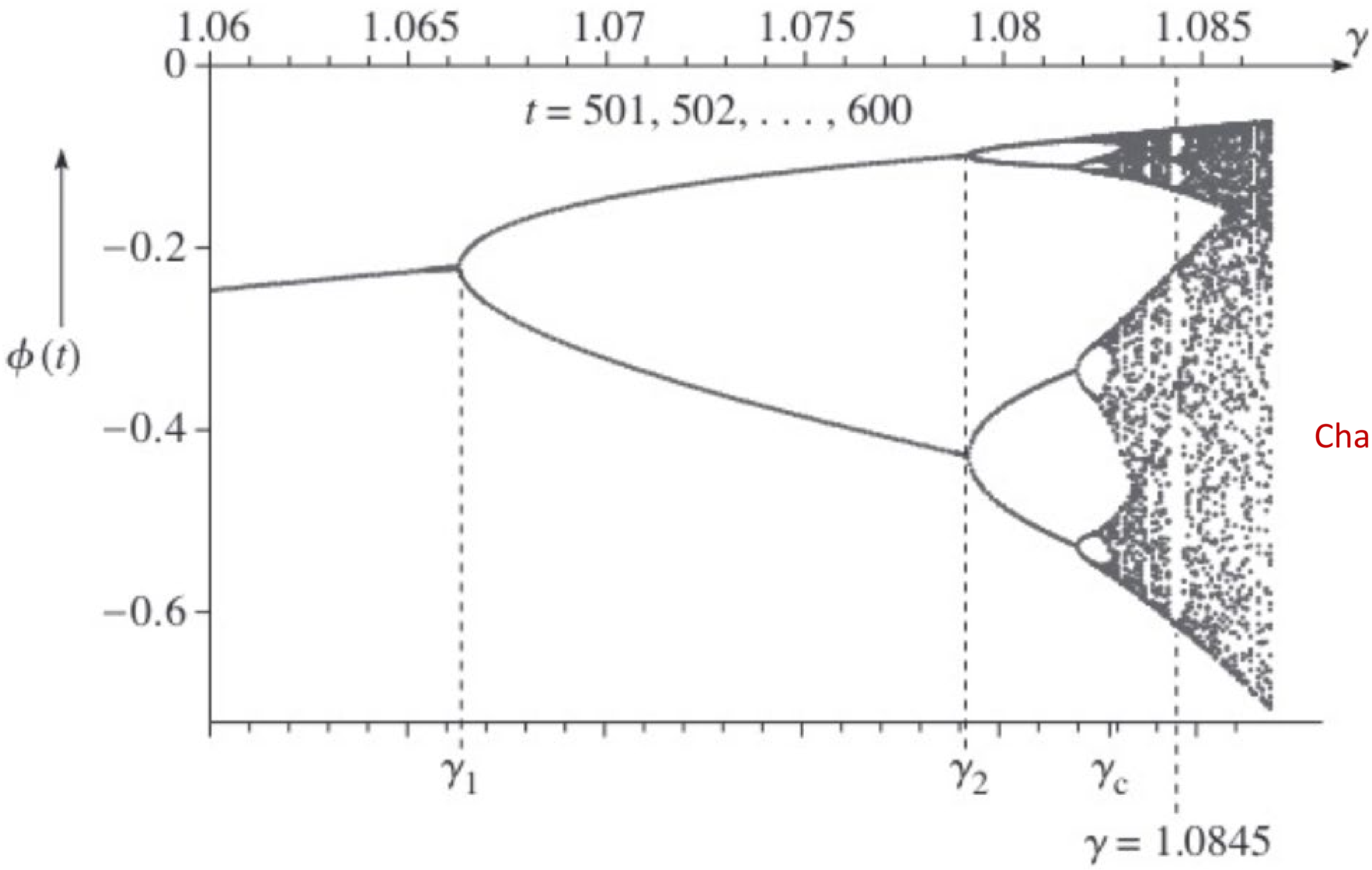
https://www.youtube.com/watch?v=PYxInARIhLY

b) Rayleigh-Bernard convection: fluid heated from below.

c) Nonlinear growth models for populations (Lokta-Volterra)

b) Logistical (Feigenbaum) map, similar nonlinear maps.

Period doubling transition to chaos: take 100 measurements of ϕ at times separated by T, 2T, ….100T for different drives γ



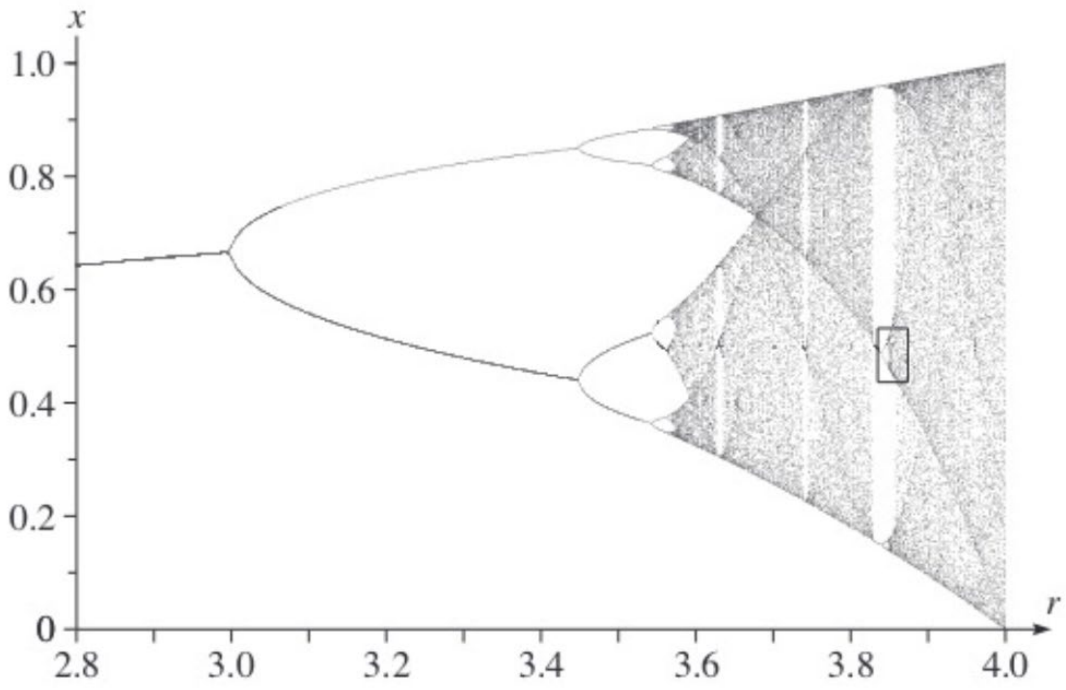
Chaos

Logistical map (discrete "time")

$$n_{t+1} = f(n_t) = r n_t (1 - n_t / N)$$

where n_t is a number that depends on integer parameter t=1,2,3,... and N is a large integer

- without the –*nt/N*, this is an exponential growth model, *f(n) = r^t n0*
- with the –*nt/N*, this is a growth model where growth is slowed when *n~N*
- Popularized by Robert May in 1975 for biology
- Period doubling and universality discovered by Mitch Feigenbaum in 1975 with HP-65 pocket calculator



Fixed points of discrete maps

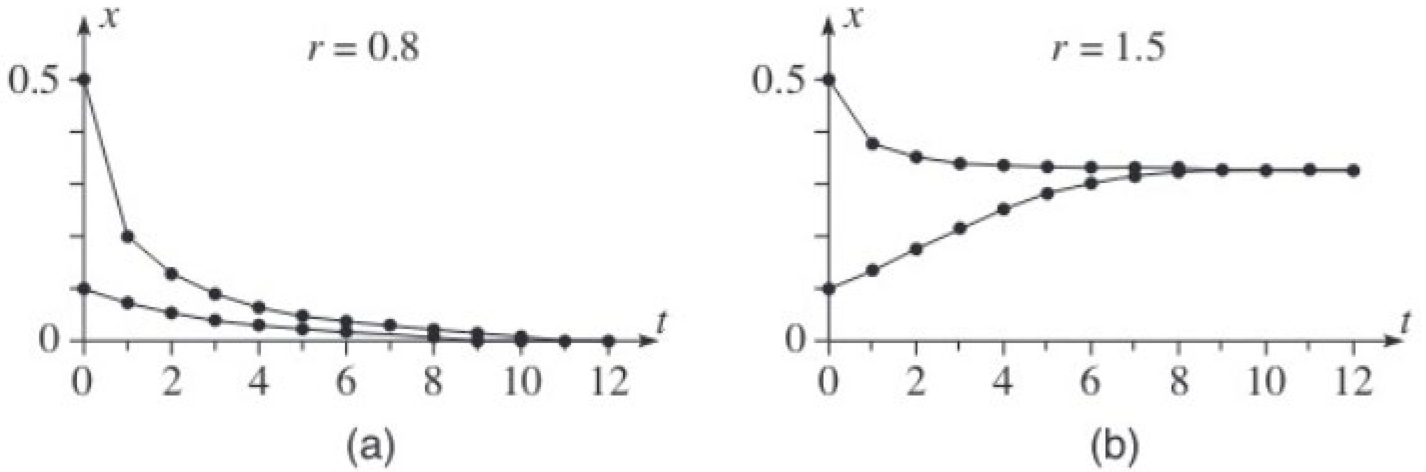
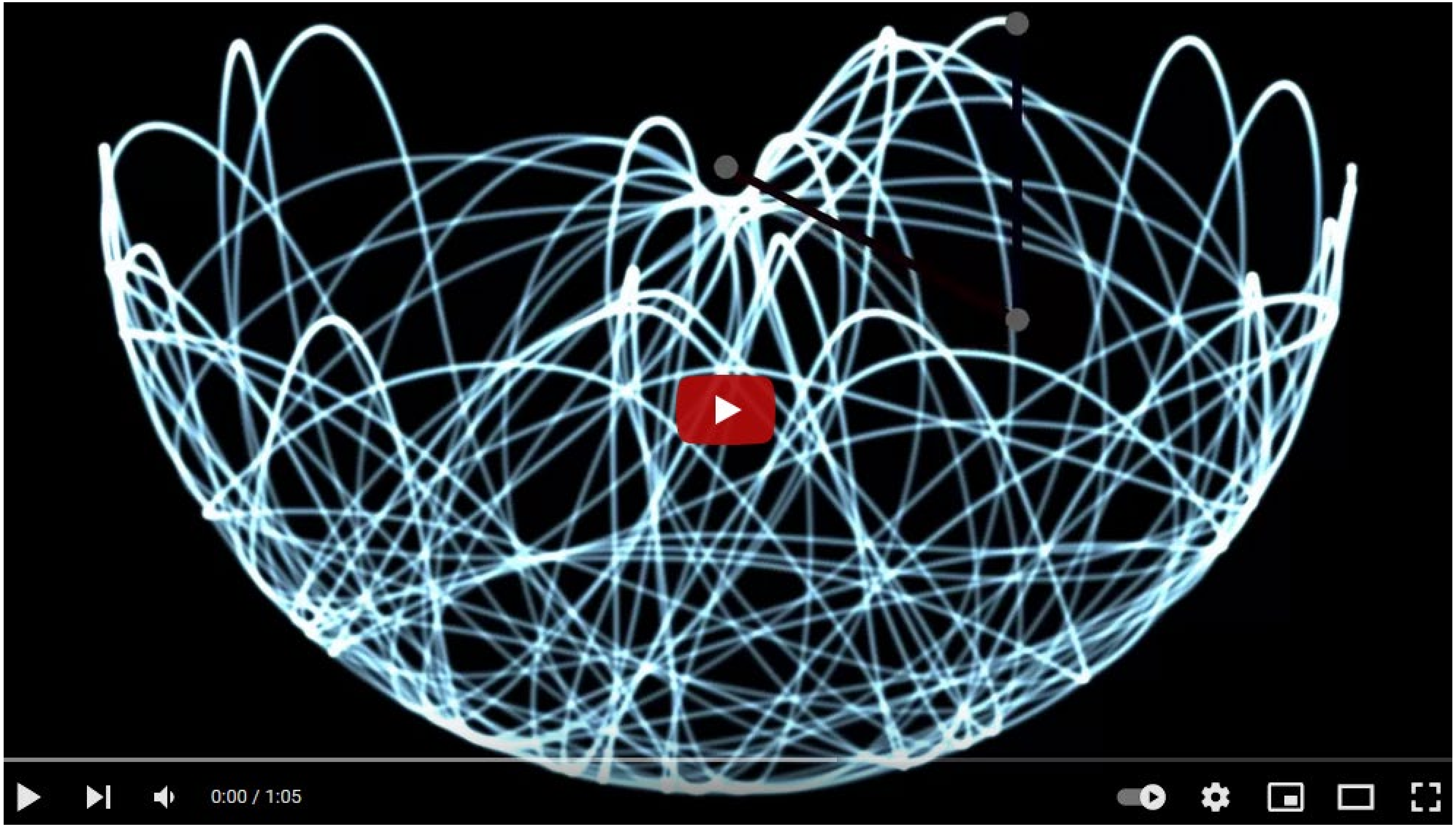


Figure 12.34 The relative population $x_t = n_t/N$ for the logisitic map (12.42), with two different initial conditions for each of two different growth rates. **(a)** With the growth parameter $r = 0.8$, the population rapidly approaches zero whether $x_0 = 0.1$ or $x_0 = 0.5$. **(b)** With $r = 1.5$ and the same two initial conditions, the population approaches the fixed value 0.33.

Note asymptotic large "time" values of x_t: "fixed points" of maps, where $x_{t+1}=x_t$.
In general, if $x_{t+1}=f(x_t)$, then fix pt. x^* defined by $f(x^*)=x^*$.
Trivial example: $x_{t+1}=x_t^2$ has fixed points 0 and 1. For logistic map, $x^*=0$ or $(r-1)/r$.

Double pendulum (no damping!)



https://www.youtube.com/watch?v=QXf95_EKS6E