PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 10

Oct. 8, 2008

Due: Wednesday, Oct. 15, 2008
Reading: Boas sec. 6-11

1. Calculate the electric field and scalar potential for a conducting sphere of radius R for all r when a charge Q is distributed over its surface. [Hint for those who have forgotten E&M: a conductor has $\phi = const.$ everywhere inside and on its surface.]

2. Show using Maxwell's equations that in free space, the electric field $\vec{E}$ satisfies a wave equation with wave speed equal to the speed of light, $c = 3.0 \times 10^8$ m/s. You may need

$$\mu_0 = 1.26 \times 10^{-6} \text{ Henry/m (SI)} \quad (1)$$
$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Farad/m (SI)}. \quad (2)$$

3. Supposing you apply a static magnetic field parallel to the surface of a metal, $B = B_0\hat{x}$ (the normal to the surface is $\hat{z}$). Make the choice of gauge corresponding to vector potential $\vec{A} = -B_0 z\hat{y}$, and verify that $\vec{\nabla} \times \vec{A} = \vec{B}$. Then assume that the metal in question is a superconductor, which means there is a very special relation between the current and the applied vector potential,

$$\vec{j} = K\vec{A}, \quad (3)$$

where K is a constant < 0. Verify using Maxwell's equations that the magnetic field B decays exponentially into the bulk of the metal, $B = B_0 \exp(-z/\lambda)$, and find λ.

4. Boas Problem 6.11.16, "What is wrong..."

5. The scalar potential due to a charge distribution, and the vector potential due to a current distribution, may be written as

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_\tau \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d\tau' \text{ and } \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_\tau \frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d\tau', \quad (4)$$

where the integration variable $\vec{r}\,'$ runs over some volume τ to which the charges and currents are confined (could be all space). Show that

$$\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = -\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = -\frac{\widehat{\vec{r}-\vec{r}\,'}}{|\vec{r}-\vec{r}\,'|^2}, \quad (5)$$

where

$$\widehat{\vec{r}-\vec{r}\,'} = \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}. \tag{6}$$

If the notation is confusing, ASK! Now calculate the fields $\vec{E}(\vec{r})$ and $\vec{B}(r)$ from the potentials using $\vec{E} = -\vec{\nabla}\Phi$ and $\vec{B} = \vec{\nabla} \times \vec{A}$.