9/2/22 ⓪

Announcements — HW1 due Sept. 7

Last time

$\dot{r} = \frac{p_r}{m}$, $\dot{\phi} = \frac{p_\phi}{mr^2}$

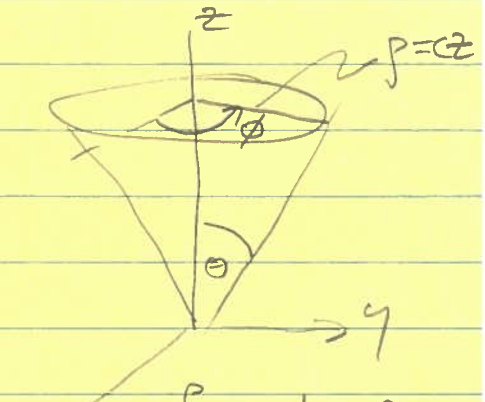
$\dot{p}_r = \frac{p_\phi^2}{2mr^3} - \nabla U$, $\dot{p}_\phi = 0$

radial $m\ddot{r} = -\nabla U + \frac{p_\phi^2}{2mr^3}$ (centrif.)

— Lagrange multipliers
— Hamilton's eqns $-\frac{\partial H}{\partial q} = \dot{p}$, $\frac{\partial H}{\partial p} = \dot{q}$
— Ex. 1 Atwood
— Ex. 2 central force

①

Ex 3. Mass on a cone in g-field



N.B. similar to problem 13.14 on HW

Use z, ϕ as gen. coords.

$\frac{\rho}{z} = \tan\theta = c$

$T = \frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\phi}^2 + \dot{z}^2\right)$

$= \frac{1}{2}m\left((1+c^2)\dot{z}^2 + c^2z^2\dot{\phi}^2\right)$

$U = mgz$ (conservative)

$H = T + U$

Note this means $T + U$ conserved.
Orbits around cone can be complicated

$p_z = \frac{\partial \mathcal{L}}{\partial \dot{z}} = m(1+c^2)\dot{z}$ $\quad p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = mc^2z^2\dot{\phi}$

$$\boxed{H = \frac{p_z^2}{2m(1+c^2)} + \frac{p_\phi^2}{2m(c^2z^2)} + mgz}$$

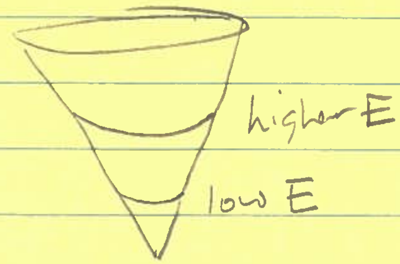
$$\dot{z} = \frac{\partial H}{\partial p_z} = \frac{p_z}{m(1+c^2)} \qquad \dot{\phi} = \frac{\partial H}{\partial p_\phi} = \frac{p_\phi}{mc^2z^2}$$

$$\dot{p}_z = -\frac{\partial H}{\partial z} = +\frac{p_\phi^2}{mc^2z^3} - mg$$

$$\dot{p}_\phi = -\frac{\partial H}{\partial \phi} = 0 \quad \text{cons a.m.}$$

Various solns circular ($\dot{z}=0$) or elliptical

$p_z = 0$ solns



fixed E

$\dot{p}_z = 0 \Rightarrow$

$$\frac{p_\phi^2}{mc^2z^3} - mg = 0 \Rightarrow \boxed{p_\phi^2 = mc^2z^3g}$$

$$\underset{\text{const.}}{E} = \frac{p_\phi^2}{2mc^2z^2} + mgz$$

$$= \frac{mgz}{2} + mgz$$

$$= \frac{3}{2}mgz$$

Should show ellipse good soln.

Note at z_{max}, z_{min} $p_z = 0$ also, so at these points

$$\underset{\text{const.}}{E} = \frac{p_\phi^2}{2mc^2z^2} + mgz \qquad (p_\phi^2 \to \text{const.})$$

See prob. 13.14

Phase space orbits

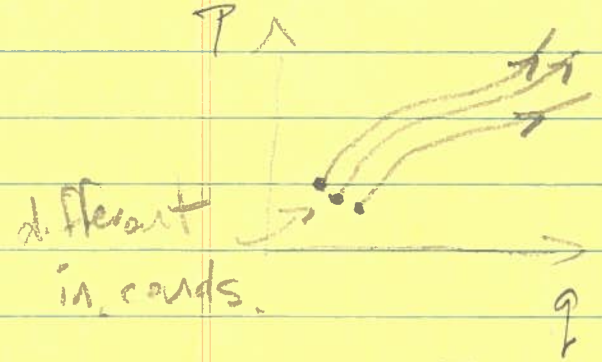
particle's classical "state" described by
(q, p) = "vector" in "phase space"

Hamilton's eqns 1st order DEQ's => unique trajectory in phase space, given initial (q_0, p_0)

N.B. in general $\vec{q} = q_1 \ldots q_N$, $\vec{p} = p_1 \ldots p_N$
many particles, spatial coords, ...

-> phase space is 2N dimensional

uniqueness => trajectories never cross
-> smooth "flow" in phase space.



Ex: SHO (Energy cons)

$$\frac{p^2}{2m} + \frac{1}{2}kx^2 = \frac{1}{2}mA^2$$

↑ ampl.