PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 1

August 25, 2008

Due: Wednesday, Sept. 3, 2008
Reading: Boas chapt. 1

1. Calculate the limits

   (a)
   $$\lim_{n\to\infty} \frac{n-1}{n+1} \tag{1}$$

   (b)
   $$\lim_{n\to\infty} \frac{a^{-n}(n+1)(n-1)}{3n^2} \quad , \quad a>1 \tag{2}$$

   (c)
   $$\lim_{n\to\infty} \left[\prod_{k=2}^{n} \left(1-\frac{1}{k^2}\right)\right] \tag{3}$$

   (d)
   $$\lim_{x\to\infty} \left(\sqrt{x(x+a)}-x\right) \tag{4}$$

2. Check the convergence of the following sums, and state which test you used.

   (a)
   $$\sum_{n=1}^{\infty} \frac{n^2}{2^n} \tag{5}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{\sin n}{n^2} \tag{6}$$

   (c)
   $$\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2} \tag{7}$$

(d)

$$\sum_{n=2}^{\infty}\left(\frac{1}{n}-\frac{1}{n-1}\right) \tag{8}$$

3. Calculate the first three terms of the Taylor series of the following functions around the point a given. Compare the result you get by plugging in $x = a+0.1$ with the exact result from a calculator.

   (a) $\sqrt{x}\quad (a = 36)$

   (b) $\tan(x)\quad (a = \pi/4)$

4.
   - Planck's theory of quantized oscillators led to an average energy

   $$\langle\epsilon\rangle = \frac{\sum_{n=1}^{\infty} n\epsilon_0 \exp(-n\epsilon_0/kT)}{\sum_{n=0}^{\infty} \exp(-n\epsilon_0/kT)}, \tag{9}$$

   where ϵ_0 was a constant energy. Identify the numerator and denominator as geometric series and show that the ratio is

   $$\langle\epsilon\rangle = \frac{\epsilon_0}{\exp(\epsilon_0/kT)-1}. \tag{10}$$

   (Hint: use $\sum_n n\exp(-nx) = -\frac{\partial}{\partial x}\sum_n \exp(-nx)$).
   - Show that the $\langle\epsilon\rangle$ of part a) reduces to kT, the classical thermal energy, for $kT \gg \epsilon_0$.

5. (conceptual question, no calculations). We argued in class that a hollow uniform shell of either charge or mass would have the field (electric or gravitational) zero everywhere inside. You may recall an interesting result in electrostatics: if a perfect conductor of any shape has a void inside of any shape, the electric field inside is zero. Does this result have a gravitational analog, i.e. if you have a planet with an arbitrarily shaped void inside, is g=0 inside? Why or why not?