PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 11

Oct. 15, 2008

Due: Friday, Oct. 17, 2008
Reading: Boas Ch. 3

1. The three Pauli matrices are

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad ; \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad ; \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{1}$$

Show they have the properties $\sigma_i^2 = 1$ for any i, and $\sigma_i \sigma_j = i\epsilon_{ijk}\sigma_k$ ($i \neq j$, k is summed, the i in front of the ϵ_{ijk} and in σ_2 is $\sqrt{-1}$).

2. Find the inverse matrices, using the formula with the cofactor matrix, and identifying the cofactor matrix along the way:

(a)

$$A = \begin{bmatrix} 8 & -\frac{2}{3} \\ -4 & \frac{1}{2} \end{bmatrix} \tag{2}$$

(b)

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{3}$$

3. (a) Find the adjoint $A^\dagger$ and transpose A^T of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1+i & 0 & 1-i \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

(b) Find a (any) 2×2 matrix which is self-adjoint $A = A^\dagger$.

4.

$$\vec{v}_1 = [3\ 0\ 4\ \ -1] \quad ; \quad A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -4 & 1 & 0 & 1 \\ 3 & 0 & -3 & 1 \\ 2 & 2 & 0 & 0 \end{bmatrix} \quad ; \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix}. \tag{5}$$

Calculate $A\vec{v}_2$ and $\vec{v}_1 \cdot A \cdot \vec{v}_2$.

5. Solve the following system of equations using Cramer's rule:

$$\begin{aligned} 3x + 3y + 3z &= 0 \qquad (6) \\ 3x - 10y + 7z &= 13 \qquad (7) \\ x + 5y + 3z &= -6 \qquad (8) \end{aligned}$$