**PHY4324–Electromagnetism II**

**Fall 2011**

**Test 1 – 55 minutes**

**Sept. 26, 2011**

*No other materials except calculators allowed. If you can't do one part of a problem, solve subsequent parts in terms of unknown answer–define clearly.* **Do 3 of the 4 problems, CLEARLY indicating which you want graded by circling the problem number!**. *Each problem is worth 10 pts., for maximum of 30 points.*

1. Consider a superposition of two linear polarized waves traveling in the $z$ direction with electric field

$$\mathbf{E}(z,t) = \mathrm{Re}\left[\hat{x}E_{10}e^{i(kz-\omega t)} + \hat{y}E_{20}e^{i(kz-\omega t)}\right], \tag{1}$$

where $E_{10} = E_1$ is real but $E_{20} = E_2 e^{i\phi}$, where $E_1$ and $E_2$ are real and positive, and the real number $\phi$ is the *phase difference* between the two components.

(a) (2 pts.) For $\phi = 0$, the resulting wave is still linearly polarized. What is the amplitude of the total electric field, and what is the direction of the polarization vector?

(b) (4 pts.) For $\phi = \pi/2$ and $E_1 = E_2$, the resulting wave is *circularly polarized.* What is the magnitude of the electric field as a function of time $t$ at the point $z = 0$? In which direction does the electric field rotate (clock- or counterclockwise) as one views the wave coming towards the observer along the $z$-axis (i.e., you are sitting way down the positive $z$ axis looking in the $-z$ direction).

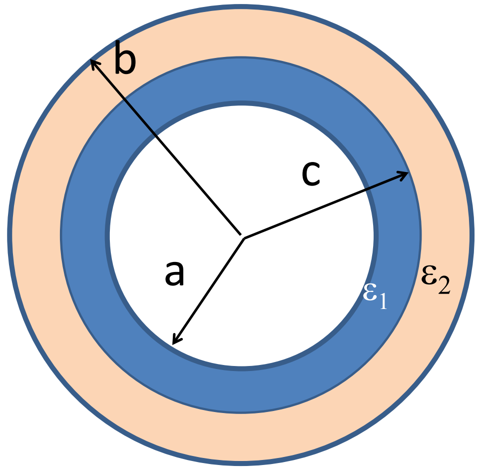
(c) (4 pts.) For $\phi = \pi/2$ and $E_1 \neq E_2$, the wave is *elliptically polarized.* Determine the electric field at $z = 0$ as a function of time.

2. A long solenoid of height $X$ is made of thin wire (diameter $d$) wrapped tightly around a cylinder of radius $a$. The density of conduction electrons in the wire is $n$, and the mean free time between collisions is $\tau_i$.

   (a) (4 pts.) In terms of the quantities given, the electron charge and mass $e$ and $m$, respectively, find the conductivity $\sigma$ and the resistance $R$ of the coil (Hint: find the total length of the wire).

(b) (4 pts.) What is the self-inductance of the coil?

(c) (2 pts.) At $t = 0$, the solenoid is connected to a battery. Find the characteristic delay time after which the current in the solenoid approaches its steady-state value.



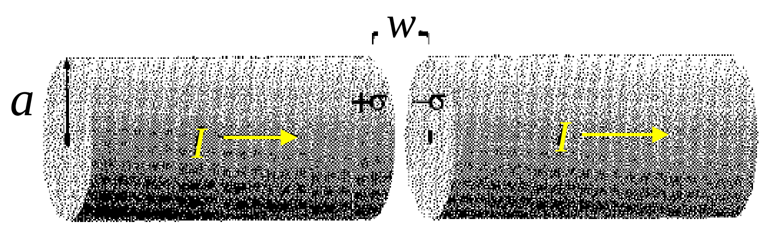

3. A $t$-dependent voltage $V(t) = V_0 \cos(\omega t)$ is applied to a capacitor, which consists of two concentric conducting spheres of radii $a$ and $b$ ($a < b$). The space in between the spheres is filled with two spherical shells made of *different* insulators, so that

$$\epsilon = \begin{cases} \epsilon_1 & \text{for } a < r < c \\ \epsilon_2 & \text{for } c < r < b. \end{cases}$$

   (a) (4 pts.) Find the capacitance $C$.

(b) (4 pts.) Find the displacement current (direction and magnitude) in terms of $C$.

(c) (2 pts.) Find the magnetic field produced by the displacement current.




4. A fat wire, radius $a$, carries a constant current $I$, uniformly distributed over its cross section. A narrow gap in the wire of width $w \ll a$, forms a parallel plate capacitor.

   (a) (4 pts.) Find $\mathbf{E}$ and $\mathbf{B}$ in the gap as functions of the distance $s$ from the axis and of the time $t$ (assume charge density $\sigma$ is zero at $t = 0$)

(b) (4 pts.) Find the energy density $u_{em}$ and the Poynting vector $\mathbf{S}$ in the gap. What are the direction and magnitude of $\mathbf{S}$?

(c) (2 pts.) Verify the conservation of energy in the gap locally. (Hint: you may need $\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{z}$.)