PHZ3113–Introduction to Theoretical Physics

Fall 2008

Problem Set 13

Oct. 29, 2008

Due: Friday, Nov. 7, 2008
Reading: Boas Ch. 2

1. A plane wave of light of angular frequency ω is represented by

$$e^{i\omega(t-nx/c)}, \tag{1}$$

where t is time, x is distance, c is the speed of light, and n is the index of refraction. In a certain medium, it is found that n is a complex quantity, $n = n' + in''$, where n' and n'' are real numbers. Find the real part of the expression above. What is the qualitative effect of n'' on the wave? What does n'' correspond to physically?

2. For the following pairs of numbers z_1 and z_2, give their polar form; their complex conjugates, their moduli (magnitudes), product, and the quotient z_1/z_2:

$$z_1 = \frac{1+i}{\sqrt{2}} \;;\quad z_2 = \sqrt{3} - i \tag{2}$$

$$z_1 = \frac{3+4i}{3-4i} \;;\quad z_2 = \left[\frac{1+2i}{1-3i}\right]^2 \tag{3}$$

3. Show using de Moivre's formula that
   (a) $\cos n\theta = \cos^n\theta - \binom{n}{2}\cos^{n-2}\theta\sin^2\theta + \binom{n}{4}\cos^{n-4}\theta\sin^4\theta \ldots$
   (b) $\sin n\theta = \binom{n}{1}\cos^{n-1}\theta\sin\theta - \binom{n}{3}\cos^{n-3}\theta\sin^3\theta + \binom{n}{5}\cos^{n-5}\theta\sin^5\theta \ldots$

   where the $\binom{n}{m}$ are binomial coefficients.

4. (a) Write out the power series for $\sin z$, $\cos z$, $\sinh z$, $\cosh z$.
   (b) Assume that these functions are defined by their power series. Show that

$$i\sin z = \sinh iz \;;\quad \sin iz = i\sinh z \tag{4}$$

$$\cos z = \cosh iz \;;\quad \cos iz = \cosh z. \tag{5}$$

   (c) Verify, using the power series, that $\cosh z = (e^z + e^{-z})/2$, i.e. that the usual relationship holds in the complex plane.

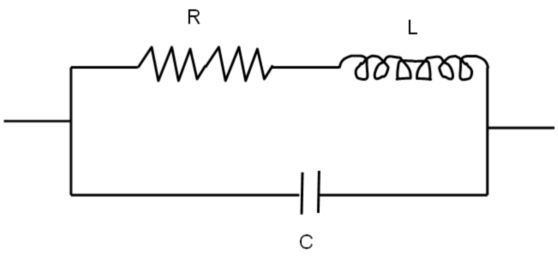

Figure 1: RLC circuit.

5. (a) Find the total effective impedance of the combination shown, to be placed in a circuit at the two end wires.
   (b) Find ω at resonance (at resonance, $\mathrm{Im}Z = 0$).