8.16 ⋆⋆ Multiplying both sides of the given equation by $(1+\epsilon\cos\phi)$ gives $r+\epsilon x = c$ (since $r\cos\phi = x$) or $r = c - \epsilon x$. Squaring both sides, setting $r^2 = x^2 + y^2$, and rearranging, we find $(1-\epsilon^2)x^2 + 2c\epsilon x + y^2 = c^2$. If we divide both sides by $(1-\epsilon^2)$ and define $d = c\epsilon/(1-\epsilon^2)$, this gives

$$(x^2 + 2dx) + \frac{y^2}{1-\epsilon^2} = \frac{c^2}{1-\epsilon^2}\,.$$

Next we can add d^2 to both sides to "complete the square" on the left, to give

$$(x+d)^2 + \frac{y^2}{1-\epsilon^2} = \frac{c^2}{1-\epsilon^2} + d^2 = \frac{c^2}{1-\epsilon^2}\left(1 + \frac{\epsilon^2}{1-\epsilon^2}\right) = \frac{c^2}{(1-\epsilon^2)^2} = a^2$$

if we define $a = c/(1-\epsilon^2)$. Finally, dividing through by a^2, we arrive at

$$\frac{(x+d)^2}{a^2} + \frac{y^2}{a^2(1-\epsilon^2)} = \frac{(x+d)^2}{a^2} + \frac{y^2}{b^2} = 1$$

where in the second expression I have introduced the definition $b = a\sqrt{1-\epsilon^2}$. Collecting our definitions of a, b, and d, we see that

$$a = \frac{c}{1-\epsilon^2}, \quad b = \frac{c}{\sqrt{1-\epsilon^2}} \quad \text{and} \quad d = a\epsilon$$

exactly as in (8.52).

8.30 ⋆⋆ If we multiply both sides of Eq.(8.49), $r = c/(1+\epsilon\cos\phi)$ by $(1+\epsilon\cos\phi)$, replace $r\cos\phi$ by x, and rearrange, we find that $r = c - \epsilon x$. Squaring both sides gives $x^2 + y^2 = c^2 - 2c\epsilon x + \epsilon^2 x^2$. We now have two cases to consider. **(a)** If $\epsilon = 1$, the terms in x^2 cancel and we're left with $y^2 = c^2 - 2cx$, a parabola. **(b)** If $\epsilon > 1$, we find $(\epsilon^2-1)x^2 - 2c\epsilon x - y^2 = -c^2$. Completing the square for x gives $(\epsilon^2-1)(x-\delta)^2 - y^2 = -c^2 + \epsilon^2 c^2/(\epsilon^2-1) = c^2/(\epsilon^2-1)$. Finally, multiplying both sides by $(\epsilon^2-1)/c^2$, we get

$$\frac{(x-\delta)^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1 \quad \text{where} \quad \alpha = \frac{c}{\epsilon^2-1},\ \beta = \frac{c}{\sqrt{\epsilon^2-1}} \text{ and } \delta = \epsilon\alpha$$

which is the equation of a hyperbola.